

Assessment of the Adaptive Pushover Analysis Using Displacement-based Loading in Prediction the Seismic Behaviour of the Unsymmetric-Plan Buildings

M.O. Makhmalbaf, F. Mohajeri Nav and M. Zabihi Samani

Abstract—The recent drive for use of performance-based methodologies in design and assessment of structures in seismic areas has significantly increased the demand for the development of reliable nonlinear inelastic static pushover analysis tools. As a result, the adaptive pushover methods have been developed during the last decade, which unlike their conventional pushover counterparts, feature the ability to account for the effect that higher modes of vibration and progressive stiffness degradation might have on the distribution of seismic storey forces. Even in advanced pushover methods, little attention has been paid to the Unsymmetric structures. This study evaluates the seismic demands for three dimensional Unsymmetric-Plan buildings determined by the Displacement-based Adaptive Pushover (DAP) analysis, which has been introduced by Antoniou and Pinho [2004]. The capability of DAP procedure in capturing the torsional effects due to the irregularities of the structures, is investigated by comparing its estimates to the exact results, obtained from Incremental Dynamic Analysis (IDA). Also the capability of the procedure in prediction the seismic behaviour of the structure is discussed.

Keywords—Nonlinear static procedures; Unsymmetric-Plan Buildings; Torsional effects; IDA.

I. INTRODUCTION

DEVELOPMENT of simple but accurate methods for estimating seismic response of structures is a serious challenge in performance based design. Hence, in comparison with non-practical and time consuming methods such as nonlinear time history analysis, using static nonlinear pushover (NSP) methods has been increased. In pushover analysis, a mathematical model directly incorporating the nonlinear load-deformation characteristics of individual components and elements of the building shall be subjected to monotonically increasing lateral loads representing inertia forces in an earthquake until a target displacement is exceeded.

In conventional pushover analyses, which have been

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introduced in ATC-40 [1] and FEMA 356 [2], lateral load is applied to the structure with a specific load pattern. The analysis will continue until the lateral displacement of control node reaches to a specific value which is called target displacement or the structure collapses. In these methods, only the effect of dominant mode is considered.

For more accurate results, especially for structures with reasonable higher modes effects, advanced pushover analyses have been developed. In these methods, major part of researchers' effort is devoted to finding a solution for considering higher modes effects [3].

A. Conventional Pushover Methods

As discussed above, in conventional pushover methods, the procedure continues either until a predefined limit state is reached or until structural collapse is detected. This target limit state may be the deformation expected for the design earthquake in case of designing a new structure, or the drift corresponding to structural collapse for assessment purposes. Furthermore, it is presumed that the finite element code has been sufficiently verified, so that numerical collapse, as opposed to structural, is not operative. The displacement coefficient method (DCM) of FEMA 356 and also the capacity spectrum method (CSM) of ATC-40 are the most popular ones [1], [2].

In the displacement coefficient method, top's maximum expected displacement is considered as structural performance point. The modified displacement of elastic response spectrum is used for estimating the maximum displacement of the equivalent nonlinear single degree of freedom system. The displacement demand of the method is determined from the elastic one by using a number of correction factors based on statistical analyses. According to FEMA 356, the target displacement, which is the maximum displacement occurring at the top of structures during a chosen earthquake, can be determined as

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g \quad (1)$$

where C_0 is the differences of displacements between the control node of MDOF (multi degree of freedom) buildings

and equivalent SDOF systems, C_1 is the modification factor for estimating the maximum inelastic deformation of SDOF systems from their maximum elastic deformation, C_2 is the response to possible degradation of stiffness and energy dissipation capacity for structural members during earthquakes, C_3 is the modification factor for including the P- Δ effects, T_e is the effective periods of evaluated structures, S_a is the spectral value of acceleration response corresponding to T_e , and g is the acceleration of gravity [1].

The capacity spectrum method (CSM) was first introduced by Freeman [4], [5] as a rapid evaluated procedure for assessing the seismic vulnerability of buildings. Afterwards, ATC-40 [2] investigated CSM procedure in details. This procedure compares the structural capacity in the form of a pushover curve with demands on the structure in the form of an elastic response spectrum. The graphical intersection of the two curves approximates the response of the structure [4]–[6]. In order to account for the effects of nonlinear behaviour of structures, equivalent viscous damping has been implemented to modify the elastic response spectrum. Implied in the capacity spectrum method is that the maximum inelastic deformation demand of a non-linear single-degree-of-freedom (SDOF) system can be approximately estimated by an iterative procedure of a series of linear secant representation systems. Therefore, it avoids dynamic analysis of inelastic systems [2].

After the capacity spectrum method was adopted by ATC-40, Fajfar [7] and Chopra and Goel [8], [9] pointed out that the ATC-40 procedure significantly underestimated the deformation demands of systems for a wide range of periods when used for the Type A idealised hysteretic damping model. Improved methods were proposed by them by implementing the inelastic design response spectrum as the demand diagram of the capacity spectrum method. ATC-40 introduces three types of structural behaviour which in actual practice and based on ATC-40 procedure, reduced Types B and C damping values should be used for evaluating the existing reinforced concrete structures [2], [7], [8], [9].

As a conclusion, the lateral load's nature (forces or displacements), its distribution pattern along the height of the structure (triangular, uniform, etc.) and also its magnitude, are critical parameters to define the characteristics of the conventional pushover. The number of load steps, the convergence criteria and the iterative strategy also play a significant role in the effectiveness and reliability of the analysis. Based on this, it is clear that conventional pushover analysis procedure does not account for the progressive changes in the modal properties during nonlinear yielding and cracking in the structure which leads also to period elongation and hence different spectral amplifications. This is due to the usage of the constant lateral load pattern, which ignores the potential redistribution of inertia forces and higher mode effects, as yielding and cracking governs the inelastic structural behaviour [3].

B. Advanced Pushover Methods

Advanced pushover methods which mainly are trying to

consider higher modes effects, can be categorized in two groups. In first group the load pattern is invariant whereas in second group the load pattern will be adapted in each step.

One of the first attempts to consider higher mode effects was made by Paret et al. [1996] and Sasaki et al. [1998], who suggested the simple, yet efficient, multi-modal pushover (MMP) procedure. This comprises several pushover analyses under forcing vectors representing the various modes deemed to be excited in the dynamic response [10], [11]. A refinement of the multi-modal pushover procedure is the PRC method (Pushover Results Combination), which has been proposed by Moghadam and Tso [2002]. According to this method, the maximum seismic response is again estimated by combining the results of several pushover analyses, which are carried out using load patterns that match the modal shapes of a predefined number of vibration modes. The final structural response is obtained as a weighed (using the respective modal participation factors) summation of the pushover results from each analysis. Usually, the first 3 or 4 modes are considered [1], [2].

A similar procedure, also based on MMP, is the MPA (Modal Pushover Analysis) method suggested by Chopra and Goel [2002]. According to MPA, the pushover curves corresponding to forcing vectors representing the various modes of vibration are idealised and transformed into bilinear curves of single-degree-of-freedom (SDOF) equivalent systems, so as to calculate the target deformation and the corresponding response parameters for each mode separately. The total demand is then determined by combining the peak modal demands using the SRSS rule. Typically, two or three modes are enough to achieve accurate results [13].

In all methods which have been mentioned above, the load pattern is invariant. As mentioned before, invariant load pattern ignores the progressive changes in the modal properties during the analysis. Hence, in last decade adaptive pushover methods have been developed. Firstly Bracci et al. [1997] introduced a procedure that utilises fully adaptive patterns and Lefort [2000] developed it [14], [15]. A different adaptive methodology was proposed by Gupta and Kunnath [2000], in which the applied load is constantly updated, depending on the instantaneous dynamic characteristics of the structure, and a site specific spectrum can be used to define the loading pattern [16].

Antoniou and Pinho [2004] investigated adaptive and non-adaptive pushover methods based on both force-based and displacement-based algorithms and concluded that if displacement loading is to be employed in adaptive pushover analysis, the results would be more accurate. Also in non-adaptive pushover methods, force based loading would reach to better results. Based on these conclusions, they adopted and developed the displacement-based adaptive pushover (DAP) analysis. In this method, lateral load distribution is not constant and based on modal shapes and modal participation factor resulting from modal analysis in each step of loading, load pattern would be updated. Besides, DAP is a multi-modal analysis considering structural stiffness reduction, period

elongation and modification of inertia forces [17]-[19].

Recently, some other adaptive pushover methods are introduced. Kalkan and Kunnath [2006] proposed the adaptive modal combination (AMC) procedure which integrates the inherent merits of the capacity spectrum method, and the modal pushover procedure adopting the energy based pushover curve [20].

Shakeri et al. [2010] proposed a story shear based adaptive pushover procedure, which predicts the peak inelastic drift response well especially when the higher mode effects are important [21].

In this article, firstly DAP method has been reviewed briefly and then, three dimensional Unsymmetric-Plan buildings are being analyzed using this method. Finally the results are compared with conventional pushover and incremental dynamic analysis (IDA). Since IDA is a reference method for seismic analysis of buildings, its results has been used as check points to control DAP analysis results.

II. THE DISPLACEMENT-BASED ADAPTIVE PUSHOVER (DAP) PROCEDURE

DAP method has been introduced and developed by Antoniou and Pinho [18]. The implementation of DAP method can be structured in four main stages, (i) definition of nominal load vector and inertia mass, (ii) computation of load factor, (iii) calculation of normalised scaling vector and (iv) update of loading displacement vector. Whilst the first step is carried out only once, at the start of the analysis, its three remaining counterparts are repeated at every equilibrium stage of the nonlinear static analysis procedure.

In DAP method, following the mentioned algorithm, the loading vector shape is automatically defined and updated at each analysis step. In order to prevent from the distortion of the load vector configuration determined in correspondence to the dynamic response characteristics of the structure at any analysis step, the nominal vector U_0 must always feature a uniform (rectangular) distribution shape in height. The magnitude of the loading vector U at any given analysis step is obtained by the product of its nominal counterpart U_0 , defined above, and the load factor λ at that step (2). The load factor is automatically increased, by means of a load control or response control incremental strategy, until a predefined analysis target, or numerical failure, is reached.

$$U = \lambda U_0 \quad (2)$$

In order to determine the shape of the load vector (or load increment vector) at each step, the normalised modal scaling vector, is used. This normalised modal scaling factor is computed at the start of every load increment. In order to compute, firstly the scaling displacement vector, D , should be determined. The scaling displacement vectors, which reflect the actual stiffness state of the structure, are obtained directly from the eigen vectors, as described in (3), where i is the storey number and j is the mode number, Γ_j is the modal participation factor for the j th mode, ϕ_{ij} is the normalised

mode shape value for the i th storey and the j th mode, and n stands for the total number of modes, calculated through an eigenvalue analysis. In the eigenvalue analysis, firstly modal shapes and modal participation factors and finally modal loads are calculated and then, SRSS or CQC combination rules are used to combine them.

$$D_i = \sqrt{\sum_{j=1}^N D_{ij}^2} = \sqrt{\sum_{j=1}^N (\Gamma_j \phi_{ij})^2} \quad (3)$$

The maximum displacement of a particular floor level (the relative maximum displacement between that floor and the ground), cannot be a good indication of the actual level of damage incurred by buildings subjected to earthquake loading. On the contrary, interstorey drifts, obtained as the difference between floor displacements at two consecutive levels, feature a much clearer and direct relationship to horizontal deformation demand on buildings. Hence, based on interstorey drifts, (3) could be written as:

$$D_i = \sum_{k=1}^i \Delta_k \quad (4)$$

$$\Delta_i = \sqrt{\sum_{j=1}^N \Delta_{ij}^2} = \sqrt{\sum_{j=1}^N [\Gamma_j (\phi_{i,j} - \phi_{i-1,j}) S_{d,j}]^2}$$

where Δ_{ij} is the interstorey drifts for each mode and D_i is the displacement pattern at the i th storey which is obtained through the summation of the modal-combined interstorey drifts of the storeys below that level. Equation (4) also includes an additional parameter $S_{d,j}$ that represents the displacement response spectrum ordinate corresponding to the period of vibration of the j th mode, which is called *spectral amplification*. It is claimed that using $S_{d,j}$ gives better results. In other words, the modal interstorey drifts are weighted by the S_d value at the instantaneous period of that mode, so as to take into account the effects that the frequency content of a particular input time-history or spectrum have in the response of the structure being analyzed.

Although using the relative displacement between floors in order to determine the floor displacement, leads to better results, however, (4) is approximate, because it is assumed that the relative maximum displacement between floors in all storeys occurs at the same time.

Since only the relative values of storey displacements (D_i) are of interests in the determination of the normalised modal scaling factor \bar{D}_i , which defines the shape, not the magnitude of the load or load increment vector, the displacements obtained by (4) are normalised so that the maximum displacement remains proportional to the load factor:

$$\bar{D}_i = \frac{D_i}{\max D_1} \quad (5)$$

Once the normalised scaling vector \bar{D}_i has been determined, knowing the value of the initial nominal load vector U_0 , the loading displacement vector U_t at a given analysis step t should be updated. Updating the loading vector could be done using one of two alternatives; total or incremental updating.

With total updating, the load vector U_t at a given analysis step, t is obtained through a full substitution of the existing balanced loads by a newly derived load vector, computed as the product between the current total load vector, λ_t , the current normalised modal scaling vector \bar{U}_t and the nominal load vector U_0 , as shown in (6) :

$$U_t = \lambda_t \cdot \bar{U}_t \cdot U_0 \quad (6)$$

With incremental updating, the load vector U_t at a given analysis step t is obtained by adding the load vector of previous step P_{t-1} a newly derived load vector increment, computed as the product between the current load factor increment $\Delta\lambda_t$, the current normalised modal scaling vector \bar{U}_t and the nominal load vector U_0 , as shown in (7) :

$$U_t = U_{t-1} + \Delta\lambda_t \cdot \bar{U}_t \cdot U_0 \quad (7)$$

III. ANALYSES AND MODELS DESCRIPTION

A. Models

The primary cause of torsional effects is the eccentricity between the mass center and stiffness center in the plan. This eccentricity may occur in two separate cases: (1) symmetrical structural plan with a shift of mass center in one or both horizontal directions and (2) asymmetrical structural plan. In this study, the first scenario is used in order to generate asymmetric plan buildings. These structural systems are variations of a 3 story 3D symmetric plan building which is adopted from the 3-story steel frame building designed for the SAC Steel Project [24]. The elevation view of the 3 story building is shown in Fig. 1.

Moreover, this symmetric-plan building was varied and modified to create three systems that are unsymmetric about the x-axis but symmetric about y-axis. While the stiffness properties were preserved, the center of mass (CM) was defined eccentric relative to the center of stiffness (CS), also the geometric center. The eccentricity between the CM and CS (e) was chosen to be along the y-axis, equal to 5%, 10% and 15% of the plan dimension in order to create three unsymmetric-plan buildings (Fig. 2).

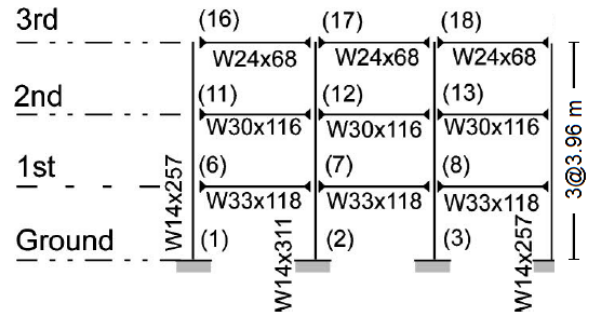


Fig. 1 The 3 story building - Elevation view

The characteristics of these models is shown in Table I. These three unsymmetric-plan systems will undergo coupled x-lateral and torsional motions due to the x-component of ground motion, which is the focus of this paper. The purely lateral response along the y-axis due to the y-component of excitation is not considered.

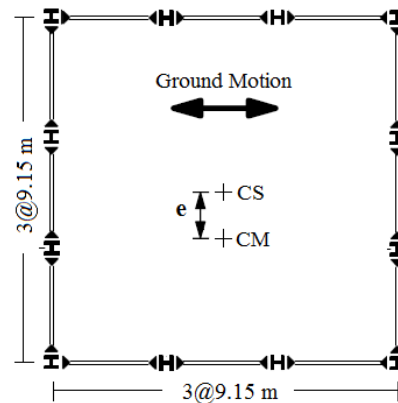


Fig. 2 The unsymmetric-plan building

B. Analyses

For every model four analyses is performed, a conventional pushover analysis with triangle (constant) load pattern, a DAP analysis and two incremental dynamic analyses (IDA). Every analysis is performed twice, once with a near fault record and then with a far fault record. The conventional analysis is performed using triangle loading. This research has been implemented using the open source finite element platform, OpenSees [22].

TABLE I
MODELS CHARACTERISTICS

| Model | e^a | Story Mass (Ton) | | |
|-------|-------|------------------|---------|----------------|
| | | Story 1 | Story 2 | Story 3 (Roof) |
| U1 | 5% | 1440 | 1560 | 4440 |
| U2 | 10% | 1440 | 1560 | 4440 |
| U3 | 15% | 1440 | 1560 | 4440 |

^a The eccentricity between the CM and CS as a percentage of the plan dimension

C. Ground Motion

Northridge ground motions (both near fault and far fault ground motions) have been used in these analyses. These ground motions are downloadable on PEER strong motion

database [23]. The ground motion's characteristics are shown in Table II.

TABLE II
 GROUND MOTIONS CHARACTERISTICS

| Earthquake | Year | M ^a | F. M. ^b | Recording Station | PGA (g) | PGV (cm/s) |
|-------------------------|------|----------------|--------------------|-------------------|---------|------------|
| Northridge (Far Fault) | 1994 | 6.7 | Thrust | Moorpark | 0.29 | 20.97 |
| Northridge (Near Fault) | 1994 | 6.7 | Thrust | Rinaldi | 0.84 | 174.79 |

^a Magnitude

^b Faulting Mechanism

IV. RESULTS

The analyses have been performed using both near fault and far fault ground motions. The eccentricity is changed in order to increase the torsional effects during the analyses, so the capability of the displacement based loading could be considered. For every model, four analyses, including IDA, DAP and conventional pushover analysis, are performed. IDA analyses are done twice using near fault and far fault ground motions. The capacity (pushover) curves resulted from these analyses are shown in Fig. (3) to Fig. (5). For every model, the IDA results, presented here, are the average results of the far fault and near fault ground motion records.

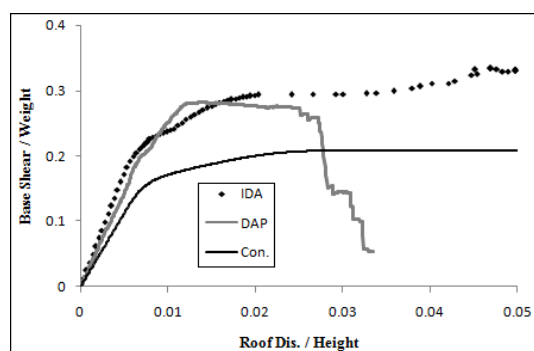


Fig. 3 Capacity curves for U1

The capacity curves for U1 are shown in Fig. (3). The eccentricity between CM and CS is considered equal to five percents of the plan dimension. It is observed that the accuracy of the DAP method seems to be satisfying up to the displacement equal to 2 percent of the total height. After that the method cannot trace the exact behaviour of the structure up to the collapse point.

Fig 4. shows the same results for U2 which has a eccentricity equal to 10 percents of the plan dimension. Similar to U1, until the roof displacement reaches to 2 percents of the total height, the results are good but after this point, the push curve descends and cannot predict the real behaviour of the structure.

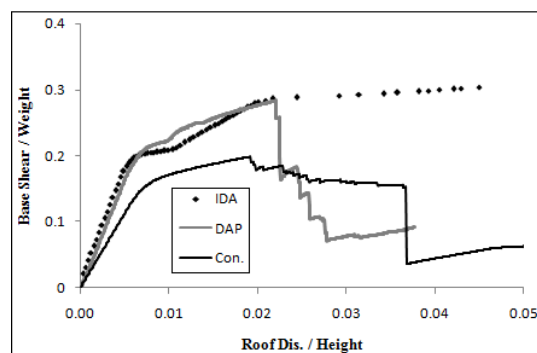


Fig. 4 Capacity curves for U2

Fig. 5 shows the pushover curves for U3, which has the biggest eccentricity in comparison to U1 and U2 (15 percents of the plan dimension). As the figure shows, DAP method cannot trace the real path of the structure up to the collapse. In contrast to U1 and U2, even in displacements which are smaller than 0.02 of the total height, the results are not acceptable. In other words, even in elastic domain, DAP procedure cannot predict the real behaviour of the structure.

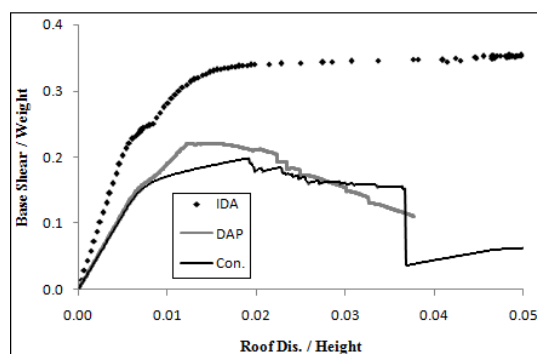


Fig. 5 Capacity curves for U3

It can be seen from the results that in structures with the eccentricity equal to or less than ten percents of the plan dimension, until the roof displacement reaches to the 0.02 of the total height of the structure, the results obtained from DAP method is acceptable.

It should be mentioned that for every model, DAP analysis is performed using the spectral amplification derived from both far fault and near fault equivalent response spectra. It is observed that changing the response spectrum based on the record, does not have a significance effect on the results of the analyses. So for every model, only one push curve is shown for DAP procedure.

Knowing the real behaviour of the structure, from the beginning of the analysis until the structure collapses, could help engineers and designers to have a comprehensive judgment about different situations which the structure is subjected during the analysis. Consequently, this could provide a better understanding of the structure's real situation during the real earthquakes. Based on this knowledge, optimal designs are reachable. Hence, push curves are useful tools in order to study the seismic behaviour of the structures.

The aim of this study was to investigate the capability of the DAP method to predict the real path of the structure until collapse happens, especially in structure with considerable torsional effects, so only the push curves has been presented here.

V.CONCLUSION

The capability of the displacement-based adaptive pushover (DAP) procedure in predicting the seismic behaviour of the structures is evaluated. A 3 story symmetric plan building is varied in order to produce three unsymmetric-plan buildings. While the stiffness properties were preserved, the center of mass (CM) was defined eccentric relative to the center of stiffness (CS), also the geometric center. The eccentricity between the CM and CS (e) was chosen to be along the y-axis, equal to 5%, 10% and 15% of the plan dimension in order to create three unsymmetric-plan buildings.

For every model, DAP and conventional pushover procedures are performed and the results are verified implementing Incremental Dynamic Analysis (IDA). The IDA analyses are performed using both far fault and near fault results. The final dynamic push curve is the average of driven results.

A summary of the main observations and general conclusions of the present study is presented below:

- In structures with a eccentricity less than 10 percents of the plan dimension, DAP method was able to predict the real responses with acceptable accuracy until the roof displacements reaches to the 0.02 of the total height.
- Even in those structures, in a situation that the roof displacement exceeds the displacement equal to 0.02 of the total height, the procedure fails to predict the exact responses.
- In a structure with a eccentricity more than 10 percents of the plan dimension, DAP method was unable to predict the exact responses of the structure. Even in elastic domain, the DAP procedure could not predict the real behaviour of the structure.
- It seems that by increment in eccentricity which produces additional torsional effects in the structure, the capability of the DAP procedure in prediction the seismic responses decreases and reaches to the point that even in elastic domain, the procedure fails to estimate the exact responses of the structure.
- The capacity (push) curve provides a meaningful tool in order to understand the seismic behaviour of the structure. A better understanding of the seismic behaviour could lead to realistic approaches in seismic design of the structures. Generally, DAP procedure seems to be unable to provide this curve accurately.

Based on these conclusions, DAP procedure seems to be

unable in realistic estimation of the seismic responses in unsymmetric-plan structures, which the torsional effects are noticeable. Although, more comprehensive investigation is needed in order to reach to more general conclusions but also development of the powerful methods in seismic analysis of the structures seems to be necessary, especially the methods which could consider the torsional effects in irregular structures.

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