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To Study the Parametric Effects on Optimality of Various Feeding Sequences of a Multi-effect Evaporators in Paper Industry using Mathematical Modeling and Simulation with MATLAB

Deepak Kumar, Vivek Kumar, and V. P. Singh

***Abstract*—**This paper describes a steady state model of a multiple effect evaporator system for simulation and control purposes. The model includes overall as well as component mass balance equations, energy balance equations and heat transfer rate equations for area calculations for all the effects. Each effect in the process is represented by a number of variables which are related by the energy and material balance equations for the feed, product and vapor flow for backward, mixed and split feed. For simulation 'fsolve' solver in MATLAB source code is used. The optimality of three sequences i.e. backward, mixed and splitting feed is studied by varying the various input parameters.

***Keywords*—**MATLAB "fsolve" solver, multiple effect evaporators, black liquor, feeding sequences.

I. INTRODUCTION

EVAPORATORS are widely used in the chemical industry to concentrate weak solution and recover solvents. It is used in industries like paper, sugar and caustic soda to concentrate black liquor, sugar cane juice and caustic soda solutions respectively. The Pulp and Paper industry, which is the focus of the present investigation predominantly, uses the Kraft Process in which black liquor is generated as spent liquor. This liquor is concentrated in multiple effect evaporator (MEE) house for further processing. Earlier, long tube vertical (LTV) types of evaporators were employed in India [21]. However, with development of flat falling film evaporators (FFFE), which claim many benefits over its counter parts LTV evaporators, most Indian paper mills have already switched to FFFE systems. In fact, it operates under low temperature drop (about 5 ◦C) across the film and thus, more evaporators can be accommodated within the total temperature difference available for evaporation to offer higher steam economy [23]. Reference [19] pointed out that the MEE house of Indian paper mills alone consumes around 24–30% of the total steam required in a large paper mill. Therefore, it calls for a thorough investigation into its analysis and various energy reduction schemes.

The black liquor contains dissolved solids ranging from 12 to 20%. So this liquor is concentrated to about 45 to 50%. The evaporation for concentration of black liquor is normally carried out in a multiple effect evaporator using low pressure steam where a series of evaporators are operated at different pressures so that the vapor from one evaporator body becomes the steam supply to the next evaporator body. This gives high steam economy.

A wide variety of mathematical models for multiple effect falling film evaporators can be found in the literature. Normally the main difference among these mathematical models is the heuristic knowledge which is incorporated in their development and simulation technique. For the analysis of MEE system mathematical models have been reported in the literature since last five decades. A few of these were developed by [1], [5], [7]-[14], [17], [20], [24] and [26]-[27]. These models are generally based on a set of linear or non-linear equations and can accommodate effects of varying physical properties of vapor/steam and liquor with change in temperature and concentration [22].

Reference [1]-[4] attempted to simulate a sextuple effect black liquor evaporator system for paper industry. This required developing a system of nonlinear simultaneous equations based on steady state mass and energy balances, heat transfer rate, equilibrium relationships and same physico-chemical/physico-thermal properties of liquor. Numerical techniques using Newton-Raphson Jacobian matrix method and method of Gauss elimination are employed to solve the problem. Reference [22]-[23] developed a nonlinear model for a Sextuple effect flat falling film evaporator (SEFEFE) system employed for concentrating weak black liquor in an Indian Kraft Paper Mill. The system incorporates different operating strategies such as condensate-, feed- and product-flashing, and

Deepak Kumar is with the Indian Institute of Technology Roorkee, Saharanpur Campus, Saharanpur, India. (phone: +91-8126421115; fax: 91-0132-2714310; dkr2009@gmail.com).

Vivek Kumar is with the Indian Institute of Technology Roorkee, Saharanpur Campus, Saharanpur, India. (vivekfpt@iitr.ernet.in).

V.P.Singh is with the Indian Institute of Technology Roorkee, Saharanpur Campus, Saharanpur, India. (singhvp3@gmail.com).

steam- and feed-splitting. Reference [18] presents the steady-state simulation and optimization of a six-effect evaporator and the provision of its relevant software package. Reference [25] developed different energy reduction schemes (ERSs) to reduce the consumption of steam for a multiple effect evaporator (MEE) system. These ERSs are condensate-, feed- and product-flashing and vapor bleeding. This process consumes more time and mathematical skills. In the present study we describe the steady state Mathematical model for six effect evaporator in the similar manner as used by [1] with some necessary modifications and used different physico-chemical/physico-thermal properties of black liquor given by [15]. The simulated results are obtained for the backward (→6→5→4→3→2→1), mixed (feed into 5th effect as, →5→6→4→3→2→1) and split feed (equal feed in 5th and 6th effects as, shown in Fig. 3) for a sextuple effect evaporator and compare with the results of previous workers [1], [3]-[4] respectively. For the solution of the system of nonlinear simultaneous equations 'fsolve' solver in MATLAB source code is used and the results show the good agreement with the above researchers. To study the optimality of the feeding sequences the effect of various input parameters are checked on the steam economy.

II. DESCRIPTION OF MATHEMATICAL MODEL

The flow diagrams of a sextuple effect evaporator system for backward, mixed and split feed sequences are shown in the Fig. 1, Fig. 2 and Fig. 3 respectively. In the backward sequence feed is introduced to the last effect and partly concentrated liquor flows to the fifth, fourth, third, second and then to the first effect from which the concentrated liquor is withdrawn. Saturated steam is fed to the shell of the first effect, and vapour generated therein flows to the shell of second effect and acts as the heating medium there. The vapour generated in the second effect supplies heat for boiling the liquor in the third effect. In this manner steam generated in each effect is supplied to the next effect as shown in the Fig. 1. The steam generated in the last effect is condensed by the condenser. But in mixed sequence feed is given in 5th effect and in split sequence the feed is given equally to both 5th and 6th effects and other process is same as shown in the Fig 2 & Fig. 3 respectively.

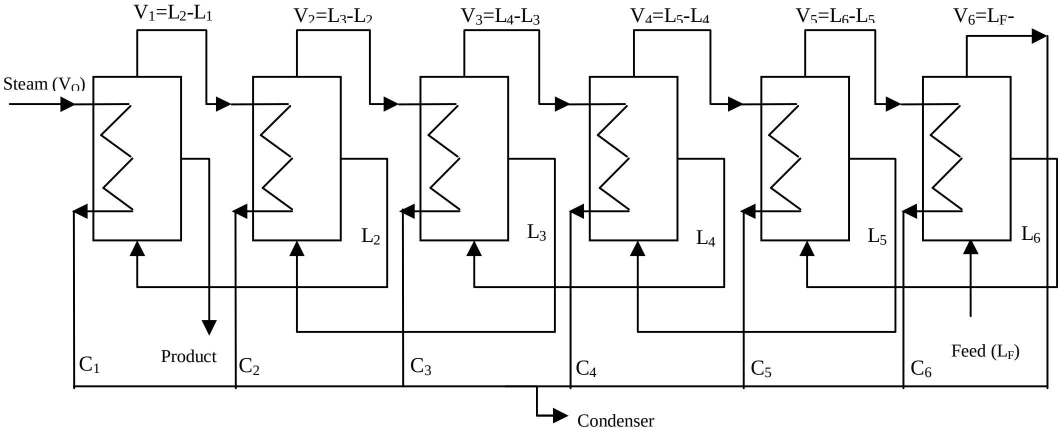

Fig. 1 Flow diagram of Sextuple backward feed evaporator (→6→5→4→3→2→1)

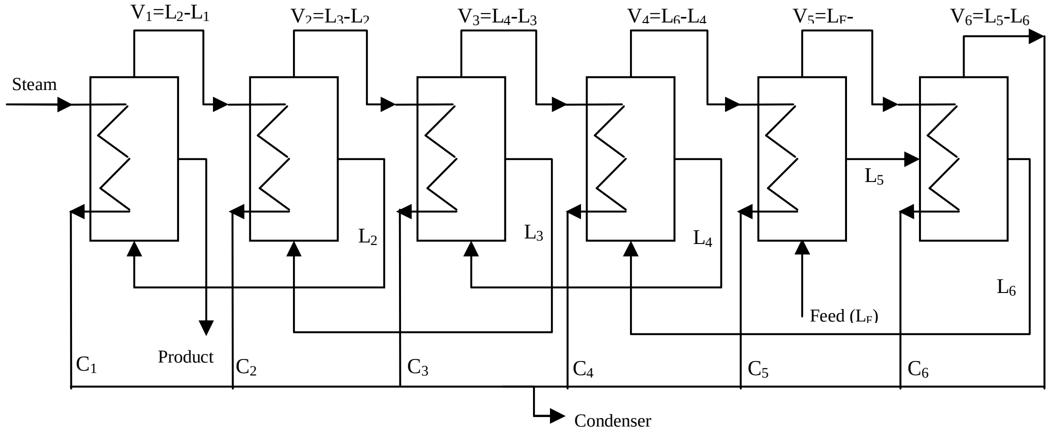

Fig. 2 Flow diagram of Sextuple mixed feed evaporator (→5→6→4→3→2→1)

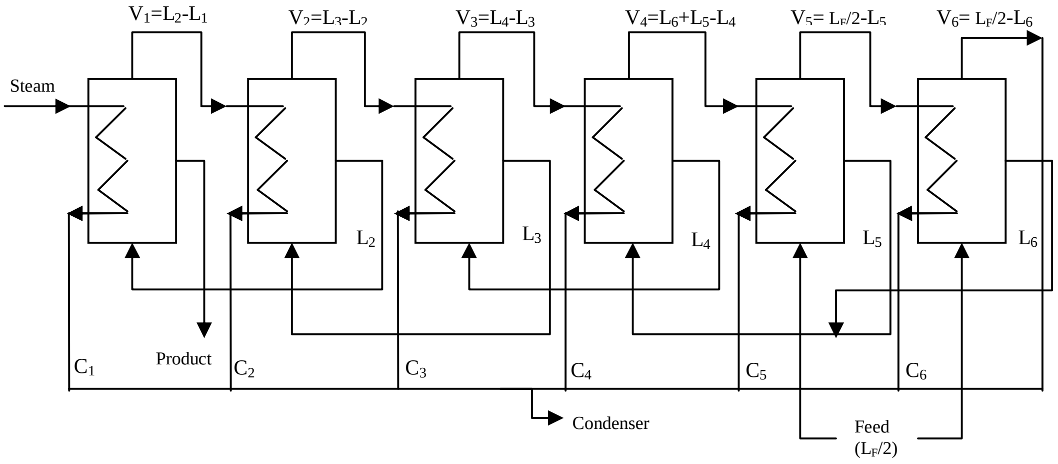

Fig. 3 Flow diagram of Sextuple split feed evaporator

→ 5
→ 6
4→3→2→1)

A. *Mathematical Formulation*

The steady state model of a multiple effect evaporator system is developed by using mass and energy balance equation for each effect of a sextuple effect evaporator. By taking the mass and energy balances over i^{th} effect of a sextuple effect evaporator system with backward, mixed and split feed model is obtained. The block diagram of the ith effect evaporator is given by Fig. 4.

Overall mass balance around evaporation section:

$$L_{i+1} = L_i + V_i \tag{1}$$

Overall balance around stem chest:

$$V_{i-1} = C_i \tag{2}$$

Partial mass balance for solids provides:

$$L_{i+1}X_{i+1} = L_iX_i = L_fX_f \tag{3}$$

An enthalpy balance

$$Q_i = H_{i-1}V_{i-1} - C_i hc_i$$

or

$$Q_i = V_{i-1}\left(hc_i + \lambda_{i-1} + Cp_v * BPR_{i-1}\right) - C_i hc_i$$

By using “(2)”

$$Q_i = V_{i-1}\left(\lambda_{i-1} + Cp_v * BPR_{i-1}\right) \tag{4}$$

Heat transfer rate

$$Q = U_iA_i(\Delta T)_{eff}$$

$$(\Delta T)_{eff} = Tv_{i-1} - Tl_i = Tv_{i-1} - Tv_i - BPR_i$$

where $Tl_i = Tv_i + BPR_i$

Hence $$Q = U_iA_i\left(Tv_{i-1} - Tv_i - BPR_i\right) \tag{5}$$

Overall energy balance gives:

$$L_{i+1}h_{i+1} + Q_i = L_ih_i + V_iH_i \tag{6}$$

Substituting the value of V_i from “(1)” and Q1 from “(4)” into "(6)” we get

$$L_{i+1}h_{i+1} + H_{i-1}V_{i-1} - C_ihc_i = L_ih_i + (L_{i+1} - L_i)H_i \tag{7}$$

Adding and subtracting $L_{i+1}h_i$ into “(7)” we get

$$L_{i+1}h_{i+1} - L_{i+1}h_i + H_{i-1}V_{i-1} - C_ihc_i$$
$$- L_ih_i + L_{i+1}h_i - (L_{i+1} - L_i)H_i = 0$$

Or it may be written as

$$L_{i+1}(h_{i+1} - h_i) + H_{i-1}V_{i-1} - C_ihc_i - (L_{i+1} - L_i)(H_i - h_i) = 0 \tag{8}$$

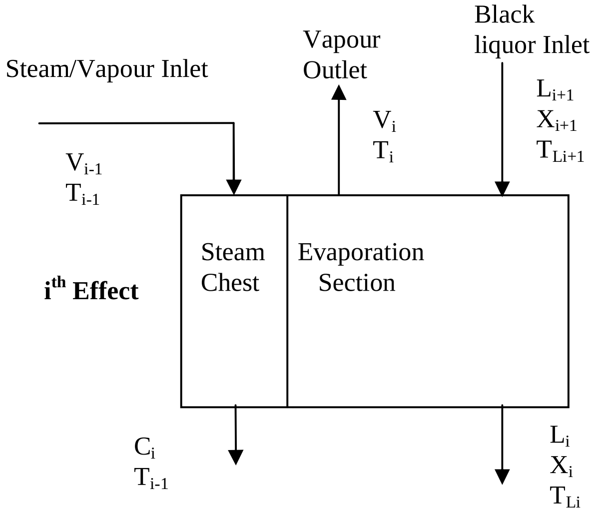

Fig. 4 Block diagram of i^{th} effect evaporator

Now since $H_i = hc_{i+1} + \lambda_i + Cp_v * BPR_i$,

$h_i = Cp_i(Tv_i + BPR_i)$ and

$hc_{i+1} = ATl_i + B$ are enthalpies of vapor, black liquor and condensate respectively. Putting these values of enthalpy into "(8)" we get

$$L_{i+1}\left\{Cp_{i+1}(Tv_{i+1} + BPR_{i+1}) - Cp_i(Tvi + BPR_i)\right\} + V_{i-1}(hc_i + \lambda_{i-1} + Cp_v * BPR_{i-1}) - C_i hc_i - (L_{i+1} - L_i)\left\{(hc_{i+1} + \lambda_i + Cp_v * BPR_i) - Cp_i(T_i + BPR_i)\right\} = 0$$

Using "(2)" and after simplifying we get

$$L_{i+1}\left\{Cp_{i+1}(Tv_{i+1} + BPR_{i+1}) - Cp_i(Tvi + BPR_i)\right\} + V_{i-1}(\lambda_{i-1} + Cp_v * BPR_{i-1}) - (L_{i+1} - L_i)\left\{\lambda_i + Cp_v * BPR_i) - Cp_i(Tv_i + BPR_i)\right\} - (L_{i+1} - L_i)(ATli + B) = 0 \qquad (9)$$

Now from "(4)" & "(5)" gives

$$U_i A_i(Tv_{i-1} - Tv_i - BPR_i) - V_{i-1}(\lambda_{i-1} + Cp_v * BPR_{i-1}) = 0 \qquad (10)$$

where i = 1, 2, 3, 4, 5 & 6.

Thus we have two nonlinear equations "(9)" & "(10)" for the i^{th} effect evaporator. Hence for i = 1 to 6 we have 12 nonlinear equations for the six effect evaporators for backward feed. To obtain the model equations for the mixed and split feed few changes are required in the feeding variables. For the mixed feed in "(9)" for 5th effect i.e. for i =5, the value L_F is used in place of L_6 and the output L_5 go into 6th effect and from 6th to 4th and from 4th to 3rd and up to the 1st effect. In this manner to obtain the model equation for the split feed the changes in the feeding variables are required. For the split feed in "(9)" for 5th and 6th effect equal feeding $L_F/2$ is given and the output from 5th and 6th effect go into 4th effect and then from 4th to 3rd effect and up to the first effect.

III. STEADY STATE SIMULATION

For obtaining the steady state solution of the sextuple effect evaporator system there is a need to solve the system of nonlinear simultaneous equations. For sextuple effect evaporator "(9)" and "(10)" give 12 nonlinear equations as obtained by [1]. Using physico-chemical properties of black liquor from TABLEII and substituting the values of enthalpies and heat transfer coefficients we get the 12 nonlinear simultaneous equations in 12 unknowns for all three feeding sequences.

Data for simulation purpose is taken from literature. The range of operating parameters is taken according to [2] and duly modified target values are considered according to Indian Paper mill evaporation conditions. The values of the operating parameters are presented in TABLEI. For falling film evaporator the range of overall heat transfer coefficient (OHTC) is given by various researchers.

TABLE I
RANGE OF OPERATIONAL PARAMETERS

S. No.	Operational Parameter	Parameter's Range	Target Value
1.	Liquor feed flow rate, Kg/sec	18.00-25.00	23.98
2.	Liquor feed Temperature, ^{0}C	70.00-90.00	80.00
3.	Liquor feed concentration	0.10-0.52	0.10
4.	Steam Temperature, ^{0}C	135-140	139
5.	Last body saturation, ^{0}C	48-52	49
6.	Liquor product concentration	0.48-0.52	0.50
7.	Heat transfer Coefficients (KCal/hm^2 ^{0}C)	1000-1200	1000, 1050,1100, 1150, 1175, 1200

Reference [6] has suggested the value of OHTC as 500-2500 KCal/h $m^2.^0C$. Reference [16] suggested the values of OHTC nearly 1300 W/m^2 ^{0}C. For evaporation of black liquor [15] gave the range of OHTC for all effects for sextuple evaporator between 500-2000 W/m^{20}C. After discussion with few Indian Paper Mills, values of OHTC were taken in the range 1000-1200 KCal/hm^2 ^{0}C for the present study in which concentration rising from 10% to 50%. The steady state solution of the system of nonlinear equations is obtained by 'fsolve' solver in MATLAB source code for the backward, mixed and split feed sequences. The solutions of backward, mixed and split feed are presented in TABLEIII, TABLEIV, and TABLEV respectively. The required steam and heat transfer area is calculated for all sequences and presented in respective TABLES. Steam economy is also calculated by the ratio of total evaporation with steam consumption is also shown in respective TABLES for each sequence.

IV. RESULTS AND DISCUSSSION

The design and operational parameters (feed rate, temperature and concentration) are equally important for modeling of MEE. A set of operational data normally employed in the paper industry globally, is given in TABLE I. With this data along with the equations for Physico-chemical/thermal properties given by [15] shown in TABLE II is used. The calculation procedure structured in this study computes design variables such as area (or area ratio between effects), externally supplied steam rate, stages temperatures, flow rates of liquid and vapors and finally steam economy.

TABLE II
PHYSICO-CHEMICAL/THERMAL PROPERTISE OF BLACK LIQUOR

S. No.	Properties	Equations	Function Parameters
1.	Boiling Point Rise	$BPR = \{6.173 X - 7.48 X^{1.5} + 32.747 X^2)^* \{1 + 0.6(Tv - 3.7316)/100\}$	X = Solid Concentration, Tv = Steam Temperature, ^{0}C
2.	Black Liquor Density, Kg/m^3	$Pl = (997 + 649 X) \{1.008 – 0.237 (Tl /1000) – 1.94(Tl/1000)^2\}$	Tl = Black Liquor Temp. in ^{0}C, X = Solid Concentration
3.	Specific Heat Capacity, kJ/kg.^{0}C,	$Cp = 4.216(1-X) + \{1.675 + (3.31Tl)/1000\} {}^*X + (4.87 - 20Tl/1000) (1-X) X^3$	Tl = Black Liquor Temp. in ^{0}C, X = Solid Concentration
4.	Enthalpy of saturated water	$hl = \alpha T + \beta$, Where $\alpha = 4.1832$ and $\beta = 0.127011$,	T = Temperature of water, ^{0}C
5.	Enthalpy of saturated steam	$hv = \gamma T + \delta$, Where $\gamma = 1.75228$ and $\delta = 2503.35$,	T = Temperature of steam, ^{0}C
6.	Latent Heat of Vaporization, kJ/kg	$\lambda = 2519.5 - 2.653 T$	T = Temperature of steam, ^{0}C

TABLE III
STEADY STATE SOLUTION OF SEXTUPLE EFFECT EVAPORATORS FOR BACKWARD FEED (→6→5→4→3→2→1)

	Effects					
Items	I	II	III	IV	V	VI
Output Liquor Concentration (kg/kg)	0.5	0.2849	0.2033	0.1608	0.1352	0.1184
Output Liquor Flow (kg/Sec)	4.796	8.417	11.7976	14.9092	17.7402	20.2613
Output Liquor Temperature (ºC)	125.0388	99.1216	83.0459	70.555	60.0491	50.125
Output Steam Temperature (ºC)	110.8637	94.065	80.2413	68.6686	58.6385	49
Output Steam Flow (kg/Sec)	3.621	3.3806	3.1116	2.831	2.5211	3.7187
Boiling Point Rise (ºC)	14.1751	5.0566	2.8046	1.8864	1.4106	1.125
Specific Heat (KJ/Kg.^{0}C)	3.3005	3.6333	3.7769	3.8569	3.9072	3.9406
Heat Transfer Area (m^2)	600.5748					
Stream Consumption (Kg/Sec)	3.9076					
Total Evaporation (Kg/Sec)	19.184					
Steam Economy	4.9094					

TABLE IV
STEADY STATE SOLUTION OF SEXTUPLE EFFECT EVAPORATORS FOR MIXED FEED (→5→6→4→3→2→1)

	Effects					
Items	I	II	III	IV	V	VI
Output Liquor Concentration (kg/kg)	0.5	0.2883	0.2067	0.1639	0.1157	0.1401
Output Liquor Flow (kg/Sec)	4.796	8.3173	11.6031	14.627	20.73	17.1105
Output Liquor Temperature (ºC)	125.2157	99.4662	83.4384	71.0077	61.3928	50.419
Output Steam Temperature (ºC)	111.0319	94.3051	80.5579	69.0649	60.2439	**49**
Output Steam Flow (kg/Sec)	3.5213	3.2858	3.0239	2.4835	3.25	3.6195
Boiling Point Rise (ºC)	14.1838	5.1611	2.8805	1.9428	1.1489	1.419
Specific Heat (KJ/Kg.^{0}C)	3.3006	3.6275	3.7704	3.8507	3.9506	3.8924
Heat Transfer Area (m^2)	591.9614					
Stream Consumption (Kg/Sec)	3.8028					
Total Evaporation (Kg/Sec)	19.184					
Steam Economy	5.04470					

TABLE V

STEADY STATE SOLUTION OF SEXTUPLE EFFECT EVAPORATORS FOR SPLIT FEED ($\begin{matrix} \to \mathbf{5} \searrow \\ \to \mathbf{6} \nearrow \end{matrix}$ **4→3→2→1**)

Items	I	II	III	IV	V	VI
				Effects		
Output Liquor Concentration (kg/kg)	0.5	0.2864	0.2045	0.1617	0.1330	0.1427
Output Liquor Flow (kg/Sec)	4.796	8.3726	11.7261	14.8293	9.0157	8.4046
Output Liquor Temperature (ºC)	126.3474	101.4446	86.2425	74.4873	65.2961	53.18
Output Steam Temperature (ºC)	112.1079	96.2997	83.3752	72.5542	61.3498	49
Output Steam Flow (kg/Sec)	3.5766	3.3535	3.1032	2.591	2.9743	3.5854
Boiling Point Rise (ºC)	14.2395	5.1449	2.8673	1.9331	3.9463	4.18
Specific Heat (KJ/Kg.^{0}C)	3.301	3.632	3.7761	3.857	3.6469	3.6044
Heat Transfer Area (m^2)	652.7756					
Stream Consumption (Kg/Sec)	3.8492					
Total Evaporation (Kg/Sec)	19.184					
Steam Economy	4.98389					

The results obtained from the simulation of models are given in TABLE III, TABLE IV, and TABLE V for the backward, mixed and split feed sequences respectively. The parametric influences of various input parameters on output parameters steam consumption (SC), steam economy (SE) and area requirement (A) for backward, mixed and split feed sequences were plotted in graphs given by Fig. 5 to Fig. 10. In this paper the effect of various input parameters on steam economy (SE) only is described for finding the optimal feeding sequence.

A. *Effect of feed temperature on steam economy*

Fig. 5 has been drawn to show that the effect of feed temperature on steam economy (SE) for backward, mixed and split feed respectively.

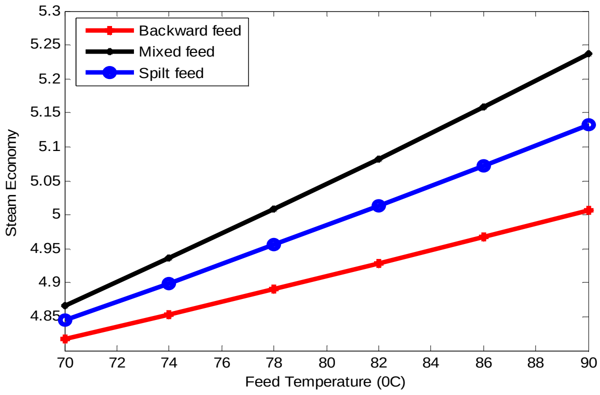

Fig. 5 Effect of feed temperature on steam economy

With the rise of feed temperature the SE is increased for all the feeding sequences. From the Fig. 5 it is clear that the mixed feed sequence is optimal for the entire range of temperature which is consider for the present study. Split feed is also more economic than simple backward feed.

B. *Effect of steam temperature on steam economy*

Fig. 6 has been drawn to show that the effect of steam temperature on steam economy (SE) for backward, mixed and split feed respectively. With the rise of steam temperature the SE is decreased for all the feeding sequences. From the Fig. 6 it is clear that the mixed is the optimal sequence.

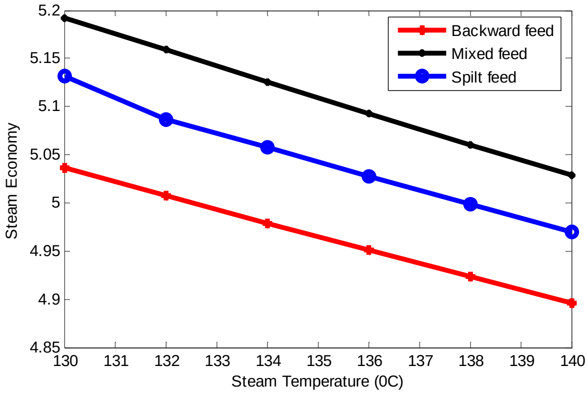

Fig. 6 Effect of steam temperature on steam economy

C. *Effect of feed flow rate on steam economy*

Fig. 7 has been drawn to show that the effect of steam temperature on steam economy (SE) for all three sequences. With varying the feed flow rate from 18-25 kg/s, the SE is decreased for split and backward feed while the SE of the mixed feed increased. In this case it is clear from the Fig. 7 that the mixed feed is more economic (optimal).

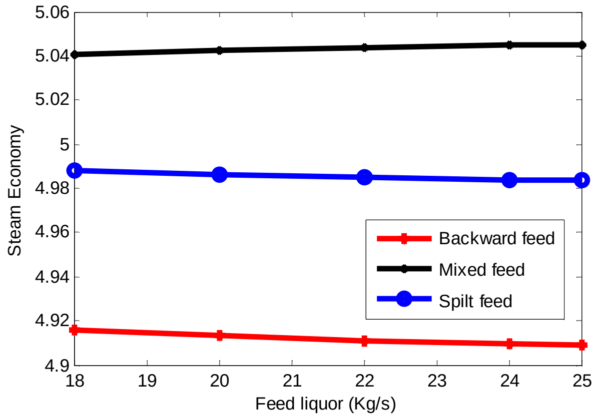

Fig. 7 Effect of feed flow rate on steam economy

D. *Effect of feed concentration on steam economy*

Fig. 8 has been drawn to show that the effect of varying feed concentration on steam economy (SE) for all three sequences. In this case it is clear from the Fig. 8 that the mixed feed is optimal for the entire range of feed flow rate.

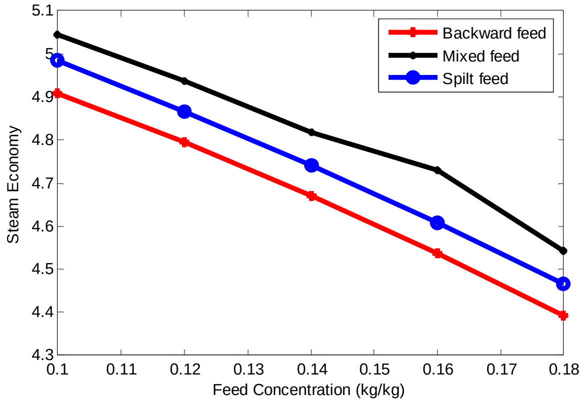

Fig. 8 Effect of feed concentration on steam economy

E. *Effect of product concentration on steam economy*

Fig. 9 has been drawn to show that the effect of varying product (i.e. output of the first effect) concentration on steam economy (SE) for all three sequences. In this case also it is clear from the Fig. 9 that the Se is increased and the mixed feed is optimal for the entire range of feed flow rate.

F. *Effect of last body temperature on steam economy*

Fig. 10 has been drawn to show that the effect of varying last body (i.e. output of the first effect) temperature on steam economy (SE) for all three sequences. In this case it is clear from the Fig. 10 that the SE is decreased for the mixed and split feed while the backward feed shows an increment in the SE for and finally the mixed feed is optimal for the entire range of feed flow rate.

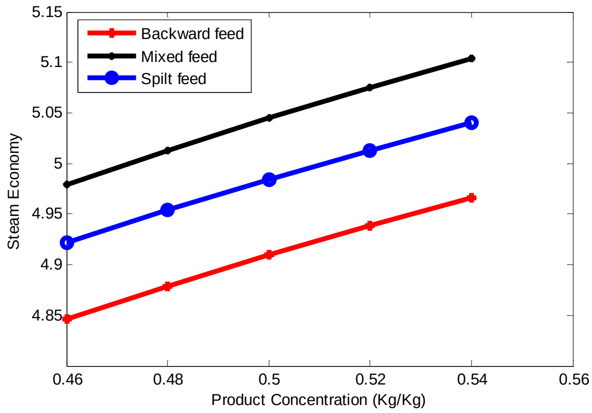

Fig. 9 Effect of product concentration on steam economy

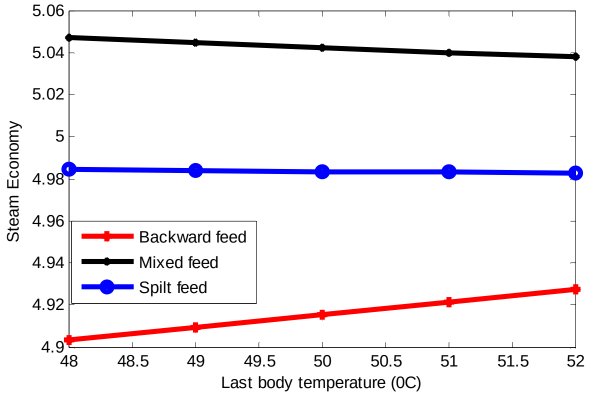

Fig. 10 Effect of last body temperature on steam economy

V. CONCLUSION

Steady state simulation is done successfully by using ‘fsolve’ solver in MATLAB source code for three feeding sequences backward, mixed and splitting. The effect of variation of various input parameters on steam economy is studied. It may be concluded that mixed feed is always optimal for entire range of each parameters.

The mathematical models developed and the solution technique employed in this present investigation can precisely evaluate data for any sequence of any MEE set up. The computational developed helps to generate large body of data and can handle many variables and the interactions and interdependence with each other. In today’s context it is extremely essential to use this software for the benefit of industry, paper mill in particular. The design procedure developed can bring accuracy in assessing the performance of an existing evaporator system or can help in designing a new system for a Greenfield pulp and paper mill.

TABLE VI
NOMENCLATURE

A	Shell area, m^2
BPR	Boiling point rise, ºC
C	Condensate flow rate
Cp	Specific heat of water at constant pressure, KJ/Kg
h	Enthalpy, KJ/Kg ºC
H	Enthalpy of saturated steam/vapour, KJ/Kg ºC
Q	Heat transfer rate, KJ
λ	Latent heat of vaporization, KJ/Kg
t	Time, sec
T	Temperature, ºC
U	Overall Heat Transfer Coefficient (OHTC), KJ/sec.m^2 ^{0}C
L	Mass flow rate, Kg/s
X	Solid content, %
V	Live steam/vapour flow rate, Kg/s
	Subscripts
c	Condensate
l	Liquor
i	Effect number
v	Vapor
f	feed

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Deepak Kumar received the Bachelor’s and Mater’s degree from M. J. P. Rohilkhand University, Bareilly, India in Mathematics in 2000 and 2002 respectively. He is pursuing Ph.D degree in Industrial Mathematics from the Indian Institute of Technology Roorkee. He is working on “Mathematical Modelling and Simulation in Pulp mill Operations”. He has published 6 research papers in which 4 are in international Journals of peer review.

Vivek kumar received the Bachelor of Engineering (Pulp and Paper Engineering) from University of Roorkee, India in 1992, Master in Engineering in Chemical Engineering from University of Roorkee (now, Indian Institute of Technology Roorkee), in 1994 and Ph.D degrees in Environmental Management from Indian Institute of Technology Delhi India, in 2000. Dr. Vivek Kumar is currently with Indian Institute of Technology Roorkee, India, where he is an Assistant Professor in the Department of Paper Technology. His special fields of interests include mathematical modeling and simulation and Environmental & Energy Management.

V.P. Singh received the Bachelor’s degree from Meerut University, India in 1970, Master’s and Ph.D degrees in Applied Mathematics from the University of Roorkee (now, Indian Institute of Technology Roorkee), India, in 1972 and 1978, respectively. Dr. V.P. Singh is currently with Indian Institute of Technology Roorkee, India, where he is a Professor in the Department of Paper Technology. His special fields of interests include applied and industrial mathematics, mathematical modeling of pulp washing problems.