# Simultaneous Determination of Reference Free-Stream Temperature and Convective Heat Transfer Coefficient

Giho Jeong, Sooin Jeong, and Kuisoon Kim

**Abstract**—It is very important to determine reference temperature when convective temperature because it should be used to calculate the temperature potential. This paper deals with the development of a new method that can determine heat transfer coefficient and reference free stream temperature simultaneously, based on transient heat transfer experiments with using two narrow band thermo-tropic liquid crystals (TLC's). The method is validated through error analysis in terms of the random uncertainties in the measured temperatures. It is shown how the uncertainties in heat transfer coefficient and free stream temperature can be reduced. The general method described in this paper is applicable to many heat transfer models with unknown free stream temperature.

*Keywords*—Heat transfer coefficient, Thermo-tropic Liquid Crystal (TLC), Free stream temperature.

# I. INTRODUCTION

T is well known that the convective heat transfer coefficient Lis a property affected only by the characteristics of flow field and should be invariant to thermal circumstances for its wide applicability. The convective heat transfer coefficient is a function only related to the flow field provided that the heating (or cooling) potential is defined well as the difference between free stream temperature and wall temperature. Unfortunately, in many convective heat transfer experiments, the free stream temperature is not only difficult to define but variant to thermal circumstances. Especially in internal flow cases such as serpentine cooling passages, pipes, ducts with small cross sectional area, heat pick-up (or heat loss) may be significant to alter the reference temperature for the stream when compared to its inlet value. One way to solve this problem is to instrument the passage in the stream-wise direction so that the reference temperatures for the stream are locally measured.

Goldstein et al.[1] has suggested to use an adiabatic wall

temperature as free stream temperature instead of jet exit temperature in impinging jet experiments. Kim et al.[2] showed that a better reference temperature for the gas stream is its corresponding '*adiabatic wall temperature*' or '*recovery wall temperature*'. But it is very difficult and cumbersome to measure the adiabatic wall temperature in many applications. To resolve this problem, many researchers have tried to obtain both free stream temperature and convective heat transfer coefficient simultaneously.

Camci [3] has obtained the adiabatic wall temperature and the film cooling heat transfer coefficient simultaneously using '*invariant h*' concept in his film cooling experiment. Yan and Owen[4] has suggested method for simultaneous determination of heat transfer coefficient and free stream temperature in transient method with two temperatures measured at different times. They also performed uncertainty analysis as well. But these methods in [3]-[4], basically based on the same concept, have large errors in the measurement of heat transfer coefficient.

In this paper the authors have described an alternative method for the simultaneous determination of local heat transfer coefficient and reference temperature in transient heat transfer experiments. The new concept introduced here is capable of producing accurate local free stream reference temperatures without performing cumbersome gas stream temperature measurements. The new method is still based on '*invariant h*' concept but this method incorporates several wall temperatures instead of two for finding heat transfer coefficient and free stream temperature through the multiple regression method.

### II. THEORETICAL DEVELOPMENT

# A. Transient Heat Transfer Experiment

The transient method proposed in [5] is a well known method for convective heat transfer measurement. The main assumption of this measurement technique is that the small penetration depth of the thermal pulse into the heat transfer model compared to the thickness of the model. This assumption allows the use of a one-dimensional heat transfer equation developed for semi-infinite bodies shown in (1). With a sudden change of the free stream temperature from  $T_i$  to  $T_{ref}$ , the local wall temperature, solution of (1), can be related to time,

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thermo-physical properties of the body and heat transfer coefficient (h) in (2)

$$\frac{\partial^2 T}{\partial n^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{1}$$

$$\Theta = (T_w - T_i)/(T_{ref} - T) = 1 - \exp(\beta^2) \cdot erfc(\beta)$$
<sup>(2)</sup>

Then the heat transfer coefficient can be calculated by measuring wall temperature at a specified time, when the reference free stream temperature, initial temperature and the thermal properties are known.

# B. Simultaneous Determination of T<sub>ref</sub> and h

The above mentioned transient method requires the measurement of free stream temperature. But, in many convective heat transfer problems, it is very difficult to define and cumbersome to measure.

This problem can be removed by using '*invariant h*' concept during transient experiments. Free stream temperature as well as heat transfer coefficient in (1) are considered as unknown values. This requires two equations for the solution of the two unknown values. The two equations for simultaneous determination of heat transfer coefficient and free stream temperature can be calculated with two wall temperatures measured at different times for a specific point [3]. Wall temperature values at different times can be measured with two different thermo-tropic liquid crystals (TLC's) [6]-[7]. But this method does not provide enough accuracy for engineering applications.

The new method proposed in this paper is still based on '*invariant h*' concept during transient experiments. But more than two wall temperatures measured at different times are incorporated into (1). Actually there are many values of wall temperature available for the measurement with two different TLC's. Then least square method provides heat transfer coefficient and free stream temperature simultaneously.

Fig. 1 shows, for instance, the time change of wall temperature at one point. In Fig. 1, solid line represents the analytic result from (1) and symbols represent the measured wall temperatures with inherent measurement errors, respectively. The measurement of wall temperature can be accomplished by using two different TLC's and five wall temperatures are measured from each TLC.

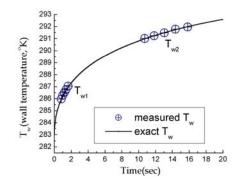


Fig. 1 The comparison of analytical temperature profile and temperature measured by TLC

TABLE I Nomenclature

NOMENCLATURE		
Symbol	Name	Definition/Units
h	Heat transfer coefficient	[W/m <sup>2</sup> K]
k	Thermal conductivity	[W/m K]
Nu	Nusselt number	Non-dimensional heat transfer coefficient
n	Normal vector	Unit vector of normal to target surface
Р	Uncertainty	Probability of measured value
$T_i$	Initial temperature	Initial temperature on target plate
$T_{ref}$	Reference temperature	Reference temperature for calculation of heating/cooling potential
$T_{rec}$	Recovery temperature	Wall temperature in steady state
Tinf	Free stream temperature	
$T_w$	Wall temperature	
erfc()	Complimentary error function	1.0-erf()
exp()	Exponential function	
α	Thermal diffusivity	Ratio of thermal conductivity to thermal capacity
β	Non-dimensional time	$h\sqrt{t/\rho C_p k}$
$\rho C_p$	Heat capacity	Density $\times$ Specific heat
$\phi^{'}$	Amplification factor	Defined as (7) and (8)
Θ	Non-dimensional temperature	Defined as Equation (2)

The least square method finds heat transfer coefficient and free stream temperature which generate a solid line most nearly passes through the ten measured points. In general the data points will not fall on the solid line. Then there is a discrepancy or error between exact wall temperatures and measured temperatures. Heat transfer coefficient and free stream temperature are determined to minimize the error in the least square sense.

The sum of square errors between measured temperature and the exact wall temperature is defined as (3). And heat transfer coefficient and reference temperature could be found by minimizing (3) as shown (4) and (5).

$$F(\varepsilon) = \sum_{P=1}^{N} [T_{w,p} - T_w]^2$$
(3)

$$\frac{\partial F(\varepsilon)}{\partial h} = 0 \tag{4}$$

$$\frac{\partial F(\varepsilon)}{\partial T_{ref}} = 0 \tag{5}$$

#### **III. VERIFICATION BY NUMERICAL EXPERIMENTS**

The simultaneous determination of heat transfer coefficient and reference temperature, derived in previous section, would be evaluated by the uncertainty analysis based on Monte-Carlo simulation. In this numerical experiment, the values of wall and initial temperature, time and any physical properties are assumed to have errors which are generated from random number generator.

#### A. Uncertainty Analysis

Any uncertainties in the measured data such as wall temperature, initial temperature, time and material properties will obviously give rise to uncertainties of the resulting free stream temperature and heat transfer coefficient. Among them, the uncertainties of wall temperature ( $P_{Tw}$ ) and initial temperature ( $P_{T\bar{t}}$ ) have dominant effects on the uncertainties of heat transfer coefficient and free stream temperature. If we assume that, for simplicity, the uncertainties in the initial and wall temperatures are same in magnitude in (6).

$$P_{T_{u}} = P_{T_{u}} = P_T \tag{6}$$

The uncertainties in heat transfer coefficient and free stream temperature can be described in terms of  $P_T$  as in (7) and (8), respectively.

$$\frac{P_h}{h} = \Phi_h \cdot \frac{P_T}{T_{ref} - T_i} \tag{7}$$

$$\frac{P_{T_{ref}}}{T_{ref} - T_i} = \Phi_T \cdot \frac{P_T}{T_{ref} - T_i}$$
(8)

The amplification factors were calculated through the following numerical experiment. Two narrow band TLC's and timer were used to measure the wall temperatures,  $T_{w1}$  and  $T_{w2}$ , and the corresponding times,  $t_1$  and  $t_2$ .

From (4) and (5), the heat transfer coefficient and reference temperature then could be calculated from wall temperatures and the initial temperature,  $T_i$ . The number of samples for calculation of *h* and  $T_{ref}$  was 1,000.

#### B. Comparison of Amplification Factor

For verification of current method, the amplification factors were compared with those presented in [4] as shown in Fig. 2. In their work, the temperature was measured only once at each point, and which means that two temperature values were used to describe the time change of wall temperature. The current results agree very well with those in [4]. This implies that the current numerical experiment based on Monte Carlo method is also reliable.

From Fig. 2, it is found that there exists minimum value of amplification factors as  $\Theta_1$  changes. It also could be found that the slope of amplification factor reduces as  $\Theta_2$  increases. But, it should be noted that large value of  $\Theta_2$  means time gap between the first and second measurements, and the assumption of one dimensional heat transfer cannot be persisted any more.

In Fig. 3, the effect of the number of temperature values on the amplification factors is shown. As the number of temperature values increases, the amplification factors decreases. However, the decreasing slope is this effect becomes small more than three values from multiple temperature values

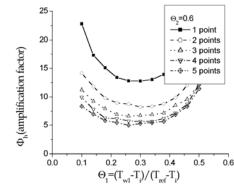


Fig. 3(a) The effect of number of data on amplification factor for h

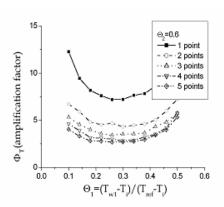


Fig. 3(b) The effect of number of data on amplification factor for T

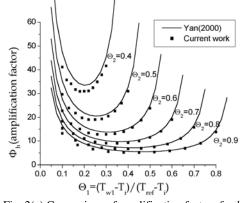


Fig. 2(a) Comparison of amplification factors for *h* for various  $\Theta_1$  and  $\Theta_2$ 

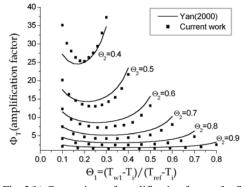


Fig. 2(b) Comparison of amplification factors for *T* for various  $\Theta_1$  and  $\Theta_2$ 

were compared with those from two temperature values. It is shown that the amplification factors calculated with multiple temperature values decreases significantly as the number of temperature values increases.

In Fig. 4, the amplification factors of heat transfer coefficient and free stream temperature calculated from multiple temperature values were compared with those from two temperature values. It is shown that the amplification factors calculated with multiple temperature values decreases significantly as the number of temperature values increases.

# IV. APPLICATION TO TRANSIENT EXPERIMENTS

The transient heat transfer experiments with circular impinging jet were performed to evaluate the feasibility of the simultaneous determination method presented in the previous section. The radial variations of non-dimensional heat transfer coefficient, Nusselt number, for three cases in which jet exit, recovery wall temperature and reference temperature were used to calculate the heating/cooling potential.

### A. Test Apparatus

In this study, the open type wind tunnel with (or including) an electric heater was designed for maximum jet exit velocity and temperature being 10m/s and 90, respectively. The fully developed turbulent flow through the 70mm diameter and 800mm long pipe impinges the target plate. The target plate made of acryl is square shape of 530mm by 530mm with 20mm thickness. We have prepared two target plates, one for conventional transient test in which jet exit temperature is used as reference temperature and the other for steady test for measurement of recovery wall temperature. Two target plates were coated with thermo-tropic liquid crystal and black paint in sequence. And eight thermocouples were set on the target plate surface to measure temperature values, prior to the coating of TLC and black paint. Especially for target plate for steady test, polystyrene foam plate was put on the opposite side of jet impingement to insulate.

The coated thermo-tropic liquid crystal was the mixture of two different commercial liquid crystals (Hallcrest, R35C1W and R46C1W). The schematic view of experiment is shown in Fig. 5.

# *B.* Evaluation of 'invariant h' with Jet Exit Temperature and Recovery Wall Temperature

During the transient test, it is possible to measure two values of temperature at one point because the mixture of R35C1W and R46CW was sprayed on the target plate. Then those two temperature values and jet exit temperature as a reference temperature were used to calculate the respective Nusselt number distribution shown in Fig. 6. The Reynolds number and the ratio of distance from nozzle exit to target plate to exit diameter, H/D, were 50000 and 6, respectively.

In Fig. 6, it is found that the local Nusselt number decreases

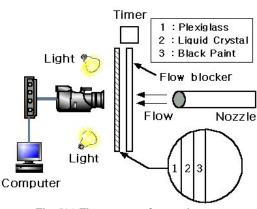


Fig. 5(a) The test setup for transient test

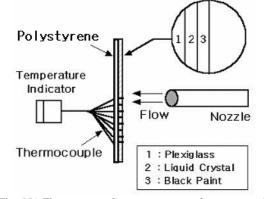


Fig. 5(b) The test setup for measurement of recovery wall temperature in steady state

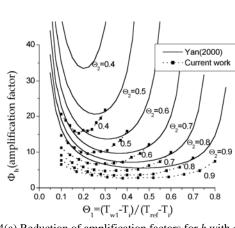


Fig. 4(a) Reduction of amplification factors for *h* with current method in which multiple points were used

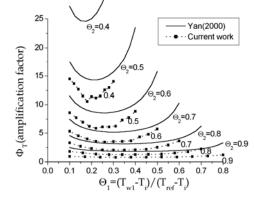


Fig. 4(b) Reduction of amplification factors for T with current method in which multiple points were used

as its position becomes away from the origin of impingement region, though there are no differences of flow condition between these two cases except wall temperature. In other word, the changes of heat transfer coefficient could be affected by the heat boundary condition when we use jet exit temperature as a reference temperature. Surely, this violates the principle of 'invariant h' that the heat transfer coefficient is function of flow conditions not heat conditions.

In contrast to that from jet exit temperature, the Nusselt number distribution calculated from recovery wall temperature converges to a value beyond the stagnation region. In addition, the Nusselt number within stagnation region is higher than those from jet exit temperature.

# C. Free Stream Temperature by Multiple Regression Method

In Fig. 7, the recovery wall temperature measured with a steady experiment and free stream temperature calculated from currently suggested method are compared with each other at specified position on target plate. It could be found that values of calculated free stream temperature coincide well with those of the recovery temperature measured by steady tests.

The 'invariant h' could be evaluated by the free stream temperature from currently suggested method shown in Fig. 8. In these figures, the effect of the definition of reference temperature on the time variations of local Nusselt number at each radial position. Three temperature values such as jet exit, recovery and free stream temperature were used to calculate the local Nusselt number shown in Fig. 8. As expected, the Nusselt number calculated from the recovery wall temperature does not vary with time regardless of position and those from jet exit temperature vary with time at each position. Though the simultaneous determination of the free stream temperature and heat transfer coefficients is much easier than the steady state for measuring recovery wall temperature, the accuracy of the resultant Nusselt number is comparable to that of steady tests.

The comparison of Nusselt number distributions from three methods mentioned in this paper, those are jet exit temperature, recovery wall temperature and reference temperature from multiple regression method, was finally shown in Fig. 9. It could be found that the Nusselt number distribution from multiple regression method shows the characteristics of 'invariant h' and coincides well with those from recovery wall temperature.

# V. CONCLUSION

In this paper, the simultaneous determination of heat transfer coefficient and free stream temperature was introduced and validated. Though the principle of 'invariant h' should be evaluated during heat transfer experiments, it is often ignored by definition of reference temperature as jet exit temperature for the simplicity. But as mentioned in this paper it does not guarantee the accuracy of heat transfer experiment at all.

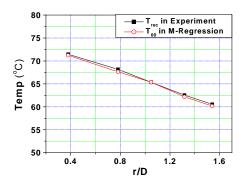


Fig. 7 Comparison of reference temperature by multiple regression method with recovery wall temperature

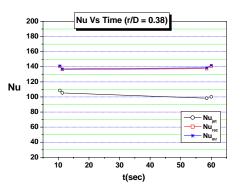


Fig. 8(a) The time variation of Nusselt number at r/D=0.38 for three methods

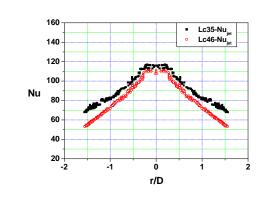


Fig. 6(a) Nusselt number distribution calculated from jet exit temperature as a reference temperature

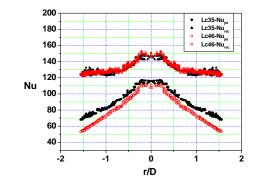


Fig. 6(b) Nusselt number distribution calculated from recovery wall temperature as a reference temperature

The authors are expecting that this study would help the researchers to define the reference temperature and understand the heat transfer characteristics of phenomena they concerned. The simultaneous determination of heat transfer coefficient and free stream temperature suggested in this paper can gives them simplicity and accuracy of their forced convective studies.

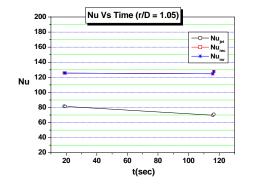


Fig. 8(b) The time variation of Nusselt number at r/D=1.05 for three methods

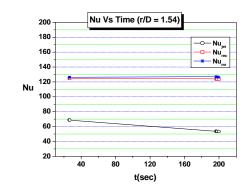


Fig. 8(c) The time variation of Nusselt number at r/D=1.64 for three methods

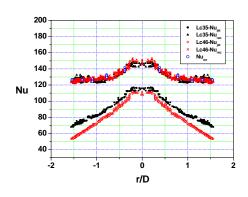


Fig. 9 The Nusselt number distribution for three method (Jet exit temperature, recovery wall temperature and reference temperature)

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