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PSO-based Possibilistic Portfolio Model with Transaction Costs

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Abstract—This paper deals with a portfolio selection problem based on the possibility theory under the assumption that the returns of assets are LR-type fuzzy numbers. A possibilistic portfolio model with transaction costs is proposed, in which the possibilistic mean value of the return is termed measure of investment return, and the possibilistic variance of the return is termed measure of investment risk. Due to considering transaction costs, the existing traditional optimization algorithms usually fail to find the optimal solution efficiently and heuristic algorithms can be the best method. Therefore, a particle swarm optimization is designed to solve the corresponding optimization problem. At last, a numerical example is given to illustrate our proposed effective means and approaches.

Keywords—Possibility theory, portfolio selection, transaction costs, particle swarm optimization.

I. Introduction

In 1952, Markowitz [1] published his pioneering work which laid the foundation of modern portfolio analysis. The basic assumption for using Markowitz's mean-variance model is that the situation of assets in the future can be correctly reflected by asset data in the past, that is to say, the means, variances and covariances in future are similar to those in the past. However, since the security market is so complex and the occurrence of new security is so quick, in many cases security returns cannot be accurately predicated by historical data. In this case, fuzzy set theory proposed by Zadeh [2] in 1965, has become a helpful tool in integrating the experts' knowledge and investors' subjective opinions into a portfolio selection problem. Since then, researchers began to employ fuzzy set theory to solve many problems including financial risk management. Watada [3], Inuiguchi and Tanino [4], and Wang and Zhu [5] discussed portfolio selection using fuzzy decision theory. Tanaka and Guo [6], [7] proposed two kinds of portfolio selection models based on fuzzy probabilities and exponential possibility distributions, respectively. Carlsson and Fullér [8] introduced the notions of lower and upper possibilistic mean values of a fuzzy number, then proposed a possibilistic approach to selecting portfolios with highest utility score in [9]. Chen [10] discussed the portfolio selection problem for bounded assets based on weighted possibilistic means and variances. Zhang et al. [11] discussed the portfolio selection problem for bounded assets with the maximum possibilistic mean-variance utility.

Transaction cost is one of the main concerns for portfolio manager. Arnott and Wagner [12] found that ignoring transaction costs would result in an inefficient portfolio. Recently, a number of researchers investigated portfolio selection problem with transaction costs. Mao [13], Brennan [14] studied the portfolio selection problem with fixed transaction costs. Yoshimoto [15], Fang et al. [16], considered portfolio optimization with changeable transaction costs. Mulvey and Vladimirou [17], Dantzig and Infanger [18] incorporated transaction costs into the multi-period portfolio selection model. However, portfolio selection problem with realistic constraints, such as transaction costs, minimum transaction lots, cardinality constrains, etc, becomes a complex nonlinear programming problem and traditional optimization algorithms fail to find the optimal solution efficiently, while heuristic algorithms can be the best method. Therefore, many researchers solve the corresponding optimization problems by using heuristic algorithms. For example, Chang et al. [19] used heuristics algorithms based upon genetic algorithms, tabu search and simulated annealing for cardinality constrained mean-variance model. Fernández and Gómez [20] used heuristics algorithms based upon neural network for the standard Markowitz mean-variance model which includes cardinality and bounding constraints. Crama and Schyns [21] applied SA to a portfolio problem with cardinality constraints, turnover and trading restrictions, etc. Lin and Liu [22] used genetic algorithms to solve portfolio problem with minimum transaction lots. Soleimani et al. [23] proposed an improved GA to solve portfolio selection model with minimum transaction lots, cardinality constraints and market capitalization. Chen and Zhang [24] proposed an improved PSO algorithm for the admissible portfolio selection problem with transaction costs. Anagnostopoulos and Mamanis [25] applied NSGA-II, PESA and SPEA2 to find an approximation of the best possible trade offs between return, risk and the number of securities included in the portfolio.

In this paper, we will discuss the portfolio selection problems with transaction costs based on possibilistic theory, and design an effective heuristic algorithm-particle swarm optimization to solve the corresponding optimization problem. The organization of this paper is as follows. In Section 2, some properties as in probability theory based on the Carlsson and Fullérs' notations are discussed. Then, we will present a possibilistic portfolio model with transaction costs in section 3. The outline procedure of the particle swarm optimization method for our proposed model is designed in Section 4. A numerical example is given to illustrate our proposed effective means and approaches in Section 5. Some concluding remarks are given in Section 6.

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II. Possibilistic mean and possibilistic variance

Let us introduce some definitions, which we need in the following section.

In 2001, Carlsson and Fullér [8] defined the notations of the lower and upper possibilistic mean values and variance of A as

$$M_*(A) = \frac{\int_0^1 a_1(\gamma)Pos[A \le a_1(\gamma)]d\gamma}{\int_0^1 Pos[A \le a_1(\gamma)]d\gamma} = 2\int_0^1 \gamma a_1(\gamma)d\gamma,$$

$$M^*(A) = \frac{\int_0^1 a_2(\gamma)Pos[A \ge a_2(\gamma)]d\gamma}{\int_0^1 Pos[A \ge a_2(\gamma)]d\gamma} = 2\int_0^1 \gamma a_2(\gamma)d\gamma.$$

Then the following lemma can directly be proved using the definition of interval-valued possibilistic mean.

Lemma 1: Let $A_1, A_2, \ldots, A_n$ be n fuzzy numbers, and let λ is a real number. Then

$$M_*(\sum_{i=1}^n A_i) = \sum_{i=1}^n M_*(A_i),$$

$$M^*(\sum_{i=1}^n A_i) = \sum_{i=1}^n M^*(A_i),$$

$$M_*(\lambda A_i) = \begin{cases} \lambda M_*(A_i) & \text{if } \lambda \ge 0, \\ \lambda M^*(A_i) & \text{if } \lambda < 0, \end{cases}$$

$$M^*(\lambda A_i) = \begin{cases} \lambda M^*(A_i) & \text{if } \lambda \ge 0, \\ \lambda M_*(A_i) & \text{if } \lambda < 0. \end{cases}$$

The following theorem obviously holds by Lemma 1.

Theorem 1: Let $A_1, A_2, \ldots, A_n$ be n fuzzy numbers, and let $\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_n$ be $n+1$ real numbers. Then

$$M_*(\lambda_0 + \sum_{i=1}^n \lambda_i A_i) = \lambda_0 + \sum_{i=1}^n |\lambda_i| M_*(sgn(\lambda_i)A_i),$$

and

$$M^*(\lambda_0 + \sum_{i=1}^n \lambda_i A_i) = \lambda_0 + \sum_{i=1}^n |\lambda_i| M^*(sgn(\lambda_i)A_i),$$

where sgn(x) is sign function of $x \in \mathcal{R}$.

Carlsson and Fullér [8] also defined the crisp possibilistic mean value of A as

$$M(A) = \int_0^1 \gamma(a_1(\gamma) + a_2(\gamma))d\gamma = \frac{M_*(A) + M^*(A)}{2}.$$

The following theorem can easily be obtained from Theorem 1.

Theorem 2: Let $A_1, A_2, \ldots, A_n$ be n fuzzy numbers, and let $\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_n$ be $n+1$ real numbers. Then

$$M(\lambda_0 + \sum_{i=1}^n \lambda_i A_i) = \lambda_0 + \sum_{i=1}^n \lambda_i M(A_i).$$

Let A with $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ and B with $[B]^\gamma = [b_1(\gamma), b_2(\gamma)] (\gamma \in [0,1])$ be two fuzzy numbers. Carlsson and Fullér [8] also introduced possibilistic variance and covariance of fuzzy numbers as

$$\mathrm{Var}(A) = \frac{1}{2}\int_0^1 \gamma(a_2(\gamma) - a_1(\gamma))^2 d\gamma$$

and

$$\mathrm{Cov}(A,B) = \frac{1}{2}\int_0^1 \gamma[(a_2(\gamma) - a_1(\gamma))(b_2(\gamma) - b_1(\gamma))]d\gamma,$$

respectively.

The following conclusions are given in [8].

Lemma 2: Let A and B be two fuzzy numbers. Then

$$\mathrm{Var}(\lambda A + \mu B) = \lambda^2 \mathrm{Var}(A) + \mu^2 \mathrm{Var}(B) + 2|\lambda\mu|\mathrm{Cov}(A,B).$$

Theorem 3 obviously holds by Lemma 2.

Theorem 3: Let $A_1, A_2, \ldots, A_n$ be n fuzzy numbers, and let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be n real numbers. Then

$$\mathrm{Var}(\sum_{i=1}^n \lambda_i A_i) = \sum_{i=1}^n \lambda_i^2 \mathrm{Var}(A_i) + 2\sum_{i<j=1}^n |\lambda_i\lambda_j|\mathrm{Cov}(A_i, A_j).$$

III. Possibilistic Portfolio Selecion Model with Transaction Costs

We consider a portfolio selection problem with n risky assets. Let $\bar{r}_j$ for asset j is a random variable with expected return $r_j = E(\bar{r}_j), j = 1, \ldots, n$, and let x_j is the proportion of capital to be invested in asset j. In order to describe conveniently, we set $\mathbf{x} = (x_1, x_2, \ldots, x_n)'$, $\bar{\mathbf{r}} = (\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n)'$, $\mathbf{r} = (r_1, r_2, \ldots, r_n)'$, and $\mathbf{e} = (1, 1, \ldots, 1)'$. Then the expected return and variance associated with the portfolio $\mathbf{x} = (x_1, x_2, \ldots, x_n)'$ are, respectively, given by

$$E(\bar{\mathbf{r}}) = \mathbf{r}'\mathbf{x}, \qquad D(\bar{\mathbf{r}}) = \mathbf{x}'\mathbf{v}\mathbf{x}$$

where $\mathbf{v} = (\sigma_{ij})_{n\times n}$ is the covariance matrix of expected returns.

Transaction cost is an important factor for an investor to take into consideration in portfolio selection. In this study, we consider portfolio selection problem with transaction costs. We assume the transaction costs is a $\mathbf{V}$-shaped function of differences between a new portfolio $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and the existing portfolio $\mathbf{x_0} = (x_1^0, x_2^0, \ldots, x_n^0)$. That's to say, for risky asset i the transaction costs $c_i = k_i|x_i - x_i^0|$, the total transaction costs is $\sum_{i=1}^n c_i = \sum_{i=1}^n k_i|x_i - x_i^0|$. So, the expected return without the transaction costs is:

$$\mathbf{r}'\mathbf{x} - \sum_{i=1}^n k_i|x_i - x_i^0|.$$

For a new investor, it can be taken that $x_i^0 = 0, i = 1, 2, \ldots, n$.

Therefore, following the idea of the mean-variance model, portfolio selection problem with transaction costs can be formulated as

$$\begin{aligned} \min \quad & \mathbf{x}'\mathbf{v}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{r}'\mathbf{x} - \sum_{i=1}^n k_i|x_i - x_i^0| \ge \mu, \\ & \mathbf{e}'\mathbf{x} = 1, \\ & \mathbf{x} \ge 0. \end{aligned} \tag{1}$$

In order to apply the model (1) in practical investment problem, we need to estimate $\bar{\mathbf{r}}$ and $\mathbf{v} = (\sigma_{ij})_{n\times n}$. It means that all the expected returns, variances, covariances of risky assets can be accurately estimated by an investor. But, it is well-known that the returns of risky assets are in a fuzzy uncertain economic environment and vary from time to time, the future states of returns and risks of risky assets cannot be predicted accurately. Fuzzy number is a powerful tool used to describe an uncertain environment with vagueness and ambiguity. In many important cases, it might be easier to estimate the possibility distributions of rates of return on risky assets, rather than the corresponding probability distributions. Based on these facts, we discuss the portfolio selection problem under the assumption that the returns of assets are LR-type fuzzy numbers.

Let $\overline{r}_j = (a_j, b_j, \alpha_j, \beta_j)$(j=1,...,n) be n LR-type fuzzy numbers, $\overline{r}_j$ can be described with the following membership function:

$$\overline{r}_j(u) = \begin{cases} L(\frac{a_j-u}{\alpha}) & \text{if } a_j-\alpha_j \le u \le a_j, \\ 1 & \text{if } a_j \le u \le b_j, \\ R(\frac{u-b_j}{\beta}) & \text{if } b_j \le u \le b_j+\beta_j, \\ 0 & \text{otherwise.} \end{cases}$$

where L, R:[0,1]→[0,1] with L(0)=R(0)=1 and L(1)=R(1)=0 are non-increasing, continuous mappings. If L and R are strictly decreasing functions then we can easily compute the γ-level sets of $\overline{r}_j$. That is,

$$[\overline{r}_j]^\gamma = [a_j - \alpha_j L^{-1}(\gamma), b_j + \beta_j R^{-1}(\gamma)],$$

for all $\gamma \in [0.1]$, j=1,...,n.

Using the definitions of the lower and upper possibilistic mean, and crisp possibilistic mean of fuzzy numbers, we easily obtain

$$M_*(\overline{r}_j) = 2\int_0^1 \gamma(a_j - \alpha_j L^{-1}(\gamma))d\gamma = a_j - 2\alpha_j E_L,$$
$$M^*(\overline{r}_j) = 2\int_0^1 \gamma(b_j + \beta_j R^{-1}(\gamma))d\gamma = b_j + 2\beta_j E_R,$$
$$M(\overline{r}_j) = \frac{a_j+b_j}{2} - \alpha_j E_L + \beta_j E_R,$$

where

$$E_L = \int_0^1 \gamma L^{-1}(\gamma)d\gamma,$$
$$E_R = \int_0^1 \gamma R^{-1}(\gamma)d\gamma.$$

Furthermore, using the definitions of the possibilistic variance and covariance of fuzzy numbers, we easily obtain

$$\begin{aligned}\mathrm{Var}(\overline{r}_j) &= \frac{1}{2}\int_0^1 \gamma[b_j + \beta_j R^{-1}(\gamma) - (a_j - \alpha_j L^{-1}(\gamma))]^2 d\gamma, \\ &= \frac{1}{2}(\beta_j^2 F_{RR} + 2\alpha_j\beta_j F_{RL} + \alpha_j^2 F_{LL}) \\ &\quad + (b_j - a_j)(\beta_j E_R + \alpha_j E_L) + \frac{1}{4}(b_j - a_j)^2,\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(\overline{r}_i, \overline{r}_j) &= \frac{1}{2}\int_0^1 \gamma[\beta_i R^{-1}(\gamma) - \alpha_i L^{-1}(\gamma) + b_i - a_i] \\ &\quad \times [\beta_j R^{-1}(\gamma) - \alpha_j L^{-1}(\gamma) + b_j - a_j]d\gamma \\ &= \frac{1}{2}[\beta_i\beta_j F_{RR} + (\alpha_i\beta_j + \alpha_j\beta_i)F_{RL} \\ &\quad + \alpha_i\alpha_j F_{LL} + (b_j - a_j)(\beta_i E_R + \alpha_i E_L) \\ &\quad + (b_i - a_i) \times (\beta_j E_R + \alpha_j E_L)] + \frac{1}{4}(b_i - a_i)(b_j - a_j),\end{aligned}$$

where

$$F_{LL} = \int_0^1 \gamma(L^{-1}(\gamma))^2 d\gamma,$$
$$F_{RR} = \int_0^1 \gamma(R^{-1}(\gamma))^2 d\gamma,$$
$$F_{RL} = \int_0^1 \gamma R^{-1}(\gamma)L^{-1}(\gamma)d\gamma.$$

By Theorem 2, the possibilistic means of the return associated with the portfolio $(x_1, x_2, \ldots, x_n)$ is given by

$$\begin{aligned}M(\mathbf{\overline{r}'x}) &= M(\sum_{i=1}^n \overline{r}_i x_i) \\ &= \sum_{i=1}^n [\frac{a_j+b_j}{2} - \alpha_j E_L + \beta_j E_R]x_i.\end{aligned}$$

By Theorem 3, the possibilistic variance of the return associated with the portfolio $(x_1, x_2, \ldots, x_n)$ is given by

$$\begin{aligned}Var(\mathbf{\overline{r}'x}) &= \frac{1}{2}F_{LL}[\sum_{i=1}^n \alpha_i x_i]^2 + \frac{1}{2}F_{RR}[\sum_{i=1}^n \beta_i x_i]^2 \\ &\quad + \frac{1}{2}F_{RL}[\sum_{i,j=1}^n (\beta_i\alpha_j + \alpha_i\beta_j)x_i x_j] \\ &\quad + \sum_{i=1}^n (b_i - a_i)(\beta_i E_R + \alpha_i E_L)x_i^2 \\ &\quad + \sum_{i\neq j=1}^n (b_i - a_i)(\beta_j E_R + \alpha_j E_L)x_i x_j \\ &\quad + \frac{1}{4}[\sum_{i=1}^n (b_i - a_i)x_i]^2 \\ &= \frac{1}{2}F_{LL}[\sum_{i=1}^n \alpha_i x_i]^2 + \frac{1}{2}F_{RR}[\sum_{i=1}^n \beta_i x_i]^2 \\ &\quad + F_{RL}[\sum_{i=1}^n (\alpha_i x_i)][\sum_{i=1}^n (\beta_i x_i)] \\ &\quad + [\sum_{i=1}^n (b_i - a_i)x_i][\sum_{i=1}^n (\beta_i E_R + \alpha_i E_L)x_i] \\ &\quad + \frac{1}{4}[\sum_{i=1}^n (b_i - a_i)x_i]^2\end{aligned}$$

Analogous to Markowitz's mean-variance methodology for the portfolio selection problem, the possibilistic mean value correspond to the return, while the possibilistic variance correspond to the risk. From this point of view, the portfolio selection model with transaction costs (1) can be formulated

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as

$$
\begin{aligned}
\min \quad & \frac{1}{2}F_{LL}[\sum_{i=1}^{n}\alpha_i x_i]^2 + \frac{1}{2}F_{RR}[\sum_{i=1}^{n}\beta_i x_i]^2 \\
& + F_{RL}[\sum_{i=1}^{n}(\alpha_i x_i)][\sum_{i=1}^{n}(\beta_i x_i)] \\
& + [\sum_{i=1}^{n}(b_i - a_i)x_i][\sum_{i=1}^{n}(\beta_i E_R + \alpha_i E_L)x_i] \\
& + \frac{1}{4}[\sum_{i=1}^{n}(b_i - a_i)x_i]^2 \\
s.t. \quad & \sum_{i=1}^{n}[\frac{a_j + b_j}{2} - \alpha_j E_L + \beta_j E_R]x_i - \sum_{i=1}^{n} k_i|x_i - x_i^0| \geq \mu, \\
& \mathbf{e}'\mathbf{x} = 1, \\
& \mathbf{x} \geq 0.
\end{aligned} \tag{2}
$$

Especially, if $\bar{r}_j, j = 1, \ldots, n$ are symmetric LR-type fuzzy numbers, that's to say, $a_j = b_j$, and $L^{-1} = R^{-1}$, then $E_L = E_R$, $F_{LL} = F_{RR} = F_{RL}$. Therefore, the possibilistic mean-variance model (2) can be described as

$$
\begin{aligned}
\min \quad & \frac{1}{2}F_{LL}[\sum_{i=1}^{n}(\alpha_i + \beta_i)x_i]^2 \\
s.t. \quad & \sum_{i=1}^{n}[\frac{a_j + b_j}{2} + (\beta_i - \alpha_i)E_L]x_i - \sum_{i=1}^{n} k_i|x_i - x_i^0| \geq \mu, \\
& \mathbf{e}'\mathbf{x} = 1, \\
& \mathbf{x} \geq 0.
\end{aligned} \tag{3}
$$

Furthermore, model (3) is equivalent to the following programming problem:

$$
\begin{aligned}
\min \quad & \frac{\sqrt{2}}{2}\sqrt{F_{LL}}\sum_{i=1}^{n}(\alpha_i + \beta_i)x_i \\
s.t. \quad & \sum_{i=1}^{n}[\frac{a_j + b_j}{2} + (\beta_i - \alpha_i)E_L]x_i - \sum_{i=1}^{n} k_i|x_i - x_i^0| \geq \mu, \\
& \mathbf{e}'\mathbf{x} = 1, \\
& \mathbf{x} \geq 0.
\end{aligned} \tag{4}
$$

IV. Particle Swarm Optimization

In this section, a modified PSO algorithm is designed to solve proposed portfolio selection problem with transaction costs.

A. Standard Particle Swarm Optimization

Particle Swarm Optimization (PSO) was first introduced by Kennedy and Eberhart [26] in 1995. PSO has many advantages over other heuristic techniques such that it can be implemented in a few lines of computer code, it requires only primitive mathematical operators, and it has great capability of escaping local optima. PSO conducts search using a population of a random solutions, corresponding to individual. In addition, each potential solution called particles is also assigned a randomized velocity. Each particle in PSO flies in the hyperspace with a velocity which is dynamically adjusted its position according to their own and their neighboring-particles experience, moving toward two points: the best position so far by itself called Pbest and by its neighbor called Gbest at every iteration. The particle swarm optimization concept consists of, at each time step, changing the velocity each particle toward its Pbest and Gbest.

Suppose that the search space is D dimensional, then the ith particle of the swarm can be represented by a D dimensional vector $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})'$. The particle velocity can be represented by another D dimensional vector $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})'$. The best previously visited position of the ith particle is denoted as $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})'$. Defining g as the index of the best particle in the swarm, and let the superscripts denote the iteration number, then the position of a particle and its velocity are updated by the following equations :

$$v_{id}^{k+1} = wv_{id}^k + c_1 r_1^k(p_{id}^k - x_{id}^k) + c_2 r_2^k(p_{gd}^k - x_{id}^k), \tag{5}$$

$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1}, \tag{6}$$

where $d = 1, 2, \ldots, D$, $i = 1, 2, \ldots, N$, and N is the size of swarm; w is called inertia weight: c_1, c_2 are two positive constants, called cognitive and social parameter respectively; r_1, r_2 are random numbers, uniformly distributed in $[0, 1]$; and $k = 1, 2, \ldots$ determines the iteration number.

B. Particle Swarm Optimization for Portfolio Selection Problem

Next, a modified PSO algorithm is designed to solve portfolio selection problem with transaction costs, which is difficult to solve with the existing traditional algorithms due to its nonconcavity and special structure. We design a PSO algorithm from the three aspects.

Firstly, a dynamic inertia weight by stages is designed according to the following equation:

$$w = w_b - \frac{\lfloor \frac{k}{N} \rfloor}{\lfloor \frac{Max}{N} \rfloor} * (w_b - w_e),$$

where w_b is the initial inertia weight, w_e is the final inertia weight, Max is the maximum iteration, k is the current iteration, N is the constant, which means how many stages are divided by Max, $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

This method ensure that w can take a larger value at the begin of searching, that's to say, particle swarms can search in a bigger space, which increase the diversities of solution spaces. Moreover, w can take a smaller value at the end of searching, that's to say, particle swarms can search in a smaller space, which increase the accuracy of solution spaces.

Secondly, in order to avoid rapid converging to local optimization, we design a random divergence method. When particles update positions, every particle changes velocity direction with a smaller probability and fly oppositely. If the particle position is beyond the boundary value, we set position value be boundary value. The random divergence method can be described as follows:

Set $p_m = \varepsilon$
for each particle
$v_{id} = wv_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id});$
if($p_m < rand(0,1)$)
$x_{id} = x_{id} - v_{id};$
else
$x_{id} = x_{id} + v_{id};$
end if
if $(x_{id} > rangR)$ $x_{id} = rangR;$
if $(x_{id} < rangL)$ $x_{id} = rangL;$
end for

where rangR is upper boundary of x_d, rangL is lower boundary of x_d.

At last, that how to deal with constraints is very important, so many methods were proposed. Koziel and Michaewicz [27] grouped them into four categories: methods based on preserving feasibility of solutions; methods based on penalty functions; methods that make a clear distinction between feasible and infeasible solutions; and other hybrid methods.

Exact penalty functions can be subdivided into two main classes: nondifferentiable exact penalty functions and continuously differentiable exact penalty functions. Nondifferentiable exact penalty functions were introduced for the first time by Zangwill [28]. Continuously differentiable exact penalty functions were introduced by Fletcher [29] for equality constrained optimization problems. In this paper, nondifferentiable exact penalty functions [30] is introduced for handling constraints. Its basic form is as follows:

$$p(x,\delta) = f(x) + \delta\{\sum_{i=1}^{J} |c_i(x)| + \sum_{i=J+1}^{K} |min(0, c_i(x))|\}$$

where δ is penalty factor.

V. Numerical example

In order to illustrate our proposed effective means and approaches, we consider a real portfolio selection example in [4]. In this example, 5 bonds whose return rates are restricted by the following type of possibility distributions with a center c_i and a spread w_i:

$$\pi_{C_i}(q) = exp(-\frac{(q-c_i)^2}{w_i}) \tag{7}$$

The parameters c_i's and w_i's are defined as

$$c_1 = 0.25, c_2 = 0.22, c_3 = 0.2, c_4 = 0.15, c_5 = 0.05,$$
$$w_1 = 0.0225, w_2 = 0.015, w_3 = 0.015, w_4 = 0.01, w_5 = 0.005.$$

Using the possibilistic portfolio selection model with transaction costs (4) in this example, it follows that $a_i = b_i = c_i, \alpha_i = \beta_i = \sqrt{w_i}, i = 1, 2, \ldots, 5$, and $L = R$. Based on possibility distribution (8), we can obtain $F_{LL} = F_{RR} = \frac{1}{4}$. Moreover, the following parameters for proposed PSO algorithm are set: the size of the population is 100, the initial inertia weight w_b is 0.75 , the final inertia weight w_e is 0.2, the maximum iteration Max is 2000, $N = 10, \delta = 50$.

Suppose that existing portfolio is $x_0 = (0.03, 0.00, 0.06, 0.00, 0.05)$, and the transaction costs rate $k_i = 0.01, i = 1, 2, \ldots, 5$. Applying the proposed PSO algorithm and LINGO software, respectively, to solve model (5), we obtain the following results listed in Tables 1-6.

Table 1: Investment Proportion with $\mu = 0.02$

	1	2	3	4	5	risk
PSO	0.0000	0.0000	0.0000	0.0000	1.0000	0.0707
LINGO	0.0000	0.0000	0.0000	0.0000	1.0000	0.0707

Table 2: Investment Proportion with $\mu = 0.05$

	1	2	3	4	5	risk
PSO	0.0001	0.0000	0.0007	0.1049	0.8942	0.07383
LINGO	0.0000	0.0000	0.0000	0.0864	0.9136	0.07324

Table 3: Investment Proportion with $\mu = 0.10$

	1	2	3	4	5	risk
PSO	0.0011	0.0065	0.0051	0.5833	0.4040	0.08848
LINGO	0.0000	0.0580	0.0000	0.4460	0.4960	0.08678

Table 4: Investment Proportion with $\mu = 0.15$

	1	2	3	4	5	risk
PSO	0.0053	0.1778	0.0143	0.7694	0.0332	0.1036
LINGO	0.0000	0.3846	0.0000	0.3846	0.2308	0.1019

Table 5: Investment Proportion with $\mu = 0.20$

	1	2	3	4	5	risk
PSO	0.0000	0.9440	0.0000	0.0000	0.056	0.1196
LINGO	0.0000	0.8333	0.0000	0.1667	0.0000	0.1187

Table 6: Investment Proportion with $\mu = 0.24$

	1	2	3	4	5	risk
PSO	1.0000	0.0000	0.0000	0.0000	0.0000	0.1500
LINGO	0.9850	0.0150	0.0000	0.0000	0.0000	0.1496

Tables 1-6 exhibit the numerical results computed by PSO and LINGO software respectively. According to the experimental results, we conclude that for proposed portfolio selection problem the PSO algorithm is an efficient method to obtain the optimal solution since the results obtained by two different methods are approximately equal, that's to say, for the same possibilistic return the possibilistic risks are approximately equal. However, there is a small difference in the obtained portfolios in both methods. For example, in the case of $\mu = 0.05, 0.15$, LINGO has three bonds while PSO has five bonds, which means that the results of PSO tend to take more distributive investment than those of LINGO.

VI. Conclusions

In this paper, we have proposed possibilistic portfolio model with transaction costs under the assumption that the returns of assets are LR-type fuzzy numbers, in which the possibilistic mean value of the return is termed measure of investment return, and the possibilistic variance of the return is termed measure of investment risk. We have designed a particle swarm optimization to solve corresponding optimization problem because traditional algorithms can not solve this problem efficiently. Results of numerical experiments show that our proposed means and approaches are effective.

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