

# Comparative Analysis of Various Multiuser Detection Techniques in SDMA-OFDM System Over the Correlated MIMO Channel Model for IEEE 802.16n

Susmita Das and Kala Praveen Bagadi

**Abstract**—SDMA (Space-Division Multiple Access) is a MIMO (Multiple-Input and Multiple-Output) based wireless communication network architecture which has the potential to significantly increase the spectral efficiency and the system performance. The maximum likelihood (ML) detection provides the optimal performance, but its complexity increases exponentially with the constellation size of modulation and number of users. The QR decomposition (QRD) MUD can be a substitute to ML detection due its low complexity and near optimal performance. The minimum mean-squared-error (MMSE) multiuser detection (MUD) minimises the mean square error (MSE), which may not give guarantee that the BER of the system is also minimum. But the minimum bit error rate (MBER) MUD performs better than the classic MMSE MUD in term of minimum probability of error by directly minimising the BER cost function. Also the MBER MUD is able to support more users than the number of receiving antennas, whereas the rest of MUDs fail in this scenario. In this paper the performance of various MUD techniques is verified for the correlated MIMO channel models based on IEEE 802.16n standard.

**Keywords**— Multiple input multiple output, multiuser detection, orthogonal frequency division multiplexing, space division multiple access, Bit error rate

## I. INTRODUCTION

THE most popular SDMA structure in MIMO system is able to increase system capacity by supporting multiple users. SDMA exploits the unique, user-specific “spatial signature”, i.e. the channel impulse response (CIR) of the individual users for differentiating amongst them. This allows the system to support multiple users within the same frequency band and/time slot, if their CIRs are sufficiently different and are accurately measured. SDMA architecture can be configured and deployed for most of the well-known mobile communication architectures such as CDMA (Code division Multiple Access), TDMA (Time Division Multiple Access) and FDMA (Frequency Division Multiple Access). However, the space domain has not been exploited within a cell so far. Equipping base stations with antenna arrays will enable beam steering in contrast to omnidirectional or sectorized single antenna systems.

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Thus, it becomes possible for base stations (BS) to radiate directed to specific users on the down-link as well as receive directed on the up-link. This can be done by reusing channels within a cell for user which are spatially separable by the antenna array mitigating the intersymbol interference (ISI) caused by multipath propagation [1]. Orthogonal frequency division multiplexing (OFDM) is effective in OFDM therefore is considered as an efficient modulation technique for broadband access in a very dispersive environment. Hence the combination of OFDM and SDMA is an efficient technique in high data rate transmission scenario [2] [3].

Research in the development of efficient signal detection algorithms for SDMA-OFDM systems have generated much interest in recent years, and several detection algorithms have been proposed in the literatures [4] [5] [6] [7] [8] [9]. Among the various MUDs, the classical linear ZF and MMSE MUDs exhibit low complexity at the cost of a limited performance. The high-complexity optimum ML MUD provided here has capable of achieving the best performance with an exhaustive search. However, the complexity of nonlinear ML detector generally avoids its use in practical systems especially with many users and large constellations. QR decomposition using tree search is a most promising algorithm, which can be implement with low complexity and also provides near optimal solution.

However, minimising mean square error (MSE) in MMSE MUD may not necessarily guarantee that the BER of the system is also minimized. But in MBER MUD minimize the error-probability or bit-error ratio (BER), rather than the minimising mean square error (MSE). In this paper, the MBER MUD weight calculation of SDMA OFDM system is described. It is also discussed that the MBER MUD may significantly perform better than the MMSE MUD in terms of minimum error probability in a two user two receiving antenna OFDM scenario. In the weight vector updating process the steepest descent gradient algorithm may converge slowly, and a Gauss-Newton algorithm is computationally expensive. The conjugate gradient method offers a better alternative. In this paper all the MUD techniques are simulated and compared based on the correlated MIMO channel models for IEEE 802.16n standards [10].

The organization of the paper is as follows. Section II describes the system models of SDMA OFDM. Section III provides various MUD techniques. Simulation results and analysis are presented in Section IV. Conclusion is given in

Section V.

II. SDMA-OFDM SYSTEM MODEL

The SDMA system with  $P$  number of users each equipped with single antenna and  $L$  number of receiving antennas at BS was depicted in Fig. 1. And each user Rx antenna pair undergoes OFDM modulation as shown in the below Fig. 2.

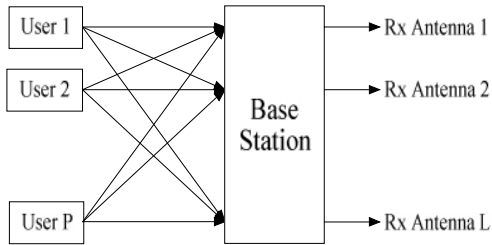


Fig. 1 Basic SDMA Structure

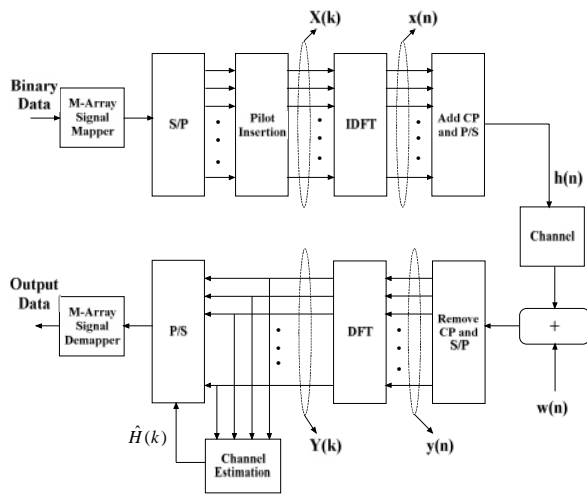


Fig. 2 OFDM Block Diagram

In the SDMA-OFDM uplink the received signal corrupted with Additive White Gaussian Noise (AWGN) at each receiving antenna can be expressed as [2]:

$$Y = H \cdot X + N \quad (1)$$

where  $Y = [Y_1, Y_2, Y_3, \dots, Y_L]^T$  is the received signal vector,  $X = [X_1, X_2, X_3, \dots, X_P]^T$  is a transmitted symbol vector,  $H$  is the  $(L \times P)$  channel frequency response matrix, and  $N = [N_1, N_2, N_3, \dots, N_L]^T$  is the noise vector whose covariance matrix is assumed to be  $\sigma_n^2 I$  in the frequency domain.

III. MULTIUSER DETECTION TECHNIQUES

A. Zero Forcing (ZF) Detection

This algorithm involves a linear transformation between the output signal and estimated channel, and the detected signal is given by [2]

$$\hat{X} = \hat{H}^{-1}Y \quad (2)$$

where  $\hat{X}$  is estimated user signal,  $\hat{H}$  is the estimated channel response and  $Y$  is the received vector.

B. Minimum mean square error (MMSE) Detection

MMSE detection scheme is also a linear process but it assumes a priori knowledge of noise variance and channel covariance. Due to this it is a more accurate detection scheme compared to ZF MUD. This algorithm detects users as [2]

$$\hat{X} = ((H^H H + \sigma^2 I_M)^{-1} H^H) Y \quad (3)$$

where  $\sigma^2$  is the variance of i.i.d. complex Gaussian random noise with zero mean  $(\cdot)^H$  indicates Hermitian.

C. Maximum Likelihood detection (ML)

The highest-complexity, highest performance optimum ML MUD uses an exhaustive search for finding the most likely transmitted users [11]. For a ML-MUD supporting  $P$  simultaneous transmitting users, a total of  $2^{mP}$  metric evaluations have to be invoked, where  $m$  denotes the number of bits per symbol, in order to detect the  $L$  user symbol vector  $\hat{X}$  that consists of the most likely transmitted symbols of the  $L$  users at a specific subcarrier, which is given by

$$\hat{X} = \arg \left\{ \min \left\| Y - H \tilde{X}^u \right\| \right\}_{u=1,2..U} \quad (4)$$

where  $Y$  is the  $(M \times 1)$ -dimensional received signal vector and  $H$  is the  $(L \times P)$ -dimensional channel matrix. The set  $U$  in (4) constitutes  $2^{mP}$  number of trial vectors.

D. QRD-M Detection

The QRD-M algorithm provides near ML detection performance with comparatively low complexity [8]. It is basically a breadth first tree traversal algorithm. At each detection layer, QRD-M algorithm keeps  $M$  reliable nodes instead of deciding the symbol. Detection is done after processing all layers. The concept of QRD-M is to apply the tree search to detect the symbols in a sequential manner. Starting from the first layer i.e.  $i = P$ , the algorithm calculates the metrics for all possible values of  $\hat{Y}_i$  from the constellation using Euclidean distance given as (12). The metrics of these points or nodes are then ordered, and only  $M$  nodes with the smallest metrics are retained and the rest of the list is deleted. The same process is applied to the next layer nodes, and this process continues to the last layer i.e.  $i = 1$ . To achieve near-ML detection performance for QRD-M algorithm,  $M$  should be large enough for the selected paths to include the correct one.

Since the complexity of ML detection depends on the constellation size and number of users, an exhaustive search required. This is infeasible in practical if either the constellation size or the signal dimension size is very large. To overcome such problem the QRD-M algorithm was proposed [12]. In this algorithm QR-decomposition of channel matrix is obtained as  $H = QR$ , where  $Q$  is a  $(L \times L)$  unitary matrix such that  $Q^H Q = I$  and  $R$  is  $(L \times P)$  upper triangular matrix. Using (1)

$$Y = Q.R.X + N \quad (5)$$

$$Q^H Y = R.X + Q^H N \quad (6)$$

$$\tilde{Y} = R.X + \tilde{N} \quad (7)$$

$$\begin{bmatrix} \tilde{Y}_1 \\ \tilde{Y}_2 \\ \vdots \\ \tilde{Y}_p \end{bmatrix} = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,L} \\ 0 & R_{2,2} & \cdots & R_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{p,L} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_L \end{bmatrix} + \begin{bmatrix} \tilde{N}_1 \\ \tilde{N}_2 \\ \vdots \\ \tilde{N}_L \end{bmatrix} \quad (8)$$

where  $R_{ij}$  is the  $(i, j)$  th component of  $R$ . The statistical properties of  $N$  and  $\tilde{N}$  are equal. Therefore, the ML detection problem (4) can be reformulated as:

$$\hat{X} = \arg \left\{ \min \left\| \tilde{Y} - R\hat{X} \right\| \right\} \quad (9)$$

$$= \arg \min \left\{ \sum_{i=1}^p \left| \tilde{Y}_i - \sum_{j=i}^p R_{i,j} X_j \right|^2 \right\} \quad (10)$$

in the above expression  $\left| \cdot \right|$  denotes the absolute value. Let us assume

$$d(\hat{X}) = \sum_{i=1}^p \left| \tilde{Y}_i - \sum_{j=i}^p R_{i,j} X_j \right|^2 \quad (11)$$

$d$  is partial Euclidian distance. To account for the case when the decision is made on symbols from  $X_p$  to  $X_k$ ,  $1 \leq k \leq P$ , (11) can be modified as

$$d_k(X) = \sum_{i=1}^p \left| \tilde{Y}_i - \sum_{j=i}^p R_{i,j} X_j \right|^2 \quad (12)$$

where  $X = [X_k, X_{k+1}, \dots, X_p]$  of length  $P-k+1$ .

A fairly large  $M$  number of branches are needed to QRD-M scheme in order to approach the MLD performance. The maximum value of  $M$  can be up to constellation size of modulation used. For example, for 16-QAM modulated systems,  $M$  can take value up to 16.

### E. Minimum Bit Error Rate (MBER) MUD

Initially we assume the channel matrix  $H$  is explicitly defined. From [13], [14] the MBER solution for BPSK modulated signal is defined as the probability of error  $P_E$  encountered at the output of the SDMA MUD characterized by the combiner weight vector  $w_p$  of user  $p$  may be expressed as:

$$P_E(w_p) = \frac{1}{N_b} \sum_{j=1}^{N_b} Q \left[ \frac{\text{sgn}(b_p^{(j)}) w_p^H \bar{Y}_j}{\sigma_n \sqrt{w_p^H w_p}} \right] \quad (13)$$

Where  $N_b$  is the number of equiprobable combinations of the binary vectors of the  $P$  users, i.e.  $N_b = 2^L$ ,

$\sigma_n$  is the variance of the noise,

$b_p^{(j)}$   $j \in 1, \dots, N_b$  is the transmitted bit of user  $p$ , and

$\bar{Y}_j$ ,  $j \in 1, \dots, N_b$  constitutes a possible value of the noiseless  $(L \times 1)$ -dimensional received signal vector  $Y$ . The MBER solution is defined as:

$$W_{p(MBER)} = \arg \min_{W_p} P_E(W_p) \quad (14)$$

where  $P_E(W_p)$  is the error probability with weight vector  $W_p$ .

In general an iterative strategy based on the steepest descent gradient method may be used for finding the MBER solution. According to this method, the linear SDMA MUD's weight vector  $W_p$  is iteratively updated, taking the MMSE weights as initial weights, until the specific SDMA MUD weight vector that exhibits the lowest BER is arrived. In each step, the weight vector is updated according to a specific step size  $\mu$  in the vectorial direction in which the BER cost function decreases most rapidly, in the direction opposite to the gradient of the BER cost function. The BER is independent of the magnitude of the MUD's weight vector, and hence the knowledge of the orientation of the detector's weight vector is sufficient for defining the decision boundary of the linear MBER OFDM/SDMA detector. And the cost function is derived as

$$\nabla_{w_p} P_E(w_p) = \frac{1}{N_b \sqrt{2\pi} \sigma_n} \left( \frac{w_p w_p^H - w_p^H w_p I}{(w_p^H w_p)^{3/2}} \right) \cdot \sum_{j=1}^{N_b} e^{\left( \frac{(\bar{Y}_j^{(j)})^2}{2\sigma_n^2 w_p^H w_p} \right)} \text{sgn}(b_p^{(j)}) \bar{Y}_j \quad (15)$$

where  $s_p^{(j)} = w_p^H \bar{Y}_j$ . In this paper conjugate gradient [8] algorithm is used for weight updation, which is given below.

### Initialization:

Choose step size  $\mu > 0$  and termination scalar  $\beta > 0$ . Set initial weight  $W(1)$  from MMSE detection and  $d(1) = -\nabla P_E(w_p(1))$  and iteration number  $i = 1$ .

### Loop:

If  $\left\| \nabla P_E(w(i)) \right\| = \sqrt{(\nabla P_E(w(i)))^T \nabla P_E(w(i))} < \beta$ : go to stop

$$W(i+1) = W(i) + \mu d(i)$$

$$W(i+1) = \frac{W(i+1)}{\|W(i+1)\|}$$

$$\phi_i = \frac{\|\nabla P_E(w(i+1))\|^2}{\|\nabla P_E(w(i))\|^2}$$

$$d(i+1) = \phi_i d(i) - \nabla P_E(w(i+1))$$

$i = i+1$ , goto Loop.

#### IV. SIMULATION ANALYSIS OF RESULTS

In this section, we have analyzed the performance of SDMA-OFDM system using the various multiuser detection techniques mentioned in previous section. In the simulation study a 2x2 SDMA OFDM system is considered. Further simulation parameters chosen are outlined in Table I. Performance carried through BER vs SNR in dB plots.

TABLE I.

PARAMETERS USED FOR SIMULATION

Parameters	Specification
No. of active Carriers	128
Guard Interval	32
No. of Frames	1000
No. of Users	2
No. of Rx antennas	2
Modulation Technique	BPSK
Optimization Used	Conjugate Gradient
Step Size	0.0975
Channel impulse response	Correlated MIMO Channel given in Table II and Gaussian Channels given in Table III.

In the simulation study the correlated MIMO channel model according to IEEE 802.16n standards is used as given in Table II [15]. To check for multiple users support we have used the unfaded dispersive Gaussian channels. In Table III the z-domain transfer function associated with the CIR of all two receiving antennas and five users are summarized in Table III [16].

TABLE II.

CORRELATED MIMO CHANNEL MODEL FOR IEEE 802.16N

	Tap 1	Tap 2	Tap 3	Units
Delay	0	0.4	0.9	μsec.
Power (30° antenna)	0	-11	-22	dB
K-Factor	3	0	0	
Doppler shift	0.4	0.3	0.5	Hz
Antenna correlation	0.4			

TABLE III.

CIRs OF THE DIFFERENT USERS AT THE DIFFERENT ANTENNAS FOR THE P=5, L=2 SYSTEM

User	Antenna	CIR
User 1	Ant. 1	$(0.5017+0.0304j)+(0.1894+0.0739j)z^{-1}+(0.1505-0.0214j)z^{-2}$
	Ant. 2	$(0.5363+0.1908j)+(0.1457-0.4261j)z^{-1}+(0.0090-0.0840j)z^{-2}$
User 2	Ant. 1	$(0.1705-0.0197j)+(0.0485+0.0660j)z^{-1}+(0.0111+0.0024j)z^{-2}$
	Ant. 2	$(0.0711-0.0121j)+(0.0182+0.0321j)z^{-1}-(0.0021-0.0030j)z^{-2}$
User 3	Ant. 1	$(0.5935+0.1160j)+(0.2535-0.0673j)z^{-1}+(0.2165-0.0209j)z^{-2}$
	Ant. 2	$(0.5671+0.0716j)+(0.2307+0.0189j)z^{-1}+(0.1975-0.0264j)z^{-2}$
User 4	Ant. 1	$(0.6747+0.1343j)-(0.1659+0.6735j)z^{-1}-(0.3611-0.2085j)z^{-2}$
	Ant. 2	$(0.5851+0.1619j)-(0.0563+0.6215j)z^{-1}-(0.2590-0.1820j)z^{-2}$
User 5	Ant. 1	$(0.4032-0.0003j)+(0.1395+0.0966j)z^{-1}+(0.0939-0.0119j)z^{-2}$
	Ant. 2	$(0.5591+0.1815j)+(0.2133-0.3020j)z^{-1}+(0.1216+0.0355j)z^{-2}$

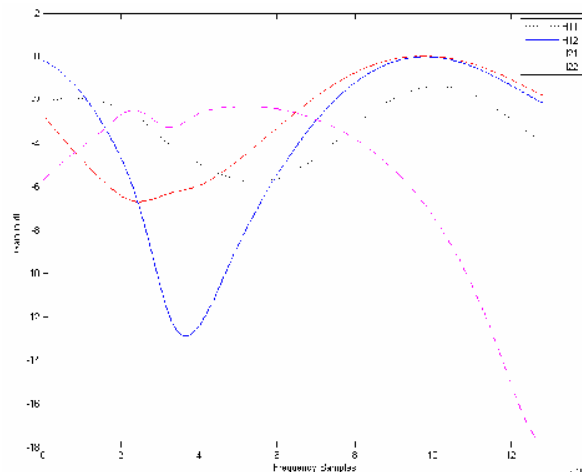


Fig. 3 Frequency response of the correlated MIMO channels in a two users two receiving antenna scenario

The frequency responses of the four channels in the 2x2 SDMA-OFDM system over correlated MIMO channel models is given in Fig. 3. In Fig. 3 due to its low channel fading effect, the system performance in Fig. 4 is quite good in low SNR itself. Fig. 4 shows Average BER performance comparison of various MUD detection schemes in a 2x2 SDMA OFDM system. In this figure the linear detection schemes like ZF and MMSE MUDs provide poor performance (9.93 and 9.29 dB SNR to achieve 10<sup>-4</sup> BER levels). In this Fig. 4 the performance of the non linear highly complex ML detection scheme provides an optimal solution. It is also observed that the average BER performance of QRD MUD performance reaches the near optimal solution with very low complexity comparing with ML detection. Also in this figure the MBER MUD achieves 10<sup>-4</sup> BER level by just losing 1dB SNR than ML detection. The advantage of the MBER MUD is that it can support more number of users compared to the number of receiving antennas, whereas all other MUD techniques are unable to support.

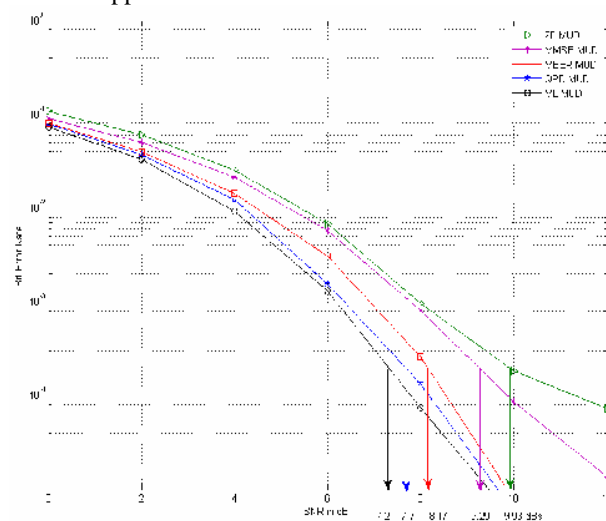


Fig. 4 BER performance of various multiuser detection schemes through the correlated MIMO channels in a two users two receiving antenna scenario

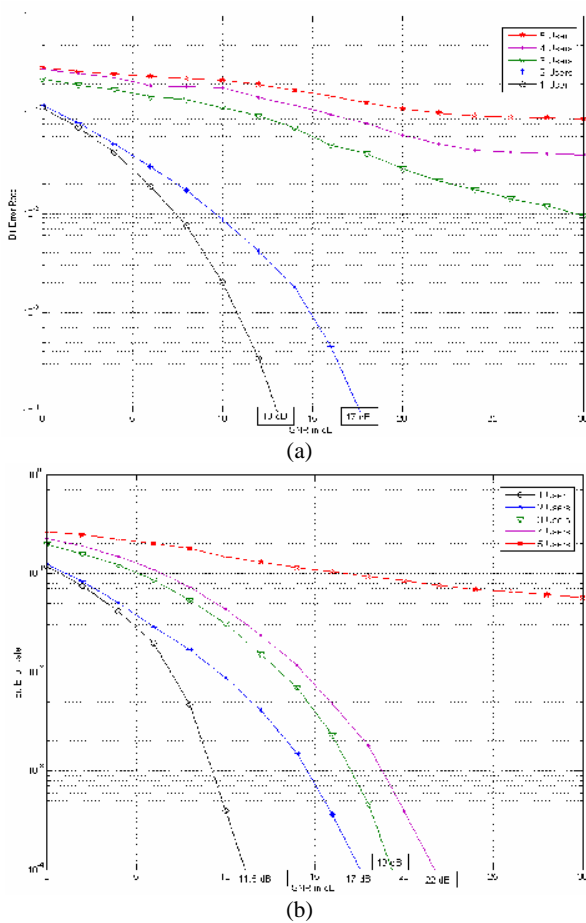


Fig. 5 The BER performance of user 1 employing both MMSE and MBER MUDs in SDMA-OFDM system equipped with two receiving antennas for different number of users over dispersive Gaussian channels given in Table III. (a). MMSE MUD (b). MBER MUD

Further a  $2 \times 5$  SDMA-OFDM system is studied over Gaussian channels given in Table III. In Fig. 5 it is seen that as the number of users increases, the BER performance degrades due to the increased multiuser interference. The MMSE MUDs in Fig. 5(a) can only support a maximum number of users that is equal to the number of receiver antennas, which is two in this case. Once the number of users exceeds the number of receiver antennas, the MMSE MUD becomes incapable of differentiating the users, and performance deteriorates. Under such case the MBER MUD as shown in Fig. 5(b) performs significantly better by supporting up to four users. Finally by comparing Fig. 5(a) and (b) we may conclude that the MBER MUD is capable of supporting more users than the number of receiver antennas. Helpful Hints

### V. CONCLUSIONS

In this paper various multiuser detection schemes like ZF, MMSE, ML, QRD, and MBER have been elaborated and compared in a SDMA-OFDM system. The MMSE estimator assumes a priori knowledge of noise variance and channel covariance, so that MMSE detects users accurately than ZF detection. The ML detection scheme provides an optimal solution for MUD, but due to its complexity its application is

prohibited in high load scenarios. The QRD detection algorithm performs close to the optimal ML detection and the computational complexity of QRD algorithm is less compared to ML detection and it eliminates exhaustive search. In this paper, it is shown that the conjugate gradient based MBER MUD is capable of achieving better performance than the conventional MMSE MUD. Unlike the MMSE MUD, the MBER MUD has compatible of supporting more users than the number of receiver antennas. The MUD simulations of SDMA-OFDM system are performed over the correlated MIMO channel for IEEE 802.16n standards, and those are studied through the BER plots.

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