Formulation, Analysis and Validation of Takagi-Sugeno Fuzzy Modeling For Robotic Monipulators

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Abstract—**This paper proposes a methodology for analysis of the dynamic behavior of a robotic manipulator in continuous time. Initially this system (nonlinear system) will be decomposed into linear submodels and analyzed in the context of the Linear and Parameter Varying (LPV) Systems. The obtained linear submodels, which represent the local dynamic behavior of the robotic manipulator in some operating points were grouped in a Takagi-Sugeno fuzzy structure. The obtained fuzzy model was analyzed and validated through analog simulation, as universal approximator of the robotic manipulator.**

Keywords: modeling of nonlinear dynamic systems, Takagi-Sugeno fuzzy model, Linear and Parameter Varying (LPV) System.

I. INTRODUCTION

Mathematical modeling is the study of methods of development and implementation of mathematical equations to real systems. The development of models that adequately represent the reality is an important task, since these mathematical models can be used for simulation, behavior analysis, design and control systems [1] [2] [9] [10]. Real systems are typically nonlinear. The linear term refers to the applicability of the *Principle of Superposition*. This means that: considering a system whick is excited by the input $u_1(t)$ produces the output $y_1(t)$ and when excited by $u_2(t)$ produces the output $y_2(t)$. Thus, if such system meets the principle of superposition, when excited by $au_1(t) + bu_2(t)$, its output is $ay_1(t) + by_2(t)$, being a and b possibly complex constants. Due to the predominance of nonlinear systems, an approach widely used is to linearize such systems around the operating points. This paper proposes a methodology to analyze the dynamic behavior of a robotic manipulator - a nonlinear system. Initially the system was analyzed by means of analog simulation using the Simulink software. This procedure revealed the nonlinear system dynamic. Then the nonlinear system was decomposed into sub-linear models and represented in the context of Linear and Parameters Varying - LPV. The LPV system was written in the form of state variables and equations were obtained from sub-models at the operation points. These equations, represented in a matricial form, were used as consequent rule base of Takagi-Sugeno (TS) fuzzy model. In [2], it is stated that a fuzzy rule-based model suitable for the to aproximation of a large class of nonlinear systems was introduced by Takagi and Sugeno (1985). The TS model obtained for the robotic manipulator in this paper was analyzed and validated through analog simulation in order to demonstrate the effectiveness of the proposed methodology to represent the nonlinear dynamic behavior of the manipulator.

II. PROBLEM FORMULATION

In this section it will be presented concepts related to the robotic manipulator, the nonlinear system analog simulation, linearization in the LPV context and representation in state space and the Takagi-Sugeno fuzzy model rule-base formulation for the manipulator.

A. Robotic Manipulator

The differential equation that represents the behavior of the manipulator is:

$$ml^2\ddot{\theta} + Bl\dot{\theta} + mgl\sin(\theta) = T_c \qquad (1)$$

where B represents the damping factor, i.e., it is considered the existence of friction at the fixation point of the manipulator; m is the mass; T_c is the torque applied at the manipulator link; T is the traction on it; W is the weight and l is its length. The angle θ is the system output, the angular position.

1) The Manipulator Analog Simulation: The analog simulation is an important tool for determining the temporal behavior of systems. The most appropriate procedure for the simulink block diagram structure to the analog simulation is as follows: first isolate the derivative of higher order in the differential equation in the first member of it and move the other terms to the second member:

$$\ddot{\theta} = \left(\frac{1}{ml^2}\right)T_c + \left(-\frac{B}{ml}\right)\dot{\theta} + \left(-\frac{g}{l}\right)\sin(\theta) \qquad (2)$$

The equation (2) allows easy structuring of Simulink blocks for the system simulation. The constants shown in parentheses in the second member of the equation represent the gains. The three parcels will be summed up by the sum

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block. Integrators and block of the sine function are used to complete the structure. The constant values used for the simalation were the following: $m = 1\,kg$, $g = 9.81\,m/s^2$, $l = 1\,m$ and $B = 1\,kgm^2/s$. In Fig. 1 it is shown the structure at the simulink to robotic manipulator simulation:
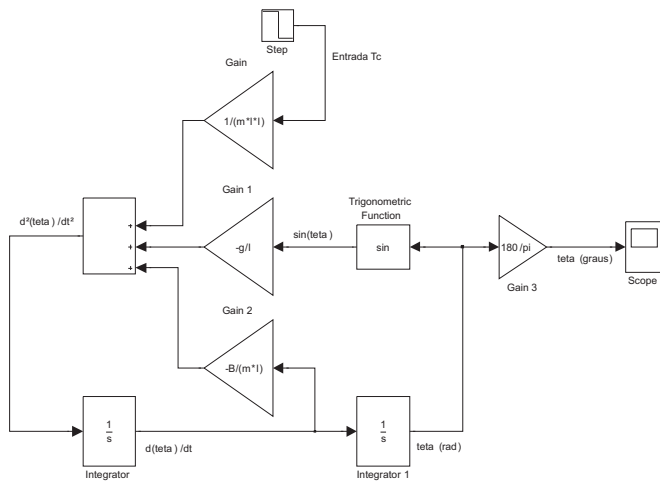

Fig. 1.   Block diagram for analog simulation

The scope block shows a graph which represents the system dynamic behavior.

2) System Linearization: The nonlinearity shown in equation (1) is due to the term $\sin(\theta)$, as the sine function does not follow the superposition principle. Thus, only this term was linearized.

A usual procedure for linearizing a characteristic such as this is to replace the characteristic with a straight line, which may give a reasonably accurate model in some small region of operation [7]. So, the function $f(\theta)$ can be written by Taylor series around the point θ_0 as follows:

$$f(\theta) = f(\theta_0) + \frac{df}{d\theta}\bigg|_{\theta=\theta_0}(\theta - \theta_0) + \frac{d^2f}{d\theta^2}\bigg|_{\theta=\theta_0}\frac{(\theta - \theta_0)}{2!} + ... \tag{3}$$

If θ is considered close to θ_0 then the terms with higher order derivatives can be ignored:

$$f(\theta) \cong f(\theta_0) + \frac{df}{d\theta}\bigg|_{\theta=\theta_0}(\theta - \theta_0) \tag{4}$$

For the case of robotic manipulator modeled in equation (1) $f(\theta) = \sin(\theta)$. Thus applying the result obtained in (4):

$$\sin(\theta) \cong \sin(\theta) + (\cos(\theta_0))(\theta - \theta_0) \tag{5}$$

The result of (5) provides the linearization of the function $\sin(\theta)$ around generic operating point θ_0. Following, the equation (5) was replaced in equation (2):

$$\ddot{\theta} = \left(\frac{1}{ml^2}\right)T_c + \left(-\frac{B}{ml}\right)\dot{\theta} + \left(-\frac{g}{l}\right)(\sin(\theta_0) + \cos(\theta_0)(\theta - \theta_0)) \tag{6}$$

Then:

$$\ddot{\theta} = \left(\frac{1}{ml^2}\right)T_c - \left(\frac{B}{ml}\right)\dot{\theta} - \left(\frac{g}{l}\right)[\sin(\theta_0) - \theta_0\cos(\theta_0)] - \frac{g\cos\theta_0}{l}\theta \tag{7}$$

In Fig. 2 it is shown the nonlinear characteristic of the function $f(\theta) = \sin(\theta)$ along with that an example of linearization around the operating point $\theta_0 = \frac{\pi}{3}\,rad$:
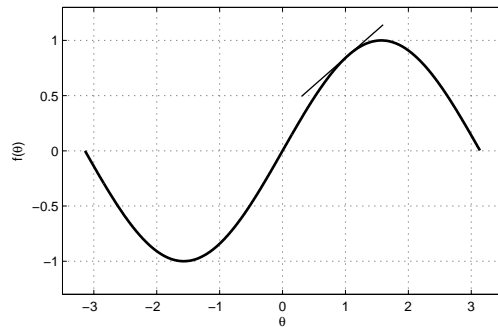

Fig. 2.   Nonlinear characteristic of $f(\theta) = \sin(\theta)$ and linearization

The equation (7) represents the LPV model, i.e., a model that is linear and varies through parameters. The follows replacements were made through simplifications: $\alpha = \frac{1}{ml^2}$, $\beta = \frac{B}{ml}$, $\gamma = \frac{g}{l}[\sin(\theta_0) - \theta_0\cos(\theta_0)]$ e $\delta = \frac{g\cos(\theta_0)}{l}\theta$. Thus:

$$\ddot{\theta} = -\beta\dot{\theta} - \delta\theta + u^* \tag{8}$$

where $u^* = \alpha T_c - \gamma$

3) State-Space Representation: Equation (8) was represented in the form of state variables as follows:

Let $x_1(t)$ and $x_2(t)$ be the state variables. So:

$$\begin{cases} x_1(t) = \theta & \Rightarrow & \dot{x}_1(t) = \dot{\theta} \\ x_2(t) = \dot{\theta} = \dot{x}_1(t) & \Rightarrow & \dot{x}_2(t) = \ddot{\theta} \end{cases} \tag{9}$$

Substituting equation (9) into equation (8), (10) is obtained:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\beta x_2(t) - \delta x_1(t) + u^* \end{cases} \tag{10}$$

Represented in a matricial form:

$$\begin{cases} \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -\delta & -\beta \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u^* \\ \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}(t) \end{cases} \tag{11}$$

where $\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$, $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ and $y(t)$ is the system output (θ).

B. Rule Base of Takagi-Sugeno Fuzzy Model for Manipulator

The fuzzy modeling is used in complex systems, nonlinear or partially unknown, whick usually have problems with conventional techniques and classic modeling. It tries to incorporate in the model the expert's knowledge, taking into account that humans are capable of dealing with relatively complex processes based on imprecise information [4][5][6][8]. Fuzzy

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systems have rules of type IF-THEN, as follows:

If <antecedent proposition> then <consequent proposition>

The types of fuzzy models are distinguished by consequent proposition. In the Sugeno-Takagi fuzzy model (Takagi and Sugeno, 1985), the consequent proposition is a crisp mathematical function of antecedent variables instead of a fuzzy proposition. The general structure of rules in the TS model with K rules is:

$$R_i : \ If \ x \ is \ A_i \ then \ y_i = f_i(x), \ i = 1, 2, 3..., K. \quad (12)$$

where x is the antecedent variable (input) and $y_i \in \mathbb{R}$ is the consequent variable (output). A_i denotes the antecedent fuzzy set of ith rule, being defined by the membership function:

$$\mu_{A_i}(x) : \mathbb{R}^p \to [0, 1] \quad (13)$$

The Fig. 3 shows a fuzzy system for a generic TS model. In this model there is no TS defuzzyfication step, since the output is a crips mathematical function.
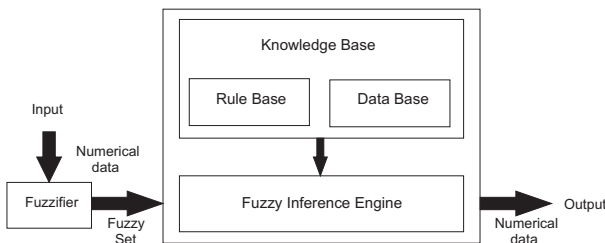

Fig. 3. Fuzzy system for the general Takagi-Sugeno model

Given the structure of the TS model and the LPV model obtained for the manipulator, it is possible to obtain the linear submodels for each operating point and group them into a TS fuzzy structure. The chosen operating points for the manipulator are $-90°$, $-85°$, $-75°$, $-60°$, $-45°$, $-30°$, $0°$, $30°$, $45°$, $60°$, $75°$, $75°$, $85°$ e $90°$, as it is shown in Fig. 4:
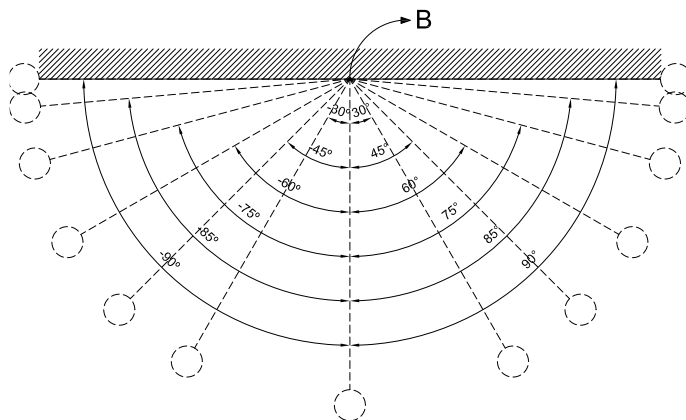

Fig. 4. Operating points of the manipulator

After the calculation of linear submodels for each operating point the following rule-base, consisting of 13 rules ($K = 13$) was obtained:

$$R_1 : \ If \ x \ is \ -90° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1{}^*$$

$$R_2 : \ If \ x \ is \ -85° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -0.8550 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2{}^*$$

$$R_3 : \ If \ x \ is \ -75° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2.5390 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_3{}^*$$

$$R_4 : \ If \ x \ is \ -60° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -4.9050 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_4{}^*$$

$$R_5 : \ If \ x \ is \ -45° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -6.9367 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_5{}^*$$

$$R_6 : \ If \ x \ is \ -30° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8.4957 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_6{}^*$$

$$R_7 : \ If \ x \ is \ 0° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -9.81 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7{}^*$$

$$R_8 : \ If \ x \ is \ 30° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8.4957 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7{}^*$$

$$R_9 : \ If \ x \ is \ 45° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -6.9367 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7{}^*$$

$$R_{10} : \ If \ x \ is \ 60° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -4.9050 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7{}^*$$

$$R_{11} : \ If \ x \ is \ 75° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2.5390 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7{}^*$$

$$R_{12} : \ If \ x \ is \ 85° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -0.8550 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7{}^*$$

$$R_{13} : \ If \ x \ is \ 90° \ then \ \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_7{}^*$$

where $u_1{}^* = T_c + 9.81$, $u_2{}^* = T_c + 8.5043$, $u_3{}^* = T_c + 6.1522$, $u_4{}^* = T_c + 3.3592$, $u_5{}^* = T_c + 1.4886$, $u_6{}^* = T_c + 0.4567$, $u_7{}^* = T_c$, $u_8{}^* = T_c - 0.4557$, $u_9{}^* = T_c - 1.4886$, $u_{10}{}^* = T_c - 3.3592$, $u_{11}{}^* = T_c - 6.1522$, $u_{12}{}^* = T_c - 8.5043$, $u_{13}{}^* = T_c - 9.81$, because $\alpha = 1$.

III. TS FUZZY MODEL FORMULATION FOR THE MANIPULATOR

In this section it will be presented the model structure Takagi-Sugeno fuzzy to the manipulator using the Simulink of the Matlab. The structure allows the analog simulation of the TS model and its comparison with the nonlinear system so that it can be validated.

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A. Inference in TS Model

According [2], the degree of fulfillment of the antecedent corresponds to the membership degree that is given to it, i.e, the credit which is granted to the antecedent. The degree of fulfillment is denoted by $\beta_i(x)$, it is simply equal to the membership degree of the given input x, i.e., $\beta_i(x) = \mu_{A_i}(x)$. The inference on Takagi-Sugeno fuzzy model is given by the simple algebraic expression:

$$y = \frac{\sum_{i=1}^{K}(\beta_i(x)y_i)}{\sum_{i=1}^{K}\beta_i(x)} \qquad (14)$$

The numerator of equation (14) indicates a sum of products between each linear submodel and its degree of activation of the rule corresponding to the submodel. The denominator indicates the sum of the degrees of fulfillment.

B. TS Model Simulation Structure Formulation

Once obtained the thirteen linear submodels (shown on the rule-base already presented), and made the structuring of membership functions, it was possible to develop completely the Takagi-Sugeno fuzzy model for the robotic manipulator in question. The chosen membership functions were gaussian. The law of formation of this membership function is given by the following expression:

$$f(x) = e^{\left(\frac{-0.5(x-c)^2}{\sigma^2}\right)} \qquad (15)$$

In equation (15), e represents the Euler number, c represents the center of the Gaussian and σ indicates the variance. As the variance has its value increased or decreased, the curve opening gets wider or narrower, respectively. The centers, for the model in question are the operation points of the manipulator, the variances were adjusted as the need for greater or lesser activation of a specific rule for better efficiency of the TS model.

The Fig. 5 shows the structure of membership functions and linear submodels in simulink. The inputs of blocks appear all interlinked: they are connected to the global output of the system, so that the fuzzyfication may happen. The outputs are connected to a sum block to generate the sum of the degrees of fulfillment, as the denominator of equation (14). The structure shown on the right in the figure are the linear submodels and product blocks to implement, together with the sum, the numerator of equation (14).

The Fig. 6 shows the general structure of linear submodels. The input of each submodel is the torque, as shown in the interconnection of all inputs of SLIT's. In the same figure it can be observed the *State-Space* block, whose function is to implement a linear system in state space.

The names of two of the blocks in Fig. 6 correspond to the constants α and γ formerly defined in this paper.
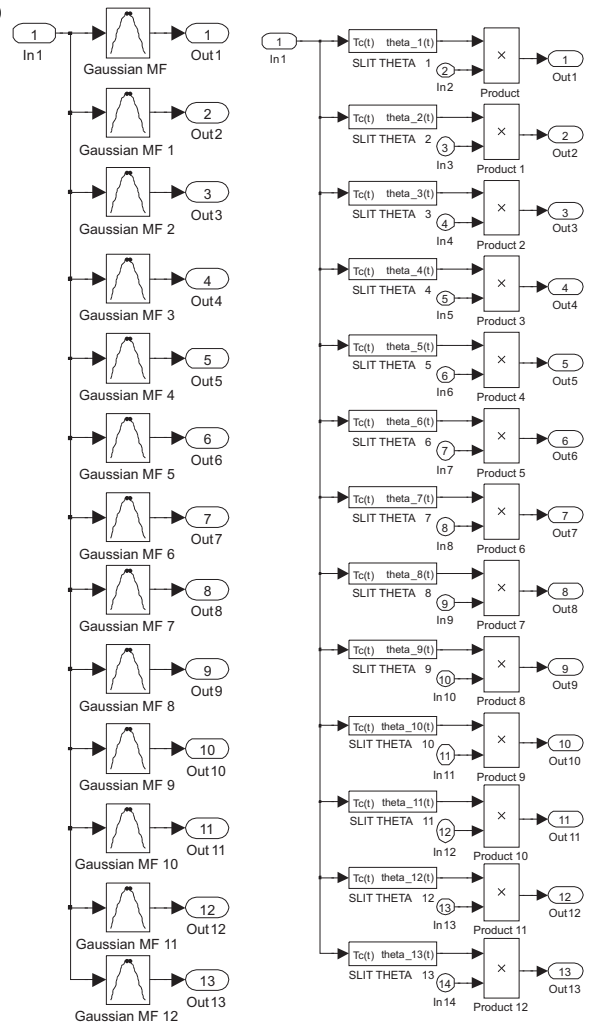

Fig. 5. Membership Functions and linear submodels of TS model
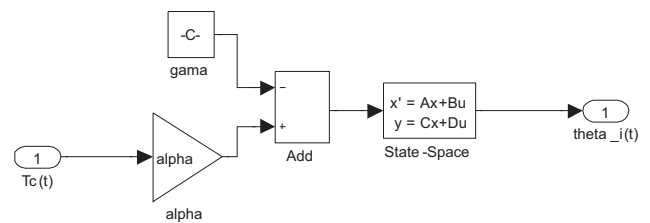

Fig. 6. General form of the linear submodel

The Fig. 7 shows the complete implementation of equation (14), in simulink. This structure also allows the analog simulation of both the TS fuzzy model and of the nonlinear model simultaneously for validation purposes. The block called Robotic Manipulator is a subsystem of the structure shown in Figure 1. The input of this subsystem is the torque and the output is the angle $\theta(t)$ (nonlinear).

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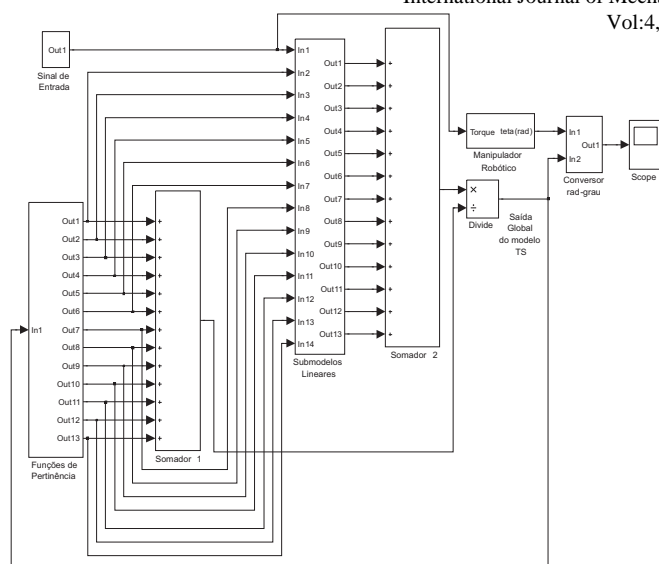Vol:4, No:10, 2010

Fig. 7.   Framework for TS model simulation

In the structure (Fig. 7) there is also a converter from radians to degrees. All two outputs (nonlinear and fuzzy) are connected to the input of this converter. The output of this block is connected to the scope, which will show the curves for the angles in degrees. The Fig. 8 shows a graph of the input signal representing the torque applied to the robotic manipulator. This is a noisy sinusoidal signal.
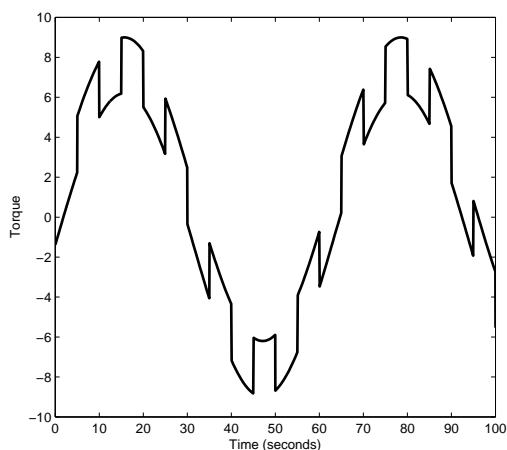

Fig. 8.   Graph of the input signal

IV. ANALISIS AND VALIDATION OF TS FUZZY MODEL OF MANIPULATOR

The analysis and validation of Takagi-Sugeno fuzzy model obtained for the robotic manipulator was made through analog simulation.

A. Simulation of TS Model obtained

The structure shown in Fig. 7 was simulated several times to obtain the values of variance of the Gaussian to generate a coherent model. These variance values were being chosen as the output of TS fuzzy model was closer to the output of the nonlinear model. Once obtained all the parameters characterizing the membership functions, it was constructed a graph of the fuzzy sets of $x_1(t)$:
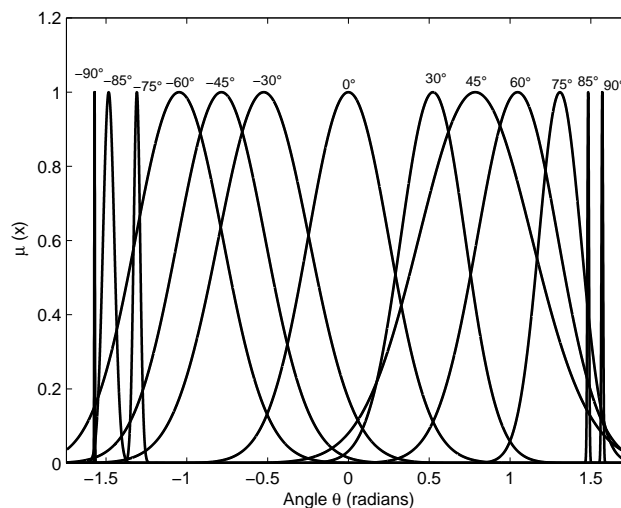

Fig. 9.   Fuzzy sets of $x_1(t)$

Fig. 9 allows to look through the membership functions which for a given angle, several rules are activated, but with different degrees of fulfillment. Table 1 shows the characterization of the membership functions showing the values of variance for each of the operating points (centers):

Center(c)	Variance(σ)
$-90°$	0.002
$-85°$	0.035
$-75°$	0.02
$-60°$	0.27
$-45°$	0.27
$-30°$	0.28
$0°$	0.25
$30°$	0.2
$45°$	0.35
$60°$	0.25
$75°$	0.13
$85°$	0.005
$90°$	0.005

TABLE I
CHARACTERIZATION OF MEMBERSHIP FUNCTIONS (GAUSSIAN)

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B. Validation of TS Fuzzy Model

After manual adjustment of membership functions of Takagi-Sugeno fuzzy model for the manipulator through the analog simulation of the structure shown in Fig. 7, it was obtained as a result, the temporal response of TS model in comparison with nonlinear model, as shown in Fig. 10:
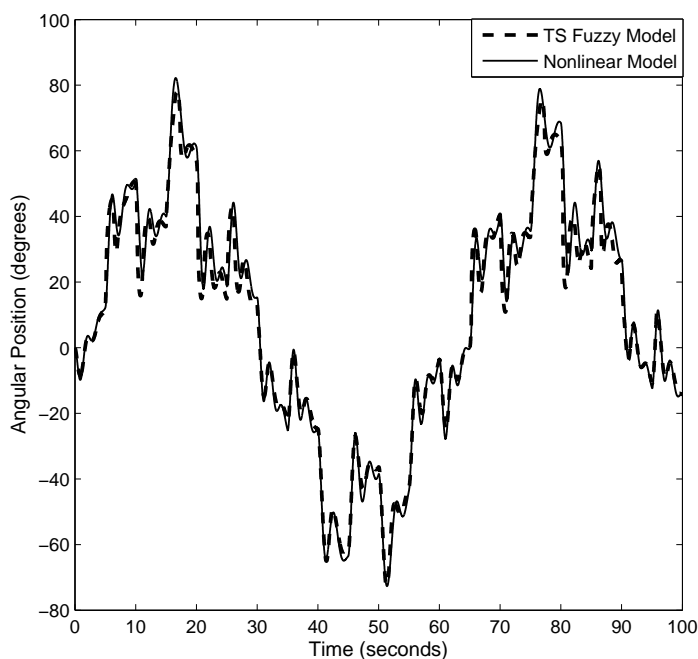

Fig. 10.   Response of the TS model and nonlinear model

As the figure caption, the curve of the fuzzy model is shown in dashed line. The nonlinear model curve is in continuous line. There's a good proximity between the two curves, indicating that the TS model obtained adequately represents the dynamic behavior of the manipulator at the operation points.

V. CONCLUSION

The methodology proposed in this paper for modeling the robotic manipulator is adequate for modeling nonlinear systems in general, since the Takagi-Sugeno fuzzy model has been proven efficient in the concatenation of linear submodels for the system global output composition. The TS model obtained for the manipulator has a reasonable number of rules due to on the characteristic behavior of the manipulator in the operating point near 90 ° and -90 °. However, the thirteen rules were needed to ensure the effectiveness of the model.

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