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The Application of Hadamard Matrixes in the SNR Enhancement of Optical Time-Domain Reflectometry(OTDR)

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Abstract—Results in one field necessarily give insight into the others, and all have much potential for scientific and technological application. The Hadamard-transform technique once been applied to the spectrometry also has its use in the SNR Enhancement of OTDR. In this report, a new set of code (Simplex-codes) is discussed and where the addition gain of SNR come from is implied.

***Keywords*—**Hadamard-transform, matrixes, averaging, optical time-domain reflectometry (OTDR).

I. Introduction

OPTICAL time-domain reflectometry is an effective tool to characterize optical fibers. By infecting an optical pulse into the fiber under test and detecting the backscattered optical signals (including Rayleigh backscattering and Fresnel reflections), we can reveal the character of the fiber and even indirectly the environment surrounding it.

As the fiber extents further, the received signal becomes much weaker and immersed in the background noises. To increase its dynamic range, an effective way to enhance the SNR of OTDR is demanding. Two methods can be conceived: One is to increase the power of input pulse by widen the pulsewidth of probe pulse. The second is to decrease the variance σ^2 of errors (under the assumption that the noise are uncorrelated zero-mean random processes with σ^2).

The first way is not effective because of the unavoidable trade-off between SNR and spatial resolution (1)

$$L = \frac{\frac{C}{n} \times T}{2} = \frac{CT}{2n} \quad (1)$$

If the pulsewidth is 500ns and n=1.5, the spatial resolution of the OTDR is L=50m

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To decrease the σ^2 of errors, the conventional methods is to repeat the test and average the result. In recent years, various coding technique [1], [2] were employed, however, experimental demonstration of SNR improvement using the code technique has been limited to the complementary correlation OTDR based on the Golay Codes, which suffers from the resolution penalty (or an equivalent 3-dB reduction in the SNR improvement at the same level of spatial resolution) inherently associated with the decoding process.

II. SNR Gain from Averaging

The received signal can be denoted as (2). $\Psi(t)$ is the ideal signal and $e(t)$ is noise, which is uncorrelated, zero-mean and random. ($D(e(t)) = \sigma^2 \quad E(e(t)) = 0$)

$$\eta(t) = \Psi(t) + e(t) \quad (2)$$

Then, by averaging $\eta(t)$: $\langle \eta(t) \rangle = \Psi(t) + \frac{1}{n}\sum_{i=1}^{n} e_i(t)$, the variance of the result is

$$\begin{aligned} D(\langle \eta(t) \rangle) &= D(\Psi(t) + \frac{1}{n}\sum_{i=1}^{n} e_i(t)) \\ &= D(\frac{1}{n}\sum_{i=1}^{n} e_i(t)) = \frac{n}{n^2} D(e(t)) = \frac{1}{n}\sigma^2 \end{aligned} \quad (3)$$

By averaging the result, the noise amplitude is reduced by $\frac{1}{\sqrt{n}}$

III. Hadamard-Transform and S-Matrixes

The S Matrix has been used for noise reduction in optical spectrometry [3]. Briefly, the S Matrix is a unipolar matrix composed of 1's and 0's, and the rows of this matrix are called scs. This matrix can be also derived from a normalized Hadamard matrix, a bipolar matrix composed of 1's and -1's. The operation using Hadamard or S matrix is called the Hadamard transform. Traditionally, the Hadamard-transform technique has been applied to the spectrometry by using a spatial mask having holes and blocks.

A. Coding Process

The coding process is to lunch several pulses into the fiber as the code sequence. And the received signal

is $$\begin{pmatrix}\eta_1(t)\\ \eta_2(t)\\ \vdots\\ \eta_n(t)\end{pmatrix} = S\begin{pmatrix}\Psi_1(t)\\ \Psi_2(t)\\ \vdots\\ \Psi_n(t)\end{pmatrix} + \begin{pmatrix}e_1(t)\\ e_2(t)\\ \vdots\\ e_n(t)\end{pmatrix} \quad (4),$$

$\Psi_l(t)$ is the ideal conventional trace of a single pulse, e(t) is the noise. We should note that $\Psi_i(t)$ and $\Psi_{i+1}(t)$ are different only with a time delay τ .

B. Decoding Process

From (4), it is easy to derived the following expression

$$\begin{pmatrix}\Psi_1\hat{}(t)\\ \Psi_2\hat{}(t)\\ \vdots\\ \Psi_n\hat{}(t)\end{pmatrix} = S^{-1}{}_l\begin{pmatrix}\eta_1(t)\\ \eta_2(t)\\ \vdots\\ \eta_n(t)\end{pmatrix} = \begin{pmatrix}\Psi_1(t)\\ \Psi_2(t)\\ \vdots\\ \Psi_n(t)\end{pmatrix} + \begin{pmatrix}e_1(t)\\ e_2(t)\\ \vdots\\ e_n(t)\end{pmatrix} \quad (5)$$

$\Psi_i\hat{}(t)$ is the estimate of $\Psi_i(t)$, it can be calculated by the received signals of previously lunched codes by multiplying S_l^{-1}. (6)

$$\Psi_i\hat{}(t) = \Psi_i(t) + \sum_{k=1}^{l} S^{-1}{}_{ik} e_k(t) \quad (6)$$

$$D(\Psi_i\hat{}(t)) = D(\sum_{k=1}^{n} S^{-1}{}_{ik} e_k(t)) = (\frac{2}{n+1})^2 N\sigma^2$$
$$= \frac{4n}{(n+1)^2}\sigma^2 \quad (7)$$
$$(S^{-1} = \frac{2}{n+1}T \ ,\ T_{ij} = 1,-1(0<i, j<=n))$$

In the code-decode process, the SNR gain is $\frac{N+1}{2\sqrt{N}}$ (Fig. 1).

C. Final Results

To obtain a conventional trace of greater SNR, we just need to inversely time-shifting each row in (5) with $(i-1)\tau$ and average each row.

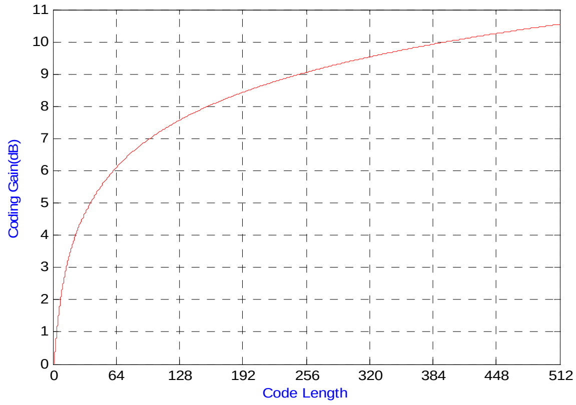

Fig. 1 SNR gain in the code-decode process

$$\begin{pmatrix}\Psi_1\hat{}(t)\\ \Psi_2\hat{}(t+\tau)\\ \vdots\\ \Psi_n\hat{}(t+(n-1)\tau)\end{pmatrix} = S^{-1}{}_n\begin{pmatrix}\eta_1(t)\\ \eta_2(t+\tau)\\ \vdots\\ \eta_n(t+(n-1)\tau)\end{pmatrix} = \begin{pmatrix}\Psi_1(t)\\ \Psi_2(t+\tau)\\ \vdots\\ \Psi_n(t+(n-1)\tau)\end{pmatrix}$$
$$+ S_n^{-1}\begin{pmatrix}e_1(t)\\ e_2(t+\tau)\\ \vdots\\ e_n(t+(n-1)\tau)\end{pmatrix} = \begin{pmatrix}\Psi_1(t)\\ \Psi_1(t)\\ \vdots\\ \Psi_1(t)\end{pmatrix} + S_n^{-1}\begin{pmatrix}e_1(t)\\ e_2(t+\tau)\\ \vdots\\ e_n(t+(n-1)\tau)\end{pmatrix} \quad (8)$$

$$\Psi_i\hat{}(t+(i-1)\tau) = \Psi_1 + \sum_{k=1}^{l} S^{-1}{}_{ik} e_k(t+(i-1)\tau)$$

$$\frac{1}{n}\sum_{i=1}^{n} \Psi_i\hat{}(t+(i-1)\tau) = \Psi_1 +$$
$$\frac{2}{n(n+1)}\sum_{j=1}^{n}\sum_{k=1}^{n} T_{jk} e_k(t+(i-1)\tau)$$

So, $D(\frac{2}{n(n+1)}\sum_{j=1}^{n}\sum_{k=1}^{n} T_{jk} e_k(t+(i-1)\tau)) = \frac{4}{(n+1)}\sigma^2$

The SNR gain is $\frac{n+1}{2}$, equal to $\frac{1}{\sqrt{n}} \times \frac{N+1}{2\sqrt{n}}$.

Note that the above results are derived under the following assumptions.

$$\begin{aligned} &E\{e_i(t+\xi)\} = 0\\ &E\{e_i{}^2(t+\xi)\} = \sigma^2\\ &E\{e_i(t)e_j(t+\xi)\} = 0\\ &E\{e_i(t)e_i(t+\xi)\} = 0 \end{aligned} \quad (9)$$

D. Illustrated Process

TABLE I
ILLUSTRATED PROCESS

S-codes ⇨ 
Optical-fiber
Receiver
$\begin{pmatrix}\Psi_1(t)\\ \ldots \\ \Psi_n(t)\end{pmatrix} = \begin{pmatrix} a_{11} & \cdots & a_{1n}\\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn}\end{pmatrix}$
$\begin{pmatrix}\Psi_1(t)\\ \ldots \\ \Psi_n(t+(n-1)\tau)\end{pmatrix} = \begin{pmatrix} 000\ldots a_{11}a_{12}a_{13}\ldots a_{1l}\\ 00\ldots a_{21}a_{22}a_{23}\ldots a_{2l}..0 \\ \vdots \\ 0\ldots a_{l1}a_{l2}a_{l3}\ldots a_{ll}\ldots 00\end{pmatrix}$
Averaging 

IV. FURTHER DISCUSSION

S-codes have two advantages: first, codes of length 2P-are easily generated as shift register sequences. Second, even if the pulse codes that are launched have different amplitudes, the true reflectivity may still be obtained.
If so, (8) is changed to

$$\begin{pmatrix}\Psi_1{}^{\wedge}(t)\\ \Psi_2{}^{\wedge}(t+\tau)\\ \vdots \\ \Psi_n{}^{\wedge}(t+(n-1)\tau)\end{pmatrix} = S^{-1}{}_n \begin{pmatrix}\eta_1(t)\\ \eta_2(t+\tau)\\ \vdots \\ \eta_n(t+(n-1)\tau)\end{pmatrix}$$

$$= \begin{pmatrix} m_1\Psi_1(t)\\ m_2\Psi_2(t+\tau)\\ \vdots \\ m_n\Psi_n(t+(n-1)\tau)\end{pmatrix} + S_n^{-1}\begin{pmatrix} e_1(t)\\ e_2(t+\tau)\\ \vdots \\ e_n(t+(n-1)\tau)\end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} m_1\Psi_1(t)\\ m_2\Psi_1(t)\\ \vdots \\ m_n\Psi_1(t)\end{pmatrix} + S_n^{-1}\begin{pmatrix} e_1(t)\\ e_2(t+\tau)\\ \vdots \\ e_n(t+(n-1)\tau)\end{pmatrix}$$

$$\frac{1}{n}\sum_{i=1}^{n}\Psi_i{}^{\wedge}(t+(i-1)\tau) = m\Psi_1 + \frac{2}{n(n+1)}\sum_{j=1}^{n}\sum_{k=1}^{n}T_{jk}e_k(t+(i-1)\tau)$$

Actually, m is the average of m_i ,and m could be 1.

V. CONCLUSION

The HADAMARD-technique is analyzed in respect of its power for noise reduction of OTDR. We deduced in step by step how a suitable noise reduction (gain) is possible. And we suggest two major advantages of HADAMARD-technique, and strongly recommend the application of this technique to related field.

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