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Evaluation of Coupling Factor in RF Inductively Coupled Systems

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***Abstract*—**This work presents an approach for the measurement of mutual inductance on near field inductive coupling. The mutual inductance between inductive circuits allows the simulation of energy transfer from reader to tag, that can be used in RFID and powerless implantable devices. It also allows one to predict the maximum voltage in the tag of the radio-frequency system.

***Keywords*—**RFID, Inductive Coupling, Energy Transfer, Implantable Device

I. INTRODUCTION

THE demand for Radiofrequency – RF applications is constantly increasing. Particularly, Radio-Frequency Identification – RFID systems finds applications in safety, transportation and most recently in health, among other areas. Therefore the Near-Field Communication is becoming widely used, thus demanding full knowledge of the behavior and interactions of tuned circuits. As a result, this work presents an approach to measure the mutual inductance between the magnetically coupled circuits. Based on the mutual inductance of near inductors it is possible to obtain the voltage at the passive tag and to predict the range of the radio-frequency system. Fig. 1 presents the simplified coupling between reader and tag [3] [7] [8] [10] [11] [12] [15] and [16], where the power is sent by the reader to the tag by RF coupling. This configuration was chosen to raise the voltage applied to the inductor, since its voltage is higher than the generator voltage V_1, at the resonant frequency.

The voltage increase is due to increase caused by the quality factor Q of the series resonant circuit, which multiplies the voltage generator V_1. The reader circuit alone can be modeled as (1).

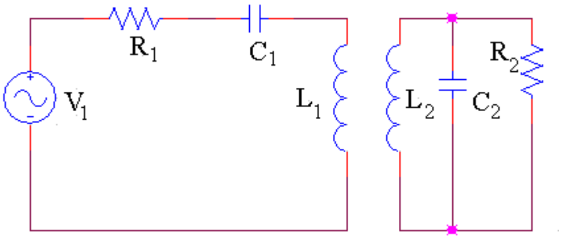

Fig. 1 Simplified coupling between reader and tag

The voltage increase is due to increase caused by the quality factor Q of the series resonant circuit, which multiplies the voltage generator V_1. The reader circuit alone can be modeled as (1).

$$Z = R_1 + (j\omega L_1 - j\frac{1}{\omega C_1}) \tag{1}$$

At the resonant frequency:

$$\omega L_1 = \frac{1}{\omega C_1}. \tag{2}$$

Therefore the impedance Z becomes resistive and equals to R, which represents the inductor series resistance. In this way, the current at the reader circuit presented in Fig. 1 will be dependent on resistance R1, and the voltage at the inductor will be given by (3).

$$E_1 = jI\omega L_1 = j\frac{V_1\omega L_1}{R_1} = jQV_1. \tag{3}$$

At this point it is important to define the quality factor $Q=wL_1/R_1$ of resonant circuit, which represents the ratio between the stored and dissipated energy at an inductor. This result shows that the reader efficiency is increased by the primary series resonance. Consequently, the reader efficiency is optimized by using series resonance.

$$|E_1| = |V_1|Q \tag{4}$$

II. MUTUAL INDUCTANCE MEASUREMENT METHOD

According to [1] and [2], the mutual inductance between two inductors can be measured as presented next. Consider two circuits linked by inductors L_1 and L_2 as shown in Fig. 2. The inductance seen on the generator is given as:

$$L_s' = L_1 + M_{12} + M_{21} \tag{5}$$

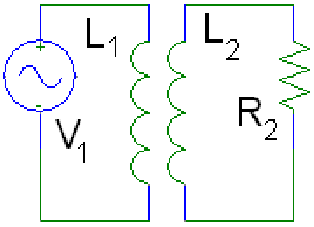

Fig. 2 Circuits linked by inductors

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Thus the total inductance seen at the generator terminals is the inductance of L_1 plus the mutual inductance of the inductor 1 on inductor 2 plus the mutual inductance of the inductor 2 on inductor 1.

Now, considering the series association presented in Fig. 3, the inductance is given by (6).

$$L_s' = L_1 + L_2 + M_{21} + M_{12} \tag{6}$$

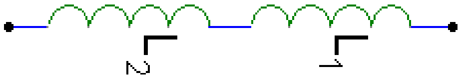

Fig. 3 Series inductors

Therefore, according to [1] and [2], if the inductances L_1 and L_2 are placed in series as shown in Fig. 3, the resulting inductance is the sum of the individual inductances plus the mutual inductances. If the measurement is performed as shown in Fig. 4, where one inductor inverted, the series inductance will be given as.

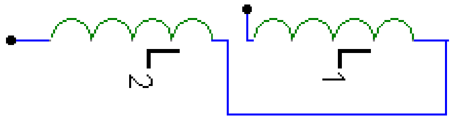

Fig. 4. Series inductors with inverted terminals

$$L_s'' = L_1 + L_2 - M_{12} - M_{21} \tag{7}$$

By subtracting L_s' from L_s'' one can obtain:

$$L_1 + L_2 + M_{12} + M_{21} - L_1 - L_2 + M_{12} + M_{21} = L_s' - L_s'' \tag{8}$$

By considering $M_{12}=M_{21}=M$, then:

$$M = \frac{L_s' - L_s''}{4} \tag{9}$$

By obtaining M and knowing the values of L1 and L2, one can calculate the coupling factor as

$$k = \frac{M}{\sqrt{L_1 L_2}} \tag{10}$$

III. TAG VOLTAGE SIMULATION

Once the mutual inductance M is known, the reader and the tag equivalent circuit can be evaluated, as indicated in Fig. 1. Observe that the tag circuit is formed by the parallel connection of L2, C2 and R2. Nevertheless the analysis can be greatly simplified by using the series equivalent circuit. Thus the quality factor of the inductor can be expressed in terms of inductive and resistive parameters, as indicated in Fig. 5.

Therefore, the transformation of a parallel circuit into a series is indicated in Fig. 5.

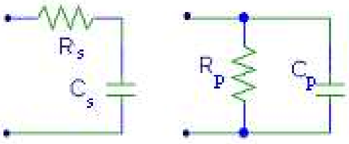

Fig. 5 Capacitive equivalent circuit

In order to find equivalence between the series and parallel representation, the real and the imaginary parts of each one must be the same [4] [5]. Therefore:

$$y_p = G_p + j\omega C_p \tag{11}$$

$$Z_s = \frac{G_p - j\omega C_p}{G_p{}^2 + (\omega C_p)^2} \tag{12}$$

From (12), the real part is R_s and the imaginary part is C_s, thus:

$$R_s = \frac{G_p}{G_p{}^2 + (\omega C_p)^2} \tag{13}$$

And

$$C_s = \frac{G_p{}^2 + (\omega C_p)^2}{\omega^2 C_p} \tag{14}$$

$$G_p = \frac{1}{R_p} \tag{15}$$

That approach allows taking the tag load as R_p, since the quality factor of the capacitor is very high. Observe also that the capacitor value is frequency dependent. Based on this approach, the obtained equivalent circuit is shown in Fig. 6.

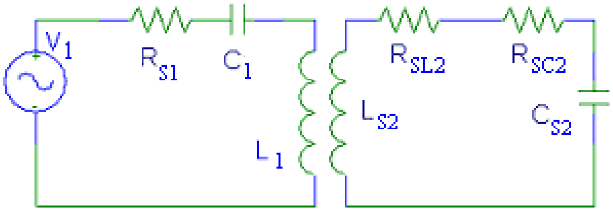

Fig. 6 Equivalent circuit modified to inductive reader-tag coupling

The circuit presented in Fig. 6 can be equated [13] as:

$$R_{s2}' = R_{sL2} + R_{sC2} \tag{16}$$

$$L_{s2} = L_2 \tag{17}$$

$$C_{s2} = C_2 \tag{18}$$

$$V_1 = I_1(R_{s1} + j\omega L_1 + \frac{1}{j\omega C_1}) - I_2 j\omega M \tag{19}$$

$$0 = -I_1 j\omega M + I_2(R_{s2}' + j\omega L_2 + \frac{1}{j\omega C_2}) \tag{20}$$

From (20):

$$I_2 = \frac{I_1 j\omega M}{R_{s2}' + j\omega L_2 + \frac{1}{j\omega C_2}} \qquad (21)$$

Voltage over capacitor C_2 is:

$$V_2 = I_2 \frac{1}{j\omega C_2} \qquad (22)$$

By replacing (21) into (19), results in:

$$V1 = I_1(R_{s1} + j\omega L_1 + \frac{1}{j\omega C_1}) - \frac{I_1(j\omega M)^2}{(R_{s2}' + j\omega L_2 + \frac{1}{j\omega C_2})} \qquad (23)$$

$$I_1 = \frac{V_1}{(R_{s1} + j\omega L_1 + \frac{1}{j\omega C_1}) + \frac{(\omega M)^2}{(R_{s2}' + j\omega L_2 + \frac{1}{j\omega C_2})}} \qquad (24)$$

Thus the voltage over capacitor C_2 can be obtained from (22) and (24) as:

$$V_2 = \frac{I_1 j\omega M}{(R_{s2}' + j\omega L_2 + \frac{1}{j\omega C_2}) j\omega C_2} \qquad (25)$$

Now, by replacing (24) into (25) results in:

$$V_2 = \frac{V_1 M}{[(\omega M)^2 + (R_{s2}' + j\omega L_2 + \frac{1}{j\omega C_2})(R_{s1} + j\omega L_1 + \frac{1}{j\omega C_1})]C_2} \qquad (26)$$

It can be observed from (26) that once the mutual inductance M between reader and tag and the quality factor of L1 and L2 are known, the voltage on the tag can be obtained.

Once the load resistance R_L is known, the equivalent loss resistance of capacitor C_2 can be found by replacing R_P with R_L, and by using the relation $G_p=1/R_p$. Therefore, by using (13), the equivalent loss resistance can be found.

Thus, by considering only the resonance frequency of 13.56 MHz [14] [17], then total series equivalent resistance will be the loss of C_2 and the series resistance of inductor L_2. Therefore, it is taken, as a first approximation, that the loss resistance of capacitor C_2 is constant with frequency. Observe also that the value of capacitor C_2 will be modified as given by (14), thus the equivalent capacitance will vary with the frequency. Consequently, the simulation will consider this frequency variation in C_2.

IV. COMPARISON OF SIMULATION AND MEASUREMENT

It was used the measurement method shown in Section II, with inductors L1=1.77 µH and L2 =5.4 µH, and the results are summarized in Table I.

It was used a network analyzer along with the auxiliary support shown in Fig. 7 to conduct the measurements. The series resistance was measured by the network analyzer, as R_{s1}=1.14Ω and R_{s2}=2.2Ω at 13.56 MHz.

TABLE I
SIMULATION MEASUREMENTS

Measurement	Distance between L_1 and L_2	M
$L_{s'}$=10.0 µH $L_{s''}$=6.33 µH	5mm	0.917 µH
$L_{s'}$=9.48 µH $L_{s''}$=6.71 µH	10mm	0.692 µH
$L_{s'}$=9.13 µH $L_{s''}$=7.0 µH	15mm	0,532 µH
$L_{s'}$=8.74 µH $L_{s''}$=7.26 µH	20mm	0.370 µH
$L_{s'}$=8.55 µH $L_{s''}$=7.40 µH	25mm	0.287 µH

Fig. 7 Mutual inductance measurement

The former results were obtained by MATLAB simulation of equations (13), (14) and (26). The simulation results are shown in Fig. 8.

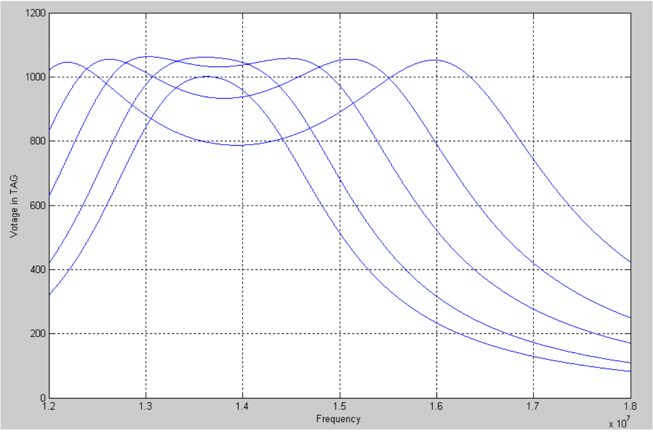

Fig. 8 Simulation in MATLAB

It can be observed that the voltage at the tag is approximately 1V. Nevertheless, the resonating frequency varies along with the mutual inductance. Therefore, for each mutual inductance, capacitors C_1 and C_2 can be adjusted for the maximum voltage at the tag.

As can be observed from Table II, the maximum error decreases to 4.5% for distances smaller than 15 mm. The error also increases as the distance increases, since the magnetic field at the inductor scatters.

TABLE II
COMPARISON RESULTS OF SIMULATION AND MEASUREMENT

Distance	Measured	Simulated	M
25mm	769mV	1001mV	0.287μH
20mm	943mV	1061mV	0.370 μH
15mm	1038mV	1063mV	0.532 μH
10mm	1026mV	1055mV	0.692 μH
5mm	1007mV	1052mV	0.917 μH

The inductor assembly may present small result deviations since the cables are subject to tiny influence through the magnetic field. Another factor is the inductor series resistance. This resistance varies with the frequency, and it is not taken account in the MATLAB simulation. The measurement set up is shown in Fig. 9.

Fig. 9 View of measurement set up

V. CONCLUSIONS

By using the proposed method to measure the mutual inductance between reader and tag, it is possible to calculate the maximum voltage at the tag of an RFID system. That information is useful to evaluate the need of high voltage protection or to evaluate the maximum distance between tag and reader. The maximum error is a 4.5 % on the tag voltage for distances of up to 15mm. For 20mm, the total error increases to approximately 30%. Therefore, the method does not work properly for large distances and it is not possible to determine precisely the maximum voltage in the tag, but it may provide a rough estimate.

It is very important to know the maximum voltage in the tag, mainly for implantable devices, so that the designer can take the proper precautions. We are now conducting a research on the influence of scattered magnetic field and the series resistance variations in the inductors.

ACKNOWLEDGMENT

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