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Performance Improvement in the Bivariate Models by using Modified Marginal Variance of Noisy Observations for Image-Denoising Applications

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Abstract**—** Most simple nonlinear thresholding rules for wavelet- based denoising assume that the wavelet coefficients are independent. However, wavelet coefficients of natural images have significant dependencies. This paper attempts to give a recipe for selecting one of the popular image-denoising algorithms based on VisuShrink, SureShrink, OracleShrink, BayesShrink and BiShrink and also this paper compares different Bivariate models used for image denoising applications. The first part of the paper compares different Shrinkage functions used for image-denoising. The second part of the paper compares different bivariate models and the third part of this paper uses the Bivariate model with modified marginal variance which is based on Laplacian assumption. This paper gives an experimental comparison on six 512x512 commonly used images, Lenna, Barbara, Goldhill, Clown, Boat and Stonehenge. The following noise powers 25dB, 26dB, 27dB, 28dB and 29dB are added to the six standard images and the corresponding Peak Signal to Noise Ratio (PSNR) values are calculated for each noise level.

Keywords**—**BiShrink, Image-Denoising, PSNR, Shrinkage function.

I. INTRODUCTION

AN image is corrupted by noise in its acquisition or transmission. The goal of denoising is to remove the noise while retaining as much as possible the important signal features. Traditionally, this is achieved by linear processing but for the past few years image- denoising has been done using nonlinear techniques. A simple denoising algorithm that uses the wavelet transform consist of the following three steps,

(1)Calculate the wavelet transform of the noisy signal
(2) Modify the noisy wavelet coefficients according to some rule.
(3) Compute the inverse transform using the modified coefficients.

One of the most well-known rules for the second step is soft thresholding analyzed by Donoho [1, 2]. Due to its effectiveness and simplicity, it is frequently used in the literature. The main idea is to subtract threshold value 'T' from all coefficients larger than 'T' and to set all other coefficients to zero. Alternative approaches can be found in, for example, [2],[3],[4] and [5]. VisuShrink[1] uses one of the well-known thresholding rules: the Universal threshold. In addition, subband adaptive systems have superior performance, such as SureShrink[3], which is a data-driven system. Recently, bayesShrink[5], which is also a data-driven subband adaptive technique, is proposed and outperforms VisuShrink and sometimes SureShrink.

The organization of this paper is as follows. In Section II, the basic idea of image–denoising using VisuShrink[2], SureShrink[3], BayesShrink[5] and OracleShrink[5] are described briefly. The PSNR values obtained for six denoised images for different algorithms are compared with Bivaraite Model 1. These models try to capture the dependencies between a coefficient and its parent. Section III, compares the Bivariate Model 1 and Bivariate Model 3. Then, Section IV describes the Bivariate model with modified marginal variance which use the Laplacian assumption.

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II. COMPARISON OF BIVARIATE MODEL WITH OTHER IMAGE-DENOISING MODELS

The following subsections explain different image denoising shrinkage functions

A. VisuShrink

For image denoising, VisuShrink [1] (Visually calibrated adaptive smoothing) is known to yield smoothed images. The threshold choice for VisuShrink is $\sigma\sqrt{2\log M}$ (called the universal threshold and σ^2 is the noise variance).

The soft-threshold function [2] (also called the Shrinkage function),

$$\eta_T(x) = \text{sgn}(x).\max(|x| - T, 0) \qquad (1)$$

takes the argument and shrinks it toward zero by the threshold 'T'.

B. SureShrink

The SureShrink[3] uses a hybrid of the universal threshold and the SURE threshold, derived from minimizing stein's unbiased risk estimator [4], and has shown to perform well. The SURE threshold choice is dependent on the energy of the particular sub band [3].

The threshold on subband S to be used with a soft shrinkage function is,

$$T_S{}^S = \arg\min_{T\geq 0}[SURE^S(T, Y_S)] \quad (2)$$

$$SURE^S(T, Y_S) = N_S + \sum_{K=1}^{Ns}[\min(|Y_K|, T)]^2 - 2[1:N_S] \quad (3)$$

where,
Y_S is the detail coefficients from subband S and N_S is the number of coefficients Y_K in $\{Y_S\}$.

C. OracleShrink

The OracleShrink, which is truly optimal soft-thresholding estimator assuming the original image is known. The threshold of OracleShrink in each sub band is

$$T_{OS} = \arg\min_T \sum_{i,j=1}^{n}(\eta_T(Y_{ij}) - X_{ij})^2 \quad (4)$$

with X_{ij} known.

D. BayesShrink

The BayesShrink rule uses a Bayesian mathematical framework for images to device subband dependent thresholds that are nearly optimal for soft threshold. The formula for the threshold on a given subband is [5],

$$T_S = \frac{\hat{\sigma}_n^2}{\hat{\sigma}_X} \quad (5)$$

where

$\hat{\sigma}_n^2$ is the estimated noise variance and $\hat{\sigma}_X^2$ is the estimated signal variance on the sub band considered.

The noise variance is estimated as the median of the absolute deviation of the diagonal detail coefficients on the finest level (i.e., subband HH1).

The estimate of the signal standard deviation on subband S is

$$\hat{\sigma}_X = \sqrt{\max(\hat{\sigma}_Y^2 - \hat{\sigma}_n^2, 0)} \quad (6)$$

where

$\hat{\sigma}_Y^2 = \frac{1}{N_S}\sum_{k=1}^{Ns} Y_K^2$ is an estimate of the variance of the observations, with N_S being the number of the wavelet coefficients Y_K on the subband under consideration. Combining equ. (5) and equ. (6), the threshold choice for BayesShrin*k* is

$$T_S = \hat{\sigma}_n^2 \big/ \hat{\sigma}_X, \quad \hat{\sigma}_n^2 < \hat{\sigma}_X^2$$
$$T_S = \max(|Y_K|), \quad \hat{\sigma}_n^2 \geq \hat{\sigma}_X^2 \quad (7)$$

E. Bivariate Model 1

Let X_{2K} represents the parent of X_{1K} (X_{2K} is the wavelet coefficient at the same position as the K th wavelet coefficient X_K, but at the next coarser scale). The problem formulated in the wavelet domain as $Y_{1K} = X_{1K} + n_{1K}$ and $Y_{2K} = X_{2K} + n_{2K}$ to take into account the statistical dependencies between a coefficient and its parent. Y_{1k} and Y_{2K} are noisy observations of X_{1K} and X_{2K} also n1k and n_{2K} are the noise samples.

The estimator of X_1 is [7]

$$\hat{X}_1 = \frac{\left(\sqrt{Y_1^2 + Y_2^2} - \sqrt{3}\frac{\sigma_n^2}{\sigma_X}\right)}{\sqrt{Y_1^2 + Y_2^2}}_+ \cdot Y_1 \quad (8)$$

which can be interpreted as a bivariate shrinkage function. Here (g) + is defined as [6]

$$(g)_+ = \begin{cases} 0, & \text{if } g < 0 \\ g, & \text{otherwise} \end{cases} \quad (9)$$

This estimator requires the prior knowledge of the noise variance and the marginal variance for each wavelet coefficient [8].

To estimate the noise variance σ_n^2 from the noisy wavelet coefficients, a robust median estimator is used from the finest scale wavelet coefficients [2].

$$\hat{\sigma}_n = \frac{median(|Y_i|)}{0.6745}, Y_i \quad \varepsilon \quad subband\ HH \quad (10)$$

The algorithm is summarized as follows:

1) Calculate the noise variance using (10).
2) Calculate $\hat{\sigma}_Y^2$ given in subsection 2.4.
3) Calculate $\hat{\sigma}_X^2$ using (6).
4) Estimate each coefficient using $\hat{\sigma}_X$ and σ_n^2 in (8).

Fig. 1 and Table I show that the BiShrink shrinkage function which gives better PSNR (Peak Signal to Noise Ratio) when compared to the models described in subsections A,B, C and D.

(a) (b)

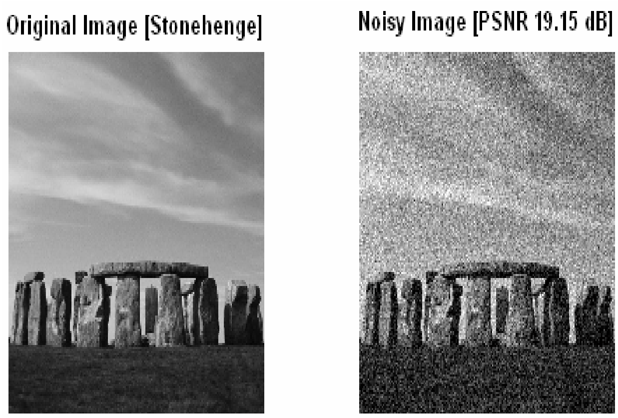


Fig. 1. (a) Original image, (b) Noisy image with PSNR 19.15dB. Power of Noise added 29dB.

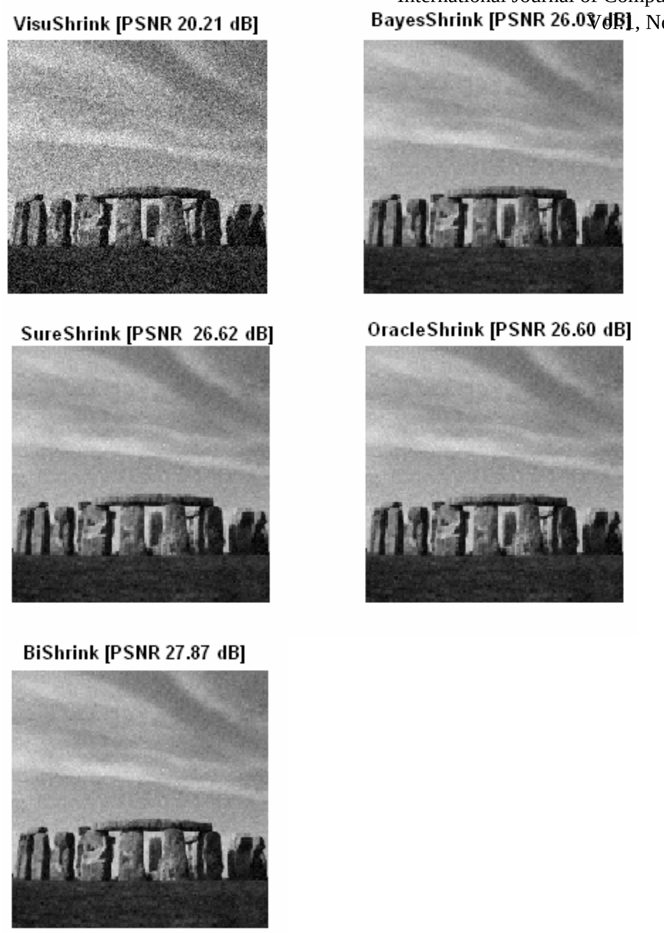


Fig.1.(c) Comparison of denoised stonehenge image obtained for different shrinkage models

III. Comparison of Different Bivariate Models

The Bivariate Model 1 already described in section II. The algorithm for Bivariate Model 3 is summarized as follows:

1) Calculate the noise variance using (10).
2) For each subband,
   (a) Calculate $\hat{\sigma}_{Y1}^{2}$ and $\hat{\sigma}_{Y2}^{2}$ using (11) and (12);
   (b) Calculate $\hat{\sigma}_{1}^{2}$ and $\hat{\sigma}_{2}^{2}$ using (13) and (14);
   (c) Estimate each coefficient using the successive approximation method.

$$\hat{\sigma}_{Y1}^{2} = \frac{1}{N_S}\sum_{k=1}^{Ns} Y_{k1}^{2} \qquad (11)$$

$$\hat{\sigma}_{Y2}^{2} = \frac{1}{N_S}\sum_{k=1}^{Ns} Y_{k2}^{2} \qquad (12)$$

$$\hat{\sigma}_1 = \sqrt{\max(\hat{\sigma}_{Y1}^{2} - \hat{\sigma}_n^{2}, 0)} \qquad (13)$$

$$\hat{\sigma}_2 = \sqrt{\max(\hat{\sigma}_{Y2}^{2} - \hat{\sigma}_n^{2}, 0)} \qquad (14)$$

$$\hat{X}_1 = \frac{\left(\sqrt{Y_1^2 + Y_2^2} - \sqrt{3}\,\dfrac{\sigma_n^2}{\sigma_1^2 r}\right)}{\sqrt{Y_1^2 + Y_2^2}} + \cdot Y_1 \qquad (15)$$

$$r = \sqrt{\left(\frac{\hat{X}_1}{\hat{\sigma}_1}\right)^2 + \left(\frac{\hat{X}_2}{\hat{\sigma}_2}\right)^2} \qquad (16)$$

Model 3 outperforms Model 1 in most of the cases. Four standard images are taken and the different bivariate models are applied to these test images. The PSNR values obtained are tabulated in table II.

IV. Model 1 with Laplacian Assumption

In our experiments, we obtained better PSNR values with Bivariate Model 1if we use Laplacian assumption. The corresponding marginal variance of noisy observations is given by [9],

$$\hat{\sigma}_Y^{2} = \frac{\sqrt{2}}{N_S}\sum_{k=1}^{Ns} Y_k^{2} \qquad (17)$$

Fig. 2 and Fig. 3 compare the Bivariate Model 1, Bivariate Model 3 and Bivariate Model 1 with Laplacian assumption.

Table II shows that the Bivariate Model 1 with Laplacian assumption outperforms Bivariate Model 1 and Bivariate Model 3.

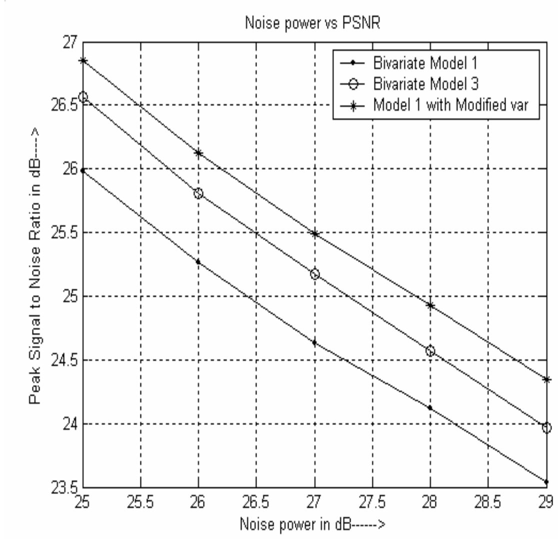
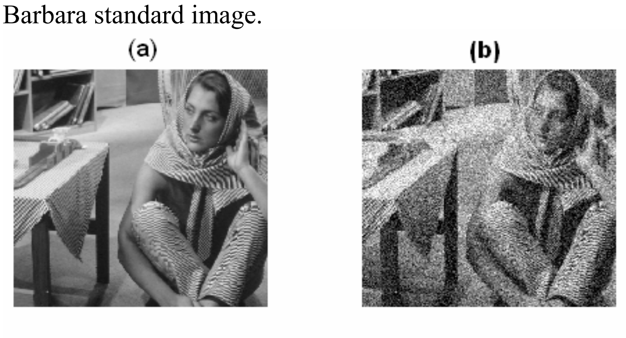
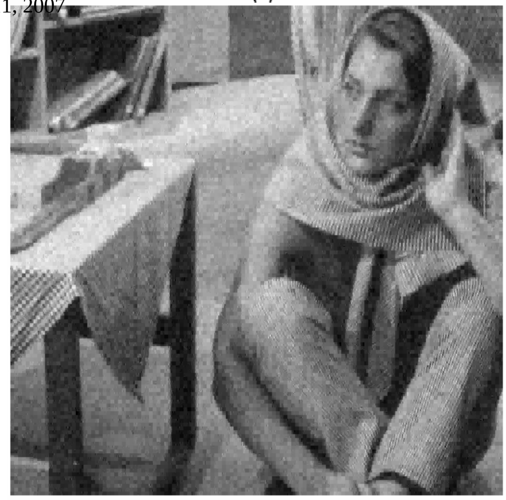


Fig.2 comparison of different Bivariate Models using Barbara standard image.



(c)

(e)



(d)

Fig.3.(a) Original image (b) Noisy image with PSNR = 18.15 dB (c) Model 1with PSNR = 23.02 dB (d) Model 3 with PSNR = 23.43 dB (e) Model 1 with Laplacian assumption PSNR = 23.77 dB.

TABLE I
FOR VARIOUS TEST IMAGES AND P VALUES, LISTS PSNR OF (1)VisuShrink, (2) SureShrink, (3) OracleShrink (4) BayesShrink and (5) BiShrink

	Noisy	VisuShrink	SureShrink	OracleShrink	BayesShrink	BiShrink
Lena						
P=25dB	23.14	24.71	26.81	26.81	26.31	29.20
P=26dB	22.13	23.54	25.55	26.56	26.17	28.64
Clown						
P=25dB	23.14	24.65	25.30	25.41	24.66	28.90
P=26dB	22.13	23.50	25.67	25.13	24.56	28.08
Boat						
P=25dB	23.14	24.62	24.88	24.88	24.23	28.05
P=26dB	22.13	23.48	24.68	24.68	24.15	27.45
Goldhill						
P=25dB	23.14	24.64	26.24	26.23	26.03	28.20
P=26dB	22.13	23.50	26.06	26.05	25.90	27.66

TABLE -II
FOR VARIOUS TEST IMAGES AND P VALUES, LISTS PSNR OF (1) Model 1, (2) Model 3, (3) Model 1 with Laplacian Assumption

	Noisy PSNR (dB)	Model 1 PSNR (dB)	Model 3 PSNR (dB)	Model 1with Laplacian Assumption PSNR(dB)
Lena				
P=27dB	21.15	28.05	28.36	28.51
P=28dB	20.14	27.52	27.75	27.89
Barbara				
P=27dB	21.15	24.63	25.17	25.49
P=28dB	20.14	24.12	24.57	24.93
Boat				
P=27dB	21.15	24.63	25.17	25.49
P=28dB	20.14	24.12	24.57	24.93
Goldhill				
P=27dB	21.15	27.20	27.48	27.64
P=28dB	20.14	26.64	26.66	27.07

V. CONCLUSION

In this paper the different shrinkage functions used for image denoising were discussed and the results were verified experimentally using MATLAB 6.5 image processing toolbox. In order to show the effectiveness of the BiShrink estimator, six examples were presented and compared with other effective techniques. The improved PSNR was obtained in the BayesShrink estimator, when simultaneous denoising and compression is used [5]. In BiShrink estimator, in addition to the orthogonal wavelet transform, dual-tree Complex Wavelet transform (CWT) [7, 8] is used to improve PSNR.

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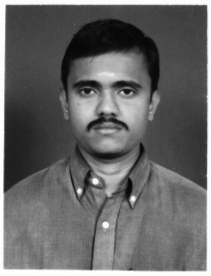

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