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A New Objective Weight on Interval Type-2 Fuzzy Sets

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***Abstract*—**The design of weight is one of the important parts in fuzzy decision making, as it would have a deep effect on the evaluation results. Entropy is one of the weight measure based on objective evaluation. Non--probabilistic-type entropy measures for fuzzy set and interval type-2 fuzzy sets (IT2FS) have been developed and applied to weight measure. Since the entropy for (IT2FS) for decision making yet to be explored, this paper proposes a new objective weight method by using entropy weight method for multiple attribute decision making (MADM). This paper utilizes the nature of IT2FS concept in the evaluation process to assess the attribute weight based on the credibility of data. An example was presented to demonstrate the feasibility of the new method in decision making. The entropy measure of interval type-2 fuzzy sets yield flexible judgment and could be applied in decision making environment.

***Keywords*—**Objective weight, entropy weight, multiple attribute decision making, type-2 fuzzy sets, interval type-2 fuzzy sets

I. Introduction

WEIGHT in multiple attribute decision making (MADM) can be divided into two groups which are subjective weight and objective weight. Subjective weight can reflect the subjective judgment or intuition of the decision makers (DM), and they can be obtained based on preference information of the attributes given by the DM through interviews, questionnaires or trade-off interrogation directly [3]. Objective weight can be obtained from the objective information such as decision matrix through mathematics models [5].

The most popular method to obtain objective weights is entropy method [5]. The conception of 'entropy' firstly appeared in thermodynamics, and was used to describe the process of a campaign irreversible phenomenon. Later 'entropy' was introduced to the information theory by Shannon to measure the disorder degree of system [13]. The entropy value as a measure of variability and randomness, high information entropy indicates high variability, which means high levels of uncertainty in a system [16]. Then, in 1965, the entropy of a fuzzy set describes the fuzziness degree of a fuzzy set and was first mentioned by Zadeh [18]. Since then, many fuzzy entropy measures have been proposed.

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De Luca and Termini [4] introduced some axioms, which captured people's intuitive comprehension to describe the fuzziness degree of a fuzzy set. Then there have been various fuzzy entropy measures ([6]; [11]; [17]), especially, Burillo and Bustince [1] introduced the concept of entropy of Antanassove's intuitionistic fuzzy set and the interval-valued fuzzy set in 1996. Then, Szmidt and Kacprzyk [15] proposed a different concept for assessing the intuitionistic fuzzy (IF) entropy.

Since fuzzy sets (FS) or usually known as type-1 fuzzy sets (T1FS) has found not be able to handle all kinds of uncertainties appearing in real life problem domain [12]. Therefore Zadeh [17] has introduced a type-2 fuzzy sets (T2FS) concepts that are more realible to handle all the uncertainties. However, T2 FS is difficult to understand and explain. Hence, Mendel and Liang [9] introduced new concepts that are more easier to calculate. These new concepts are allowing the characterization of a type-2 fuzzy set with a superior membership function and an inferior membership function; these two functions can be represented each one by a type-1 fuzzy set membership function. The interval between these two functions represent the footprint of uncertainty (FOU), which is used to characterize a type-2 fuzzy set. Then in 2006, Mendel, John and Liu are introduced new concepts of interval type-2 fuzzy sets (IT2FS). The simplest T2FS are interval type-2 fuzzy sets whose elements' degree of membership are intervals with secondary membership degree of 1.0. If we can use interval type-2 fuzzy sets [10] for handling entropy weights problems, then there is room for more flexibility due to the fact that T2FS provide more flexibility to represent uncertainties than traditional T1FS [18].

To deal with the uncertainties problems, this paper presents a new method for entropy weight based on the concepts of IT2FS. This concept is totally different with the traditional entropy method. This proposed method provides a useful way to handle entropy weight for fuzzy multiple attributes group decision-making problems in a more flexible and more intelligent manner due to the fact that it uses interval type-2 fuzzy sets rather than traditional type-1 fuzzy sets to represent the evaluating values and the weights of attributes.The rest of this paper is organized as follows: in Section 2, we briefly review basic concepts of entropy weight, type-2 fuzzy sets and interval type-2 fuzzy sets and related theory behind it. In Section 3, we briefly review the concept of entropy weight from the view of Szmidt and Kacprzyx's method. In Section 4, we introduce the new concept of entropy weight and applied it to weight measure. In Section 5, a numerical example is performed into the new concepts of entropy weight and this paper concludes at the final section.

II. Basic Concepts

A. Entropy Weight

Entropy weight is a parameter that describes how much different alternatives approach one another in respect to a certain attribute [7]. Conversely, low information entropy is a sign of a highly organized system. In information theory, the entropy value can be calculated as follows:

$$H(p_1, p_2, \ldots, p_n) = -\sum_{j=1}^{n} p_j \ln p_j$$

where, H is the level of entropy, p_j is the probability of occurrence of event.

B. Type-2 Fuzzy Sets and Interval Type-2 Fuzzy Sets

This section is briefly reviews some definitions of type-2 fuzzy sets and interval type-2 fuzzy sets from Mendel et al. [10].

Definition 1 (Mendel et al. [10])

A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$, shown as follows [10]:

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{A}}}(x,u)\right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1 \right\},$$

where J_x denotes an interval in $[0,1]$. Moreover, the type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u),$$

where $J_x \subseteq [0,1]$ and $\iint$ denotes the union over all admissible x and u.

Definition 2 [(Mendel et al. [10])

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}} = 1$, then A is called an interval type-2 fuzzy sets. An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be regarded as a special case of a type-2 fuzzy set, represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_X} 1/(x,u),$$

where $J_x \subseteq [0,1]$.

III. Szmidt and Kacprzyx's Entropy Method

Szmidt and Kacprzyk [15] developed a new entropy method for (intuitionistic fuzzy sets) IFS. In their paper, they proposed the IF entropy as a ratio of distances between the (F, F_{near}) and (F, F_{far}). The expression given as follows:

$$E_{SK}(F) = \frac{(F, F_{near})}{(F, F_{far})} \tag{1}$$

where (F, F_{near}) is the distance from F to the nearer point F_{near} among positive ideal point and negative ideal point, and (F, F_{far}) is the distance from F to the farther point F_{far} among positive ideal point and negative ideal point. Then, Szmidt and Kacprzyk [15] expressed IF entropy in the following definition:

Definition 3 (Szmidt and Kacprzyk [15])

A real function $E: IFS(X) \to [0,1]$ is called an entropy on $IFS(X)$, if E has the following properties:

P1: $E(A) = 0$ if and only if A is crisp set,
P2: $E(A) = 1$ if and only if $\mu_A(x) = \nu_A(x)$,
P3: $E(A) \le E(B)$ if A is less fuzzy than B,
i.e., $\mu_A(x) \ge \mu_B(x)$ and $\nu_A(x) \le \nu_B(x)$ for $\mu_B(x) \ge \nu_B(x)$
or
$\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for $\mu_B(x) \le \nu_B(x)$
P4: $E(A) = E(A^c)$, where A^c is the complement of A.

Because the entropy concept of Szmidt and Kacprzyk [15] constructs on distance, then the relative concepts of measuring distance for IFSs is shown. The Euclidean distance between *IFS A*, *B* belonging to *IFSs*(*X*) is defined as follows:

$$(\mu_A(x_1) - \mu_B(x_1))^2 + (\nu_A(x_1) - \nu_B(x_1))^2 + (\pi_A(x_1) - \pi_B(x_1))^2 = Y$$

$$d_{SK}^2(A,B) = \sqrt{\sum_{i=1}^{n}(Y)} \tag{2}$$

Clearly, $0 \le d_{SK}^2(A,B) \le n$

Therefore, refer to (1), the IF entropy measure that used Euclidean distance defined as follows:

Definition 4 (Szmidt and Kacprzyk [15])

Entropy for IFS A with n elements given as: \ $\tilde{\tilde{A}} = \int_{x\in X} \int_{u \in J_x} 1/(x,u),$

IF entropy measure according to Euclidean distance

$$\frac{\sqrt{(\mu_{near}(x_i) - \mu(x_i))^2 + (\nu_{near}(x_i) - \nu(x_i))^2 + (\pi_{near}(x_i) - \pi(x_i))^2}}{\sqrt{(\mu_{far}(x_i) - \mu(x_i))^2 + (\nu_{far}(x_i) - \nu(x_i))^2 + (\pi_{far}(x_i) - \pi(x_i))^2}} = X$$

$$E_{IFS}(A) = \frac{1}{n}\sum_{i=1}^{n}(X) \tag{3}$$

At this stage, we know that Szmidt and Kacprzyk [14] made use of all three parameter $(\mu(x), v(x), \mu(x))$ to defined IF distance measures. Then a ratio-based measure of entropy was proposed based on IF distance measures [15].

IV. ENTROPY WEIGHT MEASURES BASED ON INTERVAL TYPE-2 FUZZY SETS

A. Entropy Measures for Interval Type-2 Fuzzy Sets

Based on the Szmidt and Kacpryzk entropy's method, this study developed a new entropy method based on interval type-2 fuzzy set. By using the concepts of a ratio distances between the (F, F_{near}) and (F, F_{far}) as Refer to (1), therefore our new method is defined as follows:

Theorem 1. (Proposed method) A mapping $E : IT2FS \to [0,1]$. $E(\tilde{A})$ is said to be an entropy of IT2FS $\tilde{A} \in IT2FS$, if $E(\tilde{A})$ satisfies the properties condition:

P1: $E(\tilde{A}) = 0$ if and only if $\tilde{A}$ is type-1 fuzzy set,

P2: $E(\tilde{A}) = E\left(\underset{\sim}{\tilde{A}}, \tilde{\tilde{A}}\right)$ if and only if $E(\underset{\sim}{\tilde{A}}) = E\left(\tilde{\tilde{A}}\right)$,

P3: $E(\tilde{A}) \le E(\tilde{B})$ if $\tilde{A}$ is less fuzzy than $\tilde{B}$,

P4: $E(\tilde{A}) = E(\tilde{A}^C)$ if and only if $E(\underset{\sim}{\tilde{A}}) = E(\underline{\tilde{A}^C})$ and $E\left(\overline{\tilde{A}}\right) = E\left(\overline{\tilde{A}^C}\right)$.

Therefore, the proposed entropy weight method based on interval type-2 fuzzy sets is defined as follows.

$$E(\underset{\sim}{\tilde{A}}) = \frac{\sqrt{\sum_{i=1}^{n}[M_1]}}{\sqrt{\sum_{i=1}^{n}[M_2]}} \tag{4}$$

$$\begin{bmatrix} (a_{near}(x) - a_L(x))^2 + (b_{near}(x) - b_L(x))^2 \\ + (c_{near}(x) - c_L(x))^2 + (d_{near}(x) - d_L(x))^2 \end{bmatrix} = M_1 \tag{5}$$

$$\begin{bmatrix} (a_{far}(x) - a_L(x))^2 + (b_{far}(x) - b_L(x))^2 \\ + (c_{far}(x) - c_L(x))^2 + (d_{far}(x) - d_L(x)) \end{bmatrix} = M_2 \tag{6}$$

and

$$E(\underset{\sim}{\tilde{A}}) = \frac{\sqrt{\sum_{i=1}^{n}[N_1]}}{\sqrt{\sum_{i=1}^{n}[N_2]}} \tag{7}$$

$$\begin{bmatrix} (a_{near}(x) - a_U(x))^2 + (b_{near}(x) - b_U(x))^2 \\ + (c_{near}(x) - c_U(x))^2 + (d_{neat}(x) - d_U(x))^2 \end{bmatrix} = N_1 \tag{8}$$

$$\begin{bmatrix} (a_{far}(x) - a_U(x))^2 + (b_{far}(x) - b_U(x))^2 \\ + (c_{far}(x) - c_U(x))^2 + (d_{far}(x) - d_U(x))^2 \end{bmatrix} = N_2 \tag{9}$$

Therefore, we proof the following Theorem 1 as follows:

[P1] When A is a crisp set, then for all $x_i \in X$. We assume that (4) = 0 and Equation (7) = 0. Therefore,

$$\text{For } A = \left\langle \begin{matrix} ((a_L(x)), (b_L(x)), (c_L(x)), (d_L(x))); \\ ((a_U(x)), (b_U(x)), (c_U(x)), (d_U(x))) \end{matrix} \right\rangle = \langle (0,0,0,0); (1,1,1,1) \rangle \tag{10}$$

$$E(\underset{\sim}{\tilde{A}}) = \frac{\sqrt{\sum_{i=1}^{n}\left[(0 - a_L(x))^2 + (0 - b_L(x))^2 + (0 - c_L(x))^2 + (0 - d_L(x))^2\right]}}{\sqrt{\sum_{i=1}^{n}\left[(1 - a_L(x))^2 + (1 - b_L(x))^2 + (1 - c_L(x))^2 + (1 - d_L(x))^2\right]}} \tag{11}$$

Then, substitute $\left\langle \begin{matrix} ((a_L(x)), (b_L(x)), (c_L(x)), (d_L(x))); \\ ((a_U(x)), (b_U(x)), (c_U(x)), (d_U(x))) \end{matrix} \right\rangle = \langle (0,0,0,0); (1,1,1,1) \rangle$ into (11), we get

$$E(\underset{\sim}{\tilde{A}}) = \frac{-0}{4} = 0 \tag{12}$$

Next,

For $A = \left\langle \begin{matrix} ((a_L(x)), (b_L(x)), (c_L(x)), (d_L(x))); \\ ((a_U(x)), (b_U(x)), (c_U(x)), (d_U(x))) \end{matrix} \right\rangle$

$$E(\underset{\sim}{\tilde{A}}) = \frac{\sqrt{\sum_{i=1}^{n}\left[(1 - a_U(x))^2 + (1 - b_U(x))^2 + (1 - c_U(x))^2 + (1 - d_U(x))^2\right]}}{\sqrt{\sum_{i=1}^{n}\left[(0 - a_U(x))^2 + (0 - b_U(x))^2 + (0 - c_U(x))^2 + (0 - d_U(x))^2\right]}}$$

$$= \langle (0,0,0,0); (1,1,1,1) \rangle \tag{13}$$

Then, substitute $\left\langle \begin{matrix} ((a_L(x)), (b_L(x)), (c_L(x)), (d_L(x))); \\ ((a_U(x)), (b_U(x)), (c_U(x)), (d_U(x))) \end{matrix} \right\rangle = \langle (0,0,0,0); (1,1,1,1) \rangle$ into (13), we get

$$E(\underset{\sim}{\tilde{A}}) = \frac{0}{-4} = 0 \tag{14}$$

[P2] When A is a crisp set, then for all $x_i \in X$,

We know that, $x_i \in X$,

$$E(\underset{\sim}{\tilde{A}}) = \frac{-a_L(x) - b_L(x) - c_L(x) - d_L(x)}{4 - a_L(x) - b_L(x) - c_L(x) - d_L(x)}$$

and

$$E\left(\tilde{\tilde{A}}\right)=\frac{4-a_L(x)-b_L(x)-c_L(x)-d_L(x)}{-a_L(x)-b_L(x)-c_L(x)-d_L(x)}$$

Therefore, assume that

$$E\left(\underset{\sim}{\tilde{A}}\right)=\frac{-a_L(x)-b_L(x)-c_L(x)-d_L(x)}{4-a_L(x)-b_L(x)-c_L(x)-d_L(x)}=1$$

Hence

$$E\left(\underset{\sim}{\tilde{A}}\right)=-a_L(x)-b_L(x)-c_L(x)-d_L(x) \\ =4-a_L(x)-b_L(x)-c_L(x)-d_L(x) \tag{15}$$

and

$$E\left(\tilde{\tilde{A}}\right)=\frac{4-a_L(x)-b_L(x)-c_L(x)-d_L(x)}{-a_L(x)-b_L(x)-c_L(x)-d_L(x)}=1$$

Hence,

$$E\left(\tilde{\tilde{A}}\right)=4-a_L(x)-b_L(x)-c_L(x)-d_L(x) \\ =-a_L(x)-b_L(x)-c_L(x)-d_L(x) \tag{16}$$

Therefore $E\left(\underset{\sim}{\tilde{A}}\right)=E\left(\tilde{\tilde{A}}\right)$.

[P3] $E\left(\tilde{A}\right)\le E\left(\tilde{B}\right)$

To prove it, we first subdivide into two parts X and Y where: $(a_U(x), b_U(x), c_U(x), d_U(x))=X$ and $(a_L(x), b_L(x), c_L(x), d_L(x))=Y$.

i.e.

$X_A(x)\le X_B(x)$ and $Y_A(x)\ge Y_B(x)$ for $X_B(x)\le Y_B(x)$

or

$X_A(x)\ge X_B(x)$ and $Y_A(x)\le Y_B(x)$ for $X_B(x)\ge Y_B(x)$

Thus we get $E(A)\le E(B)$.

[P4] $E\left(\underset{\sim}{\tilde{A}}\right)=E\left(\underset{\sim}{\tilde{A}}^C\right)$ it is clear that
$A=((a_U(x)),(b_U(x)),(c_U(x)),(d_U(x)))$ and
$A^C=((1-a_U(x)),(1-b_U(x)),(c_U(x)),(d_U(x)))$

Thus $$E\left(\underset{\sim}{\tilde{A}}\right)=\frac{1-a_U(x)}{a_U(x)}=\frac{a_U(x)}{1-a_U(x)}=E\left(\underset{\sim}{\tilde{A}}^C\right) \tag{17}$$

and

$E\left(\tilde{\tilde{A}}\right)=E\left(\tilde{\tilde{A}}^C\right)$ it is clear that
$A=((a_L(x)),(b_L(x)),(c_L(x)),(d_L(x)))$ and
$A^C=((1-a_L(x)),(1-b_L(x)),(c_L(x)),(d_L(x)))$

Thus $$E\left(\underset{=}{\tilde{\tilde{A}}}\right)=\frac{1-a_L(x)}{a_L(x)}=\frac{a_L(x)}{1-a_L(x)}=E\left(\tilde{\tilde{A}}^C\right) \tag{18}$$

In this section, the entropy for IT2Fs has been developed. Therefore, the consideration of IT2FS entropy to weighting the attributes in MCDM will be presented in next section.

B. *Steps of Objective Weight Method Based on Interval Type-2 Fuzzy Sets*

In this paper, based on Wang and Lee [16] weighting method, we extend objective weight to our proposed IT2FS entropy method to measure the objective weight in the decision matrix. The advantage of the proposed approach is that it is involve end-users into the whole decision making process. We adopted the information entropy concept to confirm the weight of evaluating attribute which can effectively balance the influence of subjective factors. The innovative approach is capable of providing a more comprehensive methodology for decision making process. Therefore, general process of this method is listed below:

Step 1: Establish a decision matrix
Establish a decision matrix for objective weight. Weight problem can be concisely expressed in matrix format as

$$D=\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \cdots & \tilde{\tilde{f}}_{1n} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \cdots & \tilde{\tilde{f}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{m1} & \tilde{\tilde{f}}_{m2} & \cdots & \tilde{\tilde{f}}_{mn} \end{bmatrix} \end{array} \tag{19}$$

where $x_1, x_2, \ldots, x_m$ represents the alternative and $C_1, C_2, \ldots, C_n$ represents the criteria. Each entries value considered as IT2FS values, which denoted as $\tilde{\tilde{f}}_{mn}$.

Step 2: Calculate entropy value
Use IT2FS entropy formulas (Refer to (4) and (7)) to calculate the entropy value of each IT2FS in the decision matrix. Therefore, the entropy value is represented as follows:

$$\tilde{\tilde{E}}=1/FOU\left(\tilde{\tilde{E}}\right)=\left[\underset{\sim}{\tilde{\tilde{E}}}, \overline{\tilde{\tilde{E}}}\right] \tag{20}$$

$$D = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ x_1 & \tilde{\tilde{E}}_{11} & \tilde{\tilde{E}}_{12} & \cdots & \tilde{\tilde{E}}_{1n} \\ x_2 & \tilde{\tilde{E}}_{21} & \tilde{\tilde{E}}_{22} & \cdots & \tilde{\tilde{E}}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_m & \tilde{\tilde{E}}_{m1} & \tilde{\tilde{E}}_{m2} & \cdots & \tilde{\tilde{E}}_{mn} \end{matrix}$$

where E_{ij} is the IT2FS entropy value of each IT2FS in the decision matrix.

Step 3: Divide using maximal entropy value
Then, we divided all the entropy values by using the maximal entropy value and value of h_{ij} is used to represent the outcomes of maximal entropy value, and it can be defined as:

$$\tilde{\tilde{h}} = 1/FOU\left(\tilde{\tilde{h}}\right) = \left[\underline{\tilde{\tilde{h}}}, \overline{\tilde{\tilde{h}}}\right]$$

For example:

$$\tilde{\tilde{h}}_{i1} = \left[\left(\frac{\underline{\tilde{\tilde{E}}}_{i1}}{\max\left(\underline{\tilde{\tilde{E}}}_{i1}\right)}\right), \left(\frac{\overline{\tilde{\tilde{E}}}_{i1}}{\max\left(\overline{\tilde{\tilde{E}}}_{i1}\right)}\right)\right], \quad (21)$$

Then the decision matrix can be expressed as follows:

$$D = \begin{matrix} P_1 \\ P_2 \\ \vdots \\ P_m \end{matrix} \begin{bmatrix} \tilde{\tilde{h}}_{11} & \tilde{\tilde{h}}_{12} & \cdots & \tilde{\tilde{h}}_{1n} \\ \tilde{\tilde{h}}_{21} & \tilde{\tilde{h}}_{22} & \cdots & \tilde{\tilde{h}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{h}}_{m1} & \tilde{\tilde{h}}_{m2} & \cdots & \tilde{\tilde{h}}_{mn} \end{bmatrix}$$

Step 4: Weight of attributes
Finally, calculate the weight of attributes by using the weight formula. We use w_j to represent the outcome of weight value of attribute *j*, and it can be defined as:

$$\tilde{\tilde{w}}_j = 1/FOU\left(\tilde{\tilde{w}}_j\right) = \left[\underline{\tilde{\tilde{w}}}_j, \overline{\tilde{\tilde{w}}}_j\right]$$

$$\left(\underline{\tilde{\tilde{w}}}_j, \overline{\tilde{\tilde{w}}}_j\right) = \left[\left(\frac{1-\underline{\tilde{\tilde{a}}}_j}{\underline{\tilde{\tilde{T}}}}\right), \left(\frac{1-\overline{\tilde{\tilde{a}}}_j}{\overline{\tilde{\tilde{T}}}}\right)\right]$$

where $\underline{\tilde{\tilde{a}}}_j = \sum_{d=1}^{i} \underline{\tilde{\tilde{h}}}_{dj} \Big/ n$, $\overline{\tilde{\tilde{a}}}_j = \sum_{d=1}^{i} \overline{\tilde{\tilde{h}}}_{dj} \Big/ n$, $\underline{\tilde{\tilde{T}}} = \sum_{j=1}^{i} \underline{\tilde{\tilde{a}}}_j$ and $\overline{\tilde{\tilde{T}}} = \sum_{j=1}^{i} \overline{\tilde{\tilde{a}}}_j$. The $\underline{\tilde{\tilde{a}}}_j$ and $\overline{\tilde{\tilde{a}}}_j$ are represents the summmation of the normalized entropy values which are corresponding to the attribute *j*. The $\underline{\tilde{\tilde{T}}}_j$ and $\overline{\tilde{\tilde{T}}}_j$ is the summation of $\underline{\tilde{\tilde{a}}}_j$ and $\overline{\tilde{\tilde{a}}}_j$ and *n* is the number of attributes. The sum of attribute weight is $\sum\left(\underline{\tilde{\tilde{w}}}_j, \overline{\tilde{\tilde{w}}}_j\right) = (1,1), \left(\underline{\tilde{\tilde{w}}}_j, \overline{\tilde{\tilde{w}}}_j\right) \in [0,1]$.

V. NUMERICAL EXAMPLE

To demonstrate the applicability and practicability of the proposed method, an example on objective weight method with interval type-2 fuzzy sets are given in this section. In a simple decision-making, suppose that the students want to buy a new laptop. Three possible laptops $\{A_1, A_2, A_3\}$ may be considered. However, the students must make a decision according to three criteria, $\{C_1, C_2, C_3\}$: C_1 is the brand of laptop, C_2 is the price of laptop and C_3 is the design of laptop. Assume that the three decision-makers D_1, D_2 and D_3 use the linguistic terms shown in Table 1 to represent the evaluating values of the alternatives with respect to different attributes, respectively, as shown in Table 2. Based on the interval type-2 fuzzy set representation method, the linguistic terms shown in Table 1 can be represented by interval type-2 fuzzy sets, as shown in Table 3.

TABLE I
LINGUISTIC TERMS FOR THE RATINGS AND THEIR CORRESPONDING TYPE-1 FUZZY SETS [2]

Linguistic Variables for the Ratings of the Vehicles	
Very Poor (VP)	(0, 0, 0, 1; 1, 1)
Poor (P)	(0, 1, 1, 3; 1, 1)
Medium Poor (MP)	(1, 3, 3, 5; 1, 1)
Medium (M)	(3, 5, 5, 7; 1, 1)
Medium Good (MG)	(5, 7, 7, 9; 1, 1)
Good (G)	(7, 9, 9, 10; 1, 1)
Very Good (VG)	(9, 10, 10, 10; 1, 1)

TABLE II
EACH CRITERION LINGUISTIC OF DECISION MATRIX

CRITERIA	ALTERNATIVES	DECISION-MAKERS		
		D_1	D_2	D_3
C_1	A_1	VG	VG	VG
	A_2	VG	G	VG
	A_3	MG	M	MG
C_2	A_1	M	M	MP
	A_2	MP	M	M
	A_3	M	MP	MP
C_3	A_1	MG	M	MG
	A_2	MG	M	M
	A_3	M	M	M

TABLE III
LINGUISTIC TERMS FOR THE RATINGS AND THEIR CORRESPONDING TYPE-1 FUZZY SETS [3]

Linguistic Variables for the Ratings of the Vehicles	
Very Poor (VP)	((0,0,0,1;1,1), (0,0,0,1;1,1))
Poor (P)	((0,1,1,3;1,1), (0,1,1,3;1,1))
Medium Poor (MP)	((1,3,3,5;1,1), (1,3,3,5;1,1))
Medium (M)	((3,5,5,7;1,1), (3,5,5,7;1,1))
Medium Good (MG)	((5,7,7,9;1,1), (5,7,7,9;1,1))
Good (G)	((7,9,9,10;1,1), (7,9,9,10;1,1))
Very Good (VG)	((9,10,10,10;1,1), (9,10,10,10;1,1))

Therefore, using the proposed method in Section 4, the final results for the weight of attributes are shown as follows:

TABLE IV
ENTROPY-BASED WEIGHTS FOR EACH ATTRIBUTES

	$\tilde{\tilde{w}}_j$
C_1	((0.3003; 1,1), (0.3003; 1,1))
C_2	((0.0359; 1,1), (0.0359; 1,1))
C_3	((0.0184; 1,1), (0.0184; 1,1))

From the above result, all the objective weights are in interval type-2 fuzzy set concepts. Thus, the IT2FS entropy can be used for weighting attribute in MADM.

VI. CONCLUSION

In this paper we developed interval type-2 fuzzy set entropy weight for MADM problems. The concept of evaluation of two different methods which are interval type-2 fuzzy set and entropy weight were implemented to develop a new weight of interval type-2 fuzzy set entropy weight. A simple modification on properties of entropy weight has been made into interval type-2 fuzzy sets concepts. Numerical example has been given to demonstrate the proposed method. Hence, the proposed method provides us with a useful way to handle the fuzzy multiple attribute group decision-making problems in a more flexible and more intelligent manner due to the fact that it uses interval type-2 fuzzy sets rather than traditional type-1 fuzzy sets to represent the evaluating values and the weights of attributes.

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