Abstract—This paper reviews the optimization available transmission capability (ATC) of power systems using a device of FACTS named SSSC equipped with energy storage devices. So that, emplacement and improvement of parameters of SSSC will be illustrated. Thus, voltage magnitude constraints of network buses, line transient stability constraints and voltage breakdown constraints are considered. To help the calculations, a comprehensive program in DELPHI is provided, which is able to simulate and trace the parameters of SSSC has been installed on a specific line. Furthermore, the provided program is able to compute ATC, TTC and maximum value of their enhancement after using SSSC.

Keywords—available transmission capability (ATC), total transmission capability (TTC), voltage constraints, stability constraints, FACTS, SSSC.

I. INTRODUCTION

ABILITY of power systems in reliable firm transmission can be restricted by some system features like transmission line thermal limit, bus voltage magnitude and stability consideration. Available transmission capability is defined as the amount of power transmission which does not contravene the system limits [1]. Consequently, to acquire firm power transmission, it is essential to recognize the various network limits and maximum transferable power. There are several methods for ATC computations that should be selected appropriately depending on studied network.

Spread using FACTS devices over real networks, the idea to use these devices in enhancing ATC made sense [2]. Although, in FACTS devices, parallel-connected devices effect on enhancing ATC properly by correcting bus voltages and injected reactive power [3], series-connected devices demonstrate extreme capability on intensifying ATC value for critical transmission lines by modifying line reactance and injected series voltage on lines [4]. So, SSSC appears to be an extra efficient and powerful series-connected device to enhance ATC.

In this paper, the repetitive power flow method will be concisely described at first and then, important constraints will be mentioned. Then, a model of simulating SSSC equipped with energy storage devices will be presented and enhancing method of SSSC to gain the maximum value of ATC will be defined. After that, the program has been provided in DELPHI, will be concisely described and expressed its features to computing ATC and TTC values and SSSC simulation. Finally, results of ATC calculations with and without SSSC application by the provided program will be analyzed.

II. REPEITIVE POWER FLOW METHOD TO COMPUTE ATC

In RPF method, to compute the power transmission capability between generating and consuming areas, it is increased the consuming load and is solved the load flow equations repetitively. The load Increasing will be permitted until the system restrictions will not be contravened [5]. Most important advantages of using RPF method are:

1- To access P-V and V-Q curves for voltage stability studies [2], [4];
2- Ease of regulating the control parameters;
3- High speed convergence [1].

The mathematic formula to compute ATC using RPF is

TABLE I NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>QGi &amp; PGi</td>
<td>Reactive &amp; Active generation power on bus i</td>
</tr>
<tr>
<td>QDi &amp; PDi</td>
<td>Reactive &amp; Active load on bus i</td>
</tr>
<tr>
<td></td>
<td>Voltage magnitude on bus j &amp; i</td>
</tr>
<tr>
<td>Bi &amp; Gij</td>
<td>Real &amp; Image part of Ybus, i, j</td>
</tr>
<tr>
<td>δij</td>
<td>Line angle between buses i, j</td>
</tr>
<tr>
<td>Ui(max)</td>
<td>Max of voltage magnitude on bus i</td>
</tr>
<tr>
<td>Ui(min)</td>
<td>Min of voltage magnitude on bus i</td>
</tr>
<tr>
<td>Sij</td>
<td>Reactive power on line between buses i &amp; j</td>
</tr>
<tr>
<td>N</td>
<td>Number of buses</td>
</tr>
<tr>
<td>Nj</td>
<td>Number of PQ buses</td>
</tr>
<tr>
<td>M</td>
<td>Number of transmission line</td>
</tr>
<tr>
<td>Ng</td>
<td>Number of generator</td>
</tr>
<tr>
<td>Pce</td>
<td>Base active generated power relevant to generation area</td>
</tr>
<tr>
<td>Qre</td>
<td>Base Reactive Load power relevant to load area</td>
</tr>
<tr>
<td>Prx</td>
<td>Base Active Load power relevant to load area</td>
</tr>
<tr>
<td>Pa</td>
<td>Injected active power of SSSC on bus i</td>
</tr>
<tr>
<td>Qa</td>
<td>Injected reactive power of SSSC on bus i</td>
</tr>
<tr>
<td>Pa</td>
<td>Injected active power of SSSC on bus j</td>
</tr>
<tr>
<td>Qa</td>
<td>Injected reactive power of SSSC on bus j</td>
</tr>
</tbody>
</table>

1- To access P-V and V-Q curves for voltage stability studies [2], [4];
2- Ease of regulating the control parameters;
3- High speed convergence [1].

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Maximize \( \lambda \)

Where:

\[
\sum_{i=1}^{n} \left( V_i^0 \left( G_{ij} \sin \delta_j - B_{ij} \cos \delta_j \right) - V_j^0 \left( G_{ij} \sin \delta_j - B_{ij} \cos \delta_j \right) \right) = 0
\]

\[
\sum_{i=1}^{n} \left( V_i^0 \left( G_{ij} \sin \delta_j - B_{ij} \cos \delta_j \right) + V_j^0 \left( G_{ij} \sin \delta_j - B_{ij} \cos \delta_j \right) \right) = 0
\]

\[
V_i^0 \leq V_i^0 \leq V_i^0 \max
\]

In power flow equations \( P_{Gi} \), \( P_{Di} \) and \( Q_{Di} \) comments are defined as follows:

\[
P_{Gi} = P_{Gi}^0 (1 + \lambda K_{Gi})
\]

\[
P_{Di} = P_{Di}^0 (1 + \lambda K_{Di})
\]

\[
Q_{Di} = Q_{Di}^0 (1 + \lambda K_{Di})
\]

And output ATC is calculated by means of the following equation:

\[
ATC = \sum_{ic}^{n} P_{Di} (\lambda_{max}) - \sum_{ic}^{n} P_{Di}^0 - ETC
\]

III. SSSC Model and Optimizing Its Control Parameters Value

A. Model Used To Simulate SSSC

Suppose that SSSC is installed on transmission line between buses i and j on network and belongs ideal series voltage \( V_s \) and reactance \( X_s \). This model is indicated as a series voltage and reactance \( X_s \) in figure 1. As shown in figure 1:

\[
\bar{V}_s = V_s + \bar{V}_j
\]

And the controllable series voltage \( V_s \), as a phasor is:

\[
0 < \gamma < 2\pi, \quad 0 < r < r_{max}, \quad \bar{V}_s = r \bar{V}_j e^{i\gamma}
\]

\[
V_i^\angle \theta_i \quad V_j^\angle \theta_j \quad X_s \quad T_s = -j b_s \bar{V}_s, \quad b_s = \frac{1}{X_s}
\]

Fig.1 SSSC model on line between bus i & j

It is possible to switch this model with a parallel injected current instead of series injected voltage as shown in figure 2; hence

\[
T_s = -j b_s \bar{V}_s, \quad b_s = \frac{1}{X_s}
\]

Fig. 2 Replacing series voltage supply with parallel current supply

The injected power supplied by parallel current source \( I_s \) is:

\[
S_{is}^\angle \gamma = \bar{V}_j (-\bar{I}_j) = \bar{V}_j \left[ j b_s r \bar{V}_j e^{i\gamma} \right]^*
\]

\[
S_{is}^\angle \gamma = \bar{V}_j (-\bar{I}_j) = \bar{V}_j \left[ -j b_s r \bar{V}_j e^{i\gamma} \right]
\]

\[
I_s = b_s r V_{ij} \sin(\theta_j + \gamma) + j b_s r V_{ij} \cos(\theta_j + \gamma)
\]

Eventually the following model can be proposed for SSSC simulation [6]:

\[
V_j^\angle \theta_j \quad X_s \quad V_i^\angle \theta_i
\]

Fig. 3 SSSC model

Whereat

\[
P_{is} = rb_s V_j^2 \sin \gamma
\]

\[
P_{ij} = -rb_s V_j V_j^\gamma \sin(\theta_j + \gamma)
\]

\[
Q_{is} = rb_s V_j^2 \cos \gamma
\]

\[
Q_{ij} = -rb_s V_j V_j^\gamma \cos(\theta_j + \gamma)
\]

In addition to precision and accuracy close to fact, this is a suitable model for programming the simulation software conveniently. Perceiving more carefully conveys that SSSC is simulated via two PQ buses and \( X_s \) reactance. Value of \( P \) & \( Q \) on buses is determined by \( r \) and \( \gamma \) variants.

B. Enhancing control values of SSSC

The aim of installing SSSC device is to improve transmission capability of two pointed buses so that exchanged power could be expanded. Enhance gain function is defined as follow:

\[
Max(F = \sum_{h=1}^{H} (S_h - S_i))
\]

Where \( H \) is the number of path lines between two investigated nodes and \( S_h \) is the transitive complex power of line \( h \). The parameters have been distinguished by indicator * describe critical values of system. Therefore, equation (18)
means to maximize whole available transferable complex power of system in critical conditions.

Maximizing total transmission capability is a constrained optimization issue with both equality and inequality constraints.

B.1. Equality constraints

In order to assure that system moves certainly according to coefficient load from current to new operation point, current operation point is located in equality constraints as follow [7].

For bus i which is a PQ bus that SSSC is not installed on it, and for i=1,2,3,…,Nl, there are the following equations:

\[ \lambda_i (P_{G_i} - P_{D_i}) - V_i \sum_{j=1}^{Nl} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \]  
(19)

For bus i which is a PV bus and SSSC is not installed on it, and for i=1,2,3,…,Nl, there are the following equations:

\[ \lambda_i (Q_{G_i} - Q_{D_i}) - V_i \sum_{j=1}^{Nl} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \]  
(20)

For bus i which is a PQ bus and SSSC has been installed on line i-j:

\[ \lambda_i (P_{G_i} - P_{D_i}) + \lambda_{ij} (P_{ij} - P_{ij}) - V_i \sum_{j=1}^{Nl} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \]  
(21)

For bus i supposed SSSC has been installed on line i-j:

\[ \lambda_i (Q_{G_i} - Q_{D_i}) + \lambda_{ij} (Q_{ij} - Q_{ij}) - V_i \sum_{j=1}^{Nl} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \]  
(22)

For bus j:

\[ \lambda_j (P_{G_j} - P_{D_j}) - \lambda_{ij} (P_{ij} - P_{ij}) - \lambda_{ij} V_i \sum_{j=1}^{Nl} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \]  
(23)

\[ \lambda_j (Q_{G_j} - Q_{D_j}) + \lambda_{ij} (Q_{ij} - Q_{ij}) - \lambda_{ij} V_i \sum_{j=1}^{Nl} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \]  
(24)

B.2. Inequality constraints

Throughout solving the optimization issue of equation (18), five types of inequality constraints are considered:

1) **Voltage constraints**: bus voltage magnitude (for PQ bus number i and for i=1,2,3,…,Nl) is as follow [9]:

\[ V_{i,min} \leq V_i \leq V_{i,max} \]  
(26)

2) **Transmission line thermal limit**: in any condition, line transitive current should not increase more to contravene the conductor thermal limits. Line thermal restriction is as follow [9]:

\[ T_{M,min} \leq T_{M} \leq T_{M,max} \]  
(27)

3) **Generator maximum power constraint**: active and reactive powers of any generator have maximum and minimum values

\[ Q_{G_{i,max}} \leq Q_{G_{i}} \leq Q_{G_{i,min}} \]  
(28)

3) **Transmission line dynamic stability constraint**: in order to safeguard the stability of transmission lines particularly in time of occurrences like unexpected changes in line flow caused by load trip or premature load generation, discrepancy of electrical angle of voltage on both sides of transmission line is restricted regularly to a certain value. This constraint is defined for critical point as follow [9]:

\[ |\theta_i - \theta_j| \leq 45^\circ \]  
(29)

4) **Injected power limitation constraint**: SSSC is constrained legally by injected power limitation due to thermal consideration [8]:

\[ P_{se}^2 + Q_{se}^2 \leq cte \]  
(30)

As stated before, maximizing total transmission capability is a constrained optimization issue with both equality and inequality constraints which the linear programming technique is used to solve it. The aim of this paper is to gain the accurate values of ATC. In addition to have a proper algorithm for programming, linear programming technique concludes the most accurate result. The only defect of this technique is the low runtime speed but because ATC computations are considered as off-line computations, the time of computation is not usually important. Moreover, in order to optimize using memory space and reduce program run time, sparsity techniques for Ybus matrix are employed.

IV. INTRODUCING UTILIZED NETWORK

To run the present program and gain optimized SSSC values, a 6-bus network is used which is one of standard sample networks of Power World software and the specifications of this network would be found in appendix. The reason to apply this network is to demonstrate the accuracy of conclusions have been performed by provided program and compare them with programs can be provided by other software. Computations on this network are operated ‘Point to Point’ and conclusions of this scanned network are totally analyzed and will be displayed in next section.

V. RESULTS OF PROGRAM ANALYSIS

First, ATC and TTC computations are to be set up and run for 6-bus network via program. To do that, bus 1 is supposed as seller and bus 5 as buyer. Conclusions are shown in table II.

It is eligible to mention that single contingency errors, are considered when the lines are broken down. With attention to table II, it is comprehensible that transmission lines 3, 5 and 7 are in critical condition. Due to ATC negative value with considering single contingency errors and carrying very low value of TTC in normal operation, line 5 is selected as the
most critical transmission line and SSSC is installed on it to optimize ATC. The TTC of this line is 0.24 perunit in normal operation where thermal capacity rate of line is 1 perunit. The SSSC device is installed on transmission line 5 due to optimize TTC. Hence, SSSC with nominal power 0.05 and 0.1 perunit are used and it is shown that increasing the power of SSSC will provide better results.

Conclusions are shown in table III and indicate proper optimization in ATC value in line 5. Furthermore, there is no more critical line in network either. Figure 4 illustrates the comparison of ATC value before and after installing SSSC. It might be significant to detect the rate of optimizing ATC where one device is installed. It is possible to gain better result via increasing the injectable SSSC power.

In order to optimize SSSC placement, it is installed on all power lines. The results show the line 6 as the best place of SSSC. Since the ATC maximizing in the system with considering all constraints is the main issue, it has been presented only the results of ATC in this condition and compared with two conditions including with and without installing SSSC on the line 5. The results have been shown in table IV while it has been neglected the thermal capacity of the lines to achieve the better consequences.

It is obviously comprehensible of table IV that for ATC optimizing of a line, it should be installed SSSC directly on that line. After installing SSSC on the line 6, the most optimizing will be achieved. With attention to the results, it is clear that this condition is the best for ATC optimizing of the system. Since no one of the power lines is in the critical condition and the critical lines of the system are in suitable conditions, the recent result is confirmed.

VI. CONCLUSIONS

The capability of SSSC to optimize bus-bus ATC value has been illustrated on this paper. Injected power of SSSC definitely affects on the rate of optimization. In addition to optimize ATC on critical lines via SSSC installation, it’s possible to improve ATC value of all transmission lines and consequently will effect on total network.

In this paper, reliability constraints have not been considered. To obtain better results, reliability constraints must be considered together with the other constraints.

Assessment of costs of ATC, injected power of SSSC and devices will be issues of the future researches.

APPENDIX

Table V BUS DATA IN A 6 BUS NETWORK

<table>
<thead>
<tr>
<th>Bus</th>
<th>Generated Active Power (PU)</th>
<th>Load Active Power (PU)</th>
<th>Load Reactive Power (PU)</th>
<th>Bus Type</th>
<th>Bus Voltage (PU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 1</td>
<td>-</td>
<td>1</td>
<td>0.20</td>
<td>SLACK</td>
<td>1.02</td>
</tr>
<tr>
<td>Bus 2</td>
<td>2</td>
<td>1</td>
<td>0.20</td>
<td>PV</td>
<td>1.04</td>
</tr>
<tr>
<td>Bus 3</td>
<td>0.84</td>
<td>1</td>
<td>0.20</td>
<td>PV</td>
<td>1.01</td>
</tr>
<tr>
<td>Bus 4</td>
<td>2.37</td>
<td>1</td>
<td>0.20</td>
<td>PV</td>
<td>1.03</td>
</tr>
<tr>
<td>Bus 5</td>
<td>-</td>
<td>1</td>
<td>0.50</td>
<td>PQ</td>
<td>-</td>
</tr>
<tr>
<td>Bus 6</td>
<td>-</td>
<td>1</td>
<td>0.10</td>
<td>PQ</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: S_{base}=100MVA, V_{base}=110KV
### Table VI: Line Data in a 6 Bus Network

<table>
<thead>
<tr>
<th>Line</th>
<th>From</th>
<th>To</th>
<th>$R$ (PU)</th>
<th>$X$ (PU)</th>
<th>Thermal Limit (PU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bus 1</td>
<td>Bus 2</td>
<td>0.04</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Bus 1</td>
<td>Bus 5</td>
<td>0.04</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Bus 2</td>
<td>Bus 4</td>
<td>0.04</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Bus 3</td>
<td>Bus 5</td>
<td>0.04</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Bus 5</td>
<td>Bus 3</td>
<td>0.04</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Bus 5</td>
<td>Bus 4</td>
<td>0.04</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Bus 4</td>
<td>Bus 6</td>
<td>0.04</td>
<td>0.08</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: $S_{base}=100$MVA, $V_{base}=110$KV

### References


8. "Determination of Available Transfer Capability within the Western Interconnection", Approved at the October 25-26 WMIC meeting by WMIC, June 2002.


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