

Blind Identification of MA Models using Cumulants

Mohamed Boulouird and Moha M'Rabet Hassani

Abstract—In this paper, many techniques for blind identification of moving average (MA) process are presented. These methods utilize third- and fourth-order cumulants of the noisy observations of the system output. The system is driven by an independent and identically distributed (i.i.d) non-Gaussian sequence that is not observed. Two nonlinear optimization algorithms, namely the *Gradient Descent* and the *Gauss-Newton* algorithms are exposed. An algorithm based on the joint-diagonalization of the fourth-order cumulant matrices (FOSI) is also considered, as well as an improved version of the classical $C(q, 0, k)$ algorithm based on the choice of the *Best 1-D Slice* of fourth-order cumulants. To illustrate the effectiveness of our methods, various simulation examples are presented.

Keywords—Cumulants, Identification, MA models, Parameter estimation.

I. INTRODUCTION

IN this paper, we consider the following discrete time, causal, non-minimum phase, linear, time-invariant process represented on figure 1 and described by equations (1) and (2) :

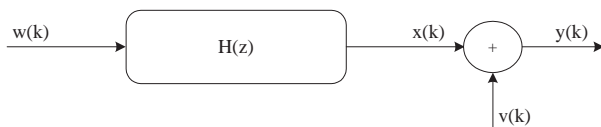


Fig. 1. Single channel system

$$x(k) = \sum_{i=0}^q h(i) w(k-i); \quad \{h(0) = 1\} \quad (1)$$

where the observations of the signal $x(k)$ are, in general, noisy :

$$y(k) = x(k) + v(k) \quad (2)$$

In the above equations, the driving sequence $w(k)$ is not observed. The following conditions are assumed to hold.

H.1. The system input $w(k)$ is zero mean, independent and identically distributed (i.i.d), non-Gaussian process, with unknown distribution, and :

$$C_{m,w}(\tau_1, \tau_2, \dots, \tau_{m-1}) = \gamma_{m,w} \delta(\tau_1, \tau_2, \dots, \tau_{m-1})$$

where :

$\diamond C_{m,w}(\tau_1, \tau_2, \dots, \tau_{m-1})$ is the m th-order cumulant of the input signal $w(k)$.

$\diamond \gamma_{m,w} = C_{m,w}(0, 0, \dots, 0) \neq 0, \quad \forall m \geq 2$

$\diamond \gamma_{2,w} = \sigma_w^2 = E \{w(k)^2\}$

$\diamond \gamma_{3,w} = E \{w(k)^3\}$ is the skewness of $w(k)$.

$\diamond \gamma_{4,w} = E \{w(k)^4\} - 3 [E \{w(k)^2\}]^2$ is the kurtosis of $w(k)$.

H.2. The measurement noise sequence $v(k)$ is assumed to be zero mean, i.i.d, Gaussian sequence with unknown variance, and independent of $w(k)$.

H.3. The order q of the model is known.

The focus of this paper is the problem of recovering the coefficients $\{h(i)\}_{i=1, \dots, q}$ of the MA model from the cumulant statistics [15]-[16] of a sample function of the observations $\{y(k), 1 \leq k \leq N\}$ over N consecutive instants.

Blind identification of MA models using Higher-Order Statistics, especially third- and fourth-order cumulants, has a wide applicability in many fields; e.g., sonar, radar, seismic data processing, adaptive filtering, blind equalization, array processing, data communication, time daily estimation, image and speech processing [12]-[14]. These statistics are very useful in problems where either non-Gaussianity, non-minimum phase assumptions, and additive Gaussian noise are present [12].

Signal processing techniques using Higher-Order Statistics (HOS) or cumulants have attracted considerable attention in the literature [2]-[9]-[10]-[13]-[17]-[18]-[19]-[20]. There are several motivations behind this interest [1]-[7]. First, higher-order cumulants are blind to all kinds of Gaussian noise; that is, HOS for a Gaussian process are identically zero. Hence, when the processed signal is non-Gaussian and the additive noise is Gaussian, the noise will vanish in the cumulants domain. Thus, a greater degree of noise immunity is possible. Second, cumulants are useful in identifying non-minimum phase systems and in reconstructing non-minimum phase signals when the signals are non-Gaussian. That is because cumulants preserve the phase information of the signal. Third, cumulants are useful in detecting and characterizing the properties of nonlinear systems.

In this paper, we compare blind identification methods using the nonlinear optimization algorithms proposed in [4]-[5], with the well known *Fourth-Order System Identification* algorithm proposed in [3]. The first approach has the advantage of estimating a non redundant parameters vector, while the second one exploits all the fourth-order cumulants through a Joint-diagonalization procedure. A third approach consists in

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selecting the best 1-D slice of fourth-order cumulants in order to improve estimation quality using the classical $C(q, 0, k)$ algorithm [6]. These algorithms are used to identify some communication channels and also solar processes.

This paper is organized as follows : A fundamental relationship linking cumulants of the output signal of the model and the coefficients is dealt in Section II. In Section III, we expose the solutions using Gradient Descent and Gauss-Newton algorithms. FOSI and modified $C(q, 0, k)$ algorithms are introduced in Sections IV and V, respectively. In Section VI, simulation results are discussed. Finally, conclusions are drawn in Section VII.

II. A FUNDAMENTAL RELATIONSHIP

The starting point for all linear algebraic methods of MA parameter estimation is the Brillinger and Rosenblatt [8] formula. It is repeated here for convenience :

$$C_{m,x}(\tau_1, \tau_2, \dots, \tau_{m-1}) = \gamma_{m,w} \times \sum_{k=k_{min}}^{k_{max}} h(k) h(k + \tau_1) \dots h(k + \tau_{m-1}) \quad (3)$$

$$\text{where } \begin{cases} k_{min} = \max(0, -\tau_1, \dots, -\tau_{m-1}) \\ k_{max} = \min(q, q - \tau_1, \dots, q - \tau_{m-1}) \end{cases}$$

and $C_{m,x}(\cdot)$ is the m th-order cumulant of the output process $x(k)$. Equation (3) gives the expression of cumulants of an MA model with respect to the parameters of the process, but it does not show what is the direct relation between two different order cumulants of the same MA process or between different cumulant slices of the same order. Equation (3) can be used to obtain such formulas that can then be used for MA parameter estimation.

For the MA system depicted in figure 1 and described by equations (1) and (2) with the assumptions **H.1**, **H.2**, and **H.3**; the m th- and n th-order cumulants of the system output $y(k)$ are linked by the following relation :

$$\begin{aligned} & \sum_{i=i_{min}}^{i_{max}} h(i) \left[\prod_{k=1}^{m-s-1} h(i + \tau_k) \right] \times \\ & C_{n,y}(\beta_1, \beta_2, \dots, \beta_{n-s-1}, i + \alpha_1, i + \alpha_2, \dots, i + \alpha_s) = \\ & \epsilon_{n,m} \sum_{j=j_{min}}^{j_{max}} h(j) \left[\prod_{k=1}^{n-s-1} h(j + \beta_k) \right] \times \\ & C_{m,y}(\tau_1, \tau_2, \dots, \tau_{m-s-1}, j + \alpha_1, j + \alpha_2, \dots, j + \alpha_s) \quad (4) \end{aligned}$$

where $m > 2$, $n > 2$, $\epsilon_{n,m} = \frac{\gamma_{n,w}}{\gamma_{m,w}}$, and s is an arbitrary integer satisfying $1 \leq s \leq \min(m, n) - 2$,

$$\text{and } \begin{cases} i_{min} = \max(0, -\tau_1, \dots, -\tau_{m-s-1}) \\ i_{max} = \min(q, q - \tau_1, \dots, q - \tau_{m-s-1}) \\ j_{min} = \max(0, -\beta_1, \dots, -\beta_{n-s-1}) \\ j_{max} = \min(q, q - \beta_1, \dots, q - \beta_{n-s-1}) \end{cases}$$

Proof :

Let :

$$R_{mn} = \sum_{ij} h(i)h(j) \left[\prod_{k=1}^{m-s-1} h(i + \tau_k) \right] \times \left[\prod_{k=1}^{n-s-1} h(j + \beta_k) \right] \left[\prod_{k=1}^s h(i + j + \alpha_k) \right] \quad (5)$$

Firstly, if we sum on j afterwards on i in (5), we will find :

$$R_{mn} = \sum_i h(i) \left[\prod_{k=1}^{m-s-1} h(i + \tau_k) \right] \times \sum_j h(j) \left[\prod_{k=1}^{n-s-1} h(j + \beta_k) \right] \left[\prod_{k=1}^s h(i + j + \alpha_k) \right] \quad (6)$$

If we multiply (6) by $\gamma_{n,w}$ and take the Brillinger and Rosenblatt formula (3) into account, we will obtain :

$$\begin{aligned} \gamma_{n,w} R_{mn} &= \sum_i h(i) \left[\prod_{k=1}^{m-s-1} h(i + \tau_k) \right] \times \\ & C_{n,x}(\beta_1, \beta_2, \dots, \beta_{n-s-1}, i + \alpha_1, i + \alpha_2, \dots, i + \alpha_s) \quad (7) \end{aligned}$$

Changing the order of summation in (5) yields :

$$R_{mn} = \sum_j h(j) \left[\prod_{k=1}^{n-s-1} h(j + \beta_k) \right] \times \sum_i h(i) \left[\prod_{k=1}^{m-s-1} h(i + \tau_k) \right] \left[\prod_{k=1}^s h(i + j + \alpha_k) \right] \quad (8)$$

If we multiply the right and left sides of (8) by $\gamma_{m,w}$ and take the relation (3) into account, we will obtain :

$$\begin{aligned} \gamma_{m,w} R_{mn} &= \sum_j h(j) \left[\prod_{k=1}^{n-s-1} h(j + \beta_k) \right] \times \\ & C_{m,x}(\tau_1, \tau_2, \dots, \tau_{m-s-1}, j + \alpha_1, j + \alpha_2, \dots, j + \alpha_s) \quad (9) \end{aligned}$$

From (7) and (9), we obtain the relation (4) between the m th- and n th-order cumulants of the MA process output.

III. METHODS BASED ON NONLINEAR OPTIMIZATION ALGORITHMS

Setting $n = 3$, $m = 4$, and $s = 1$ in equation (4), yields

$$\begin{aligned} & \sum_{i=i_{min}}^{i_{max}} h(i)h(i + \tau_1)h(i + \tau_2)C_{3,y}(\beta_1, i + \alpha_1) = \\ & \epsilon_{3,4} \sum_{j=j_{min}}^{j_{max}} h(j)h(j + \beta_1)C_{4,y}(\tau_1, \tau_2, j + \alpha_1) \quad (10) \end{aligned}$$

$$\text{where } \begin{cases} i_{min} = \max(0, -\tau_1, -\tau_2) \\ i_{max} = \min(q, q - \tau_1, q - \tau_2) \\ j_{min} = \max(0, -\beta_1) \\ j_{max} = \min(q, q - \beta_1) \end{cases}$$

By setting $\tau_1 = \tau_2 = 0$ in (10), we get the relation used in this paper for estimating the parameters $\{h(i)\}_{i=1,2,\dots,q}$ of the considered MA model.

$$\epsilon_{4,3} \sum_{i=0}^q h^3(i) C_{3,y}(\beta_1, i + \alpha_1) = \sum_{j=j_{min}}^{j_{max}} h(j)h(j + \beta_1) C_{4,y}(0, 0, j + \alpha_1) \quad (11)$$

It is important to determine the range of values of α_1 and β_1 so that the cumulants $\{C_{3,y}(\beta_1, i + \alpha_1)\}_{i=0,\dots,q}$, $\{C_{4,y}(0, 0, j + \alpha_1)\}_{j=j_{min},\dots,j_{max}}$, and the coefficients $\{h(j + \beta_1)\}$ are not all zero for each equation.

By taking into account the property of causality of the model and the domain in which third- and fourth-order cumulants of an MA(q) process are non-zero [12], we obtain :

$$\begin{cases} -q \leq \beta_1 \leq q \\ -2q \leq \alpha_1 \leq q \\ -2q + \beta_1 \leq \alpha_1 \leq q + \beta_1 \end{cases} \quad (12)$$

Using the symmetry properties of cumulants ([7]-[14]) the set of values for α_1 and β_1 is defined by :

$$\begin{cases} -q \leq \beta_1 \leq 0 \\ -2q \leq \alpha_1 \leq q + \beta_1 \end{cases} \quad (13)$$

Concatenating (11) for all values of α_1 and β_1 defined by (13), we obtain the following system of equations :

$$M\theta = r \quad (14)$$

where :

$$\theta = [h(1) \dots h(q) \quad h^2(1) \quad h(1)h(2) \quad \dots \quad h(1)h(q) \quad h^2(2) \quad \dots \quad h(2)h(q) \quad h^2(3) \quad \dots \quad h^2(q) \quad \epsilon_{4,3} \quad \epsilon_{4,3}h^3(1) \quad \dots \quad \epsilon_{4,3}h^3(q)]^T \quad (15)$$

$$\diamond \epsilon_{4,3} = \gamma_{4,w} / \gamma_{3,w}$$

$$\diamond M \text{ is a matrix of dimension } \left[\frac{5q^2+7q+2}{2}, \frac{q^2+5q+2}{2} \right]$$

$$\diamond \theta \text{ is a vector of dimension } \left[\frac{q^2+5q+2}{2}, 1 \right]$$

$$\diamond r \text{ is a vector of dimension } \left[\frac{5q^2+7q+2}{2}, 1 \right]$$

A. Gradient Descent Algorithm (GDA)

The idea of this algorithm is to reduce the dimension of the estimated parameter vector θ which has $\left(\frac{q^2+5q+2}{2}\right)$ components, as seen in (15). The new parameters vector θ_{NL} is a $(q + 1)$ length vector :

$$\theta_{NL} = [h(1), \dots, h(q), \epsilon_{4,3}]^T \quad (16)$$

The criterion to be minimized is :

$$J_{LS} = \|r - \phi(\theta_{NL})\|^2$$

The GDA solution has the following form :

$$\hat{\theta}_{NL_{gr}}^{i+1} = \hat{\theta}_{NL_{gr}}^i + \lambda J^T (r - \phi(\hat{\theta}_{NL_{gr}}^i)) \quad (17)$$

where :

- $\diamond \phi$ is the system of equations obtained by concatenating (11) for all values of α_1 and β_1 defined by (13) :

$$\phi(\theta_{NL}) = M\theta$$

- $\diamond J$ is the Jacobian matrix of ϕ ,

$$J = \left[\frac{\partial \phi_k}{\partial \theta_{NL_l}} \right]_{(k,l)}$$

where $k = 1, \dots, \frac{5q^2+7q+2}{2}$, and $l = 1, \dots, q + 1$.

- $\diamond \lambda$ is the step-size.

B. Gauss-Newton Algorithm (GNA)

This algorithm can be written as :

$$\hat{\theta}_{NL_{gn}}^{i+1} = \hat{\theta}_{NL_{gn}}^i + \mu (J^T J)^{-1} J^T (r - \phi(\hat{\theta}_{NL_{gn}}^i)) \quad (18)$$

where :

- $\diamond r$, ϕ , and J are defined in section III-A.
- $\diamond \hat{\theta}_{NL_{gn}}$ has the form (16).
- $\diamond \mu$ is the step-size.

The parameter $\epsilon_{4,3}$ must be estimated since we suppose we do not know the nature of the distribution of the input signal $w(k)$.

IV. A JOINT DIAGONALIZATION-BASED ALGORITHM

The *Fourth-Order System Identification* (FOSI) algorithm [3] proposes a solution to the blind identification problem of MA models based on the joint diagonalization of a set of fourth-order cumulant matrices via a Jacobi technique. The existing relationships between the taps of an MA system driven by a non-Gaussian input $w(k)$, and the (sample) fourth-order cumulant matrices of the output process $x(k)$ make possible the recovery of the parameters $h(i)$ of the system.

The procedure of joint-diagonalization exploits the fact that any orthonormalized fourth-order cumulant matrix is diagonal in the basis of the columns of a unitary matrix \mathbf{Q} , which under certain conditions is unique (up to a permutation matrix and phase factors). Moreover, it is easy to show that the entire set of orthonormalized fourth-order cumulant matrices can be *approximately* simultaneously diagonalized under the same unitary transformation \mathbf{Q} . So, after a preliminary orthonormalization step, a new set of orthonormalized matrices is simultaneously diagonalized, giving rise to the determination of the matrix \mathbf{Q} .

The solution of this joint-diagonalization problem is equivalent to the minimization of the following criterion :

$$\phi(\mathbf{Q}, \mathcal{M}) \stackrel{\text{def}}{=} \sum_{k=1}^K |\text{diag}(\mathbf{Q}^H \bar{\mathbf{M}}(k) \mathbf{Q})|^2, \quad (19)$$

where $\mathcal{M} = \{\bar{\mathbf{M}}(k) | k = 1, \dots, K\}$ is the set of orthonormalized cumulant matrices. The system parameters estimates are obtained from an estimate unitary matrix $\hat{\mathbf{Q}}$ minimizing the criterion (19), plus the orthonormalizing matrix, determined from the eigen-decomposition of a positive definite fourth-order cumulant matrix.

V. BEST 1-D SLICE $C(q, 0, k)$ ALGORITHM

The classical $C(q, 0, k)$ algorithm is written as follows [11] :

$$h(k) = \frac{C_{4,y}(q, 0, k)}{C_{4,y}(q, 0, 0)}, \quad k = 1, \dots, q. \quad (20)$$

This algorithm is very sensitive to cumulants estimation errors and requires exact knowledge about the system order q . Nonetheless, the amount of statistical information required is very small, which makes it a very simple and attractive estimation method. Actually, all the needed information may be arranged into a vector \mathbf{C}_0 , defined entry wise as $\mathbf{C}_{\tau_2}(k) = C_{4,y}(q, \tau_2, k)$, $k = 1, \dots, q$, where τ_2 is fixed to zero. Thus, (20) may be rewritten as

$$\mathbf{h} = \mathbf{C}_0 / \mathbf{C}_0(0), \quad (21)$$

where $\mathbf{h} = [h(1) \dots h(q)]^T$.

We note that it should be possible to change \mathbf{C}_0 in (21) by any other \mathbf{C}_j , $j = 1, \dots, q$ in order to find different parameter estimations $\hat{\mathbf{h}}_j$. Indeed, it is known that the smaller estimation error ($\epsilon_j = |\mathbf{h} - \hat{\mathbf{h}}_j|^2$) is obtained by replacing \mathbf{C}_0 in (21) by the vector \mathbf{C}_λ with the maximum two-norm ($\max[\mathbf{C}_j^H \mathbf{C}_j]$) [6]. This procedure consists in an improved algorithm that uses $\tau_2 = \lambda$ instead of $\tau_2 = 0$ in (21). The new identification formula is then written as

$$h(k) = \frac{\mathbf{C}_\lambda(k)}{\mathbf{C}_\lambda(0)} = \frac{C_{4,y}(q, \lambda, k)}{C_{4,y}(q, \lambda, 0)}, \quad k = 1, \dots, q. \quad (22)$$

This method makes use of only $(q+1)^2$ statistical information, providing a reduction rate bounded by $8q$ regarding the amount of statistics used by FOSI.

VI. SIMULATIONS

We now present the simulation examples to compare the performance of those algorithms given in Sections III-A, III-B, IV and V.

In the simulations presented in this Section, the available data $\{y(k)\}$ was generated by two different models, shown below. In both models the input signal $w(k)$ is a zero mean exponentially distributed i.i.d sequence with $\gamma_{2,w} = \sigma_w^2 = 1$ and $\gamma_{3,w} = 2$. The additive noise sequence $\{v(k)\}$ is an i.i.d, zero mean, Gaussian sequence.

We carried out 200 Monte Carlo simulations with different noise sequences. For each run, we computed the Normalized Mean Square Error (NMSE) defined as :

$$NMSE = \frac{\sum_{i=1}^q (h(i) - \hat{h}(i))^2}{\sum_{i=1}^q h^2(i)}$$

where $h(i)$ and $\hat{h}(i)$ are respectively the actual and the estimated impulse responses. The results present the fluctuations of the mean NMSE, in dB, against the noise level (SNR).

Example 1 :

$$y(k) = w(k) - 2.333w(k-1) + 0.667w(k-2) + v(k)$$

The zeros of the system transfer function $H(z)$ are located at 1.9994 and 0.3336. This model has also been used in [4]. In this case $N = 10240$ samples for each run. The simulation results are summarized in figure 2.

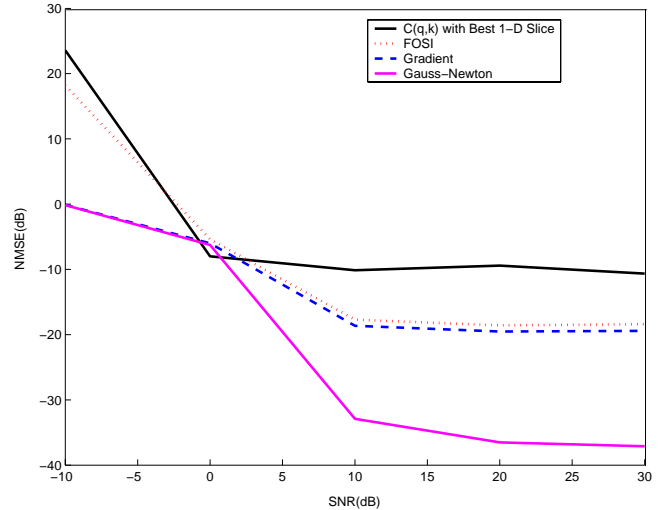


Fig. 2. Performance of the blind identification methods for the model 1

Example 2 :

$$y(k) = w(k) + 0.1w(k-1) - 1.87w(k-2) + 3.02w(k-3) - 1.435w(k-4) + 0.49w(k-5) + v(k)$$

The zeros of the system transfer function $H(z)$ are located at -2 , $0.7 \pm j0.7$ and $0.25 \pm j0.433$. This model has also been used in [4]. In this case $N = 40960$. The simulation results are given in figure 3.

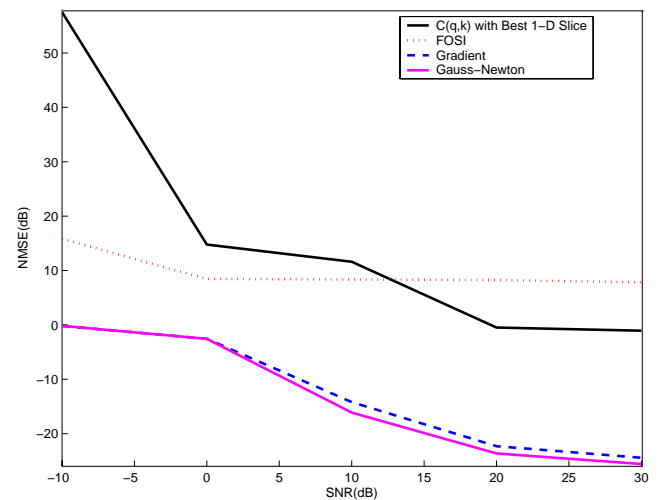


Fig. 3. Performance of the blind identification methods for the model 2

Figures 2 and 3 show the results of our simulations for the two examples. The SNR is varied from -10dB to 30dB in steps of 10dB . For each value of the SNR, 200 realizations of the noisy signal are generated.

The figures 2 and 3 demonstrate the effectiveness of the first approach, concerning the methods using nonlinear optimization techniques. In figure 3, the Gradient and Gauss-Newton algorithms are much more powerful than FOSI and $C(q, 0, k)$ with Best 1-D Slice. Notice, however, that these latter ones use only fourth-order cumulants, while the nonlinear optimization approaches utilize both third- and fourth-order cumulants. We note that increasing model order q severely affects the performance of these algorithms.

VII. CONCLUSIONS

We have presented and compared four different algorithms for consistent parameter estimation in i.i.d non-Gaussian measurement noise under the assumption that the system order is known. The methods of blind identification of MA systems use third- and fourth-order cumulants.

Simulation examples are presented in Section VI to compare these algorithms. In terms of quality of parameter estimation, the nonlinear optimization-based methods over performed the Joint-diagonalization approach as well as the $C(q, 0, k)$ algorithm incorporating Best 1-D Slice, especially for higher order models.

Nevertheless, complexity of nonlinear optimization-based algorithms remain much higher comparatively to the other two methods.

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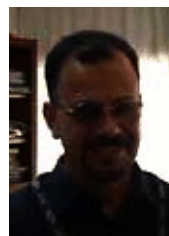
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