

# Local Curvelet Based Classification Using Linear Discriminant Analysis for Face Recognition

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*Abstract*—In this paper, an efficient local appearance feature extraction method based the multi-resolution Curvelet transform is proposed in order to further enhance the performance of the well known Linear Discriminant Analysis(LDA) method when applied to face recognition. Each face is described by a subset of band filtered images containing block-based Curvelet coefficients. These coefficients characterize the face texture and a set of simple statistical measures allows us to form compact and meaningful feature vectors. The proposed method is compared with some related feature extraction methods such as Principal component analysis (PCA), as well as Linear Discriminant Analysis LDA, and independent component Analysis (ICA). Two different multi-resolution transforms, Wavelet (DWT) and Contourlet, were also compared against the Block Based Curvelet-LDA algorithm. Experimental results on ORL, YALE and FERET face databases convince us that the proposed method provides a better representation of the class information and obtains much higher recognition accuracies.

*Keywords*—Curvelet, Linear Discriminant Analysis (LDA) , Contourlet, Discreet Wavelet Transform, DWT, Block-based analysis, face recognition (FR).

## I. INTRODUCTION

**A**MONG face recognition algorithms, the most popular are appearance-based approaches [20], [4], [12]. These approaches exhibit good performance and robustness against noise in controlled environments but still do not perform well in many real-world situations, where the query test face appearance is significantly different from the training face data, due to variations in pose, lighting and expression. In order to address this issue, many researchers propose to deploy a pre-processing step in order to capture more discriminant features for use in the recognition step. Worth noting, these observations motivated the relatively recent interest in using multi-resolution feature extraction algorithms, with successful pattern recognition applications in many challenging tasks including character recognition [2] and face recognition [1], [3].

Over the past two decades, Multi-resolution analysis [8], [7], [6], [18] has been successfully used in image processing with the recent emergence of applications to texture classification. Several studies have investigated the discriminating power of wavelet-based features in various applications including: image compression [23], image denoising [10]. However, images do not always exhibit isotropic scaling (horizontal,

vertical and diagonal) and thus call for other kinds of multi-scale representation. Recently, the finite Contourlet and Curvelet transforms have emerged as a new multi-resolution analysis tool. These tools have better directional decomposition capabilities than wavelets. The Contourlet techniques were used also for image processing problems like image compression [9] and denoising [10], but very few studies have been addressing problems related to computer vision. Applications of Curvelet to pattern recognition have been investigated in work presented by Majumdar who showed that using Curvelets, one can obtain very good results for character recognition [2]. El Aroussi et al [18] propose new approach based on local presentation of Curvelet transform, They have evaluated the proposed method on two different databases (Yale Face Database and FERET Database) and they obtain high recognition accuracy. Boukabou et al [3] proposes to employ Contourlet with PCA in order to extract discriminant features and obtain higher recognition rates. They have evaluated the proposed method on two different databases (Yale Face Database and FERET Database). The authors state that the Contourlet Transform outperforms the original PCA method. More experiments have to be performed on large database to asses these conclusion. Mandal et al [1] proposes Curvelet based face recognition system by fusing results from multiple SVM classifiers trained with Curvelets coefficients from images having different gray scale resolutions (2, 4 and 8 bits). This technique appears to be robust to the changes in facial expression as it shows good results for the Essex and the ORL database. But for Georgia Tech database which contains tilted faces with different facial expressions, lighting conditions and scale, the recognition accuracy is not satisfying and needs to be improved. However this algorithm is computationally expensive since it requires taking the Curvelet transform of the original image and its quantized representations.

In this paper, we present a novel face recognition approach based on Block Based Curvelet decomposition. In order to capture multi-orientation information in face images better. Therefore, each face image is described by a subset of band filtered images containing curvelet coefficients which characterize the face textures. We divide the Curvelet sub-bands into small sub-blocks, from which we extract compact and meaningful feature vectors using simple statistical measures. these feature sets are used in order to create templates with different information content for face recognition (Curvelet database). Once done, an Fisherface algorithm is carried out on the Curvelet database in which faces with similar statistics

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will be grouped together by LDA rules, where the difference between classes is maximized while the difference within classes is minimized. For the purpose of classification we use the city-block distance, We design experiments specifically to investigate the improvement in robustness against illumination and facial expression changes. Experimental results are presented using images from the FERET, ORL and the YALE databases. The efficiency of our approach is analyzed by comparing the results with those obtained using the well-known subspace reduction based methods PCA, LDA and ICA.

The remainder of the paper is organized as follows. In Section 2, Curvelet transform used in the study is explained. Section 3 describes the computation of the proposed face representation in detail and how to recognize faces based on Curvelet with LDA are presented in Section 4 Experimental results are presented and discussed in Section 5 conclusions and future recommendations are given.

## II. CURVELET TRANSFORM

Multi-resolution analysis [6] allows for the preservation of an image according to certain levels of resolution or blurring. Broadly speaking, multi-resolution analysis allows for the zooming in and out on the underlying texture structure. Therefore, the texture extraction is not affected by the size of the pixel neighborhood. This multi-resolution quality is one of the reasons why wavelets have been useful in so many applications from image compression to image de-noising and edge detection [21].

Wavelets and related classical multi-resolution methods use a limited dictionary made up of roughly isotropic elements occurring at all scales and locations. Despite the success of the classical wavelet viewpoint, there are objects, e.g. images that do not exhibit isotropic scaling and, thus, call for other types of multi-scale representation. Candes and Donoho [11] introduced a new system of multi-resolution analysis called the Curvelet transform. This system differs from wavelet and related systems. Curvelets take the form of basis elements, which exhibit a very high directional sensitivity and are highly anisotropic. Curvelet is one such transform that can efficiently represent edge discontinuities in images. Hence, due to its anisotropic behavior, Curvelets are elongated needle shaped structures. Owing to this needle shaped structures, Curvelets approximate edges by contiguous elongated structures rather than fat dots-as was the case with wavelets. Consequently, edges can be represented by far less Curvelet coefficients compared to wavelets. In other words, Curvelets should be sparser than wavelets for images.

The continuous Curvelet transform can be defined by a pair of windows  $W(r)$  (a radial window) and  $V(t)$  (an angular window), with variables  $W$  as a frequency-domain variable, and  $r$  and  $\theta$  as polar coordinates in the frequency-domain.

$$\sum_{j=-\infty}^{\infty} W^2(2^j r) = 1, \quad r \in \left(\frac{3}{4}, \frac{3}{2}\right) \quad (1)$$

$$\sum_{l=-\infty}^{\infty} V^2(t-l) = 1, \quad t \in \left(-\frac{1}{2}, \frac{1}{2}\right) \quad (2)$$

A polar 'wedge' represented by  $U_j$  is supported by  $W$  and  $V$ , the radial and angular windows.  $U_j$  is defined in the Fourier domain by

$$U_j(r, \theta) = 2^{-3j/4} W(2^{-j} r) V\left(\frac{2^{j/2} \theta}{2\pi}\right) \quad (3)$$

The Curvelet transform can be defined as a function of  $x = (x_1, x_2)$  at scale  $2^{-j}$ , orientation  $\theta_t$ , and position  $x_k^{(j,l)}$  by

$$\phi_{j,l,k}(x) = \phi_j(R_{\theta_t}(x - x_k^{(j,l)})), \quad (4)$$

where  $R_{\theta}$  is the rotation in radians. Fig. 1 illustrates the polar wedges represented by  $U_j$ . The interested reader is asked to refer to [11] for delving into the implementation details of the transform.

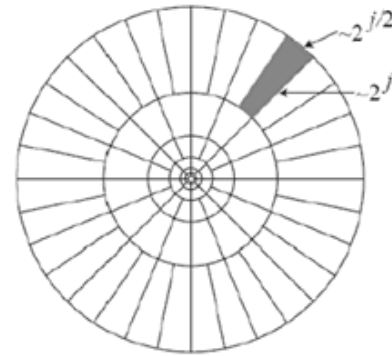


Fig. 1. Example of polar wedges

## III. PROPOSED METHOD

### A. Feature vectors

For gaining the best feature vectors from the face databases, at first, we perform Fast Curvelet Transform on each of normalized images from the databases. As a result, Curvelet sub-bands in different scales and various angles will be obtained. Figure 2 shows the Curvelet coefficients of an image from FERET database.

The Curvelet coefficients in this figure are described as follows

- 1) The low frequency (coarse scale) coefficients are stored at the center of the display.
- 2) The Cartesian concentric coronae show the coefficients at different scales; the outer coronae correspond to higher frequencies.
- 3) There are four strips associated to each corona, corresponding to the four cardinal points; these are further subdivided in angular panels.
- 4) Each panel represent coefficients at a specified scale and along the orientation suggested by the position of the panel.

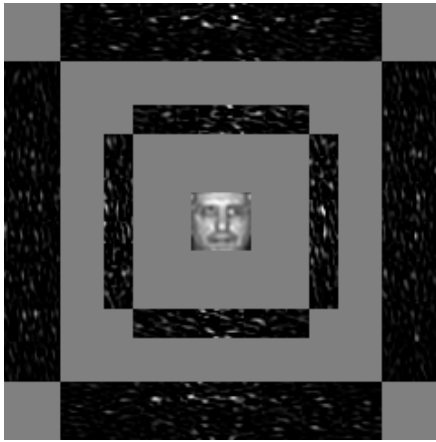


Fig. 2. Example of Curvelet coefficients

To generate the image database, each image is decomposed into 3-level and 4-orientation sub-bands. The direct use of S-P coefficients may not extract the most discriminative features as these coefficients contain much redundant and irrelevant information. For an efficient and local representation of the face image, first each S-P sub-band is partitioned into a set of equally-sized blocks in a non-overlapping way. Based on common belief, the statistical measures such as mean, variance and entropy of the energy distribution of the S-P coefficients for each sub-band at each decomposition level can be used to identify a texture. Let  $I_{ij}(x, y)$  be the image at the specific block  $j$  of sub-band  $i$ , the resulting feature vector  $\nu_{ij} = \{\mu_{ij}, \sigma_{ij}^2, e_{ij}\}$ , where  $\mu_{ij}$  =mean,  $\sigma_{ij}^2$  =variance and  $e_{ij}$  =entropy.

$$\mu_{ij} = \frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N |I_{ij}(x, y)| \quad (5)$$

$$\sigma_{ij} = \frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N |I_{ij}(x, y) - \mu_{ij}|^2 \quad (6)$$

where  $M$  and  $N$  is the size of  $I_{ij}(x, y)$ . Entropy is a statistical measure of randomness that can be used to characterize the texture of the input image. Entropy is defined as

$$e_{ij} = - \sum_p (p \times \log(p)) \quad (7)$$

where  $p$  contains the histogram counts.

The feature vector of a face is then constructed by concatenating each block measure to one big feature vectors  $V = \bigcup_{i=1}^k \bigcup_{j=1}^{k_i} \{\nu_{ij}\}$ ,  $k$  is the number of S-P sub-bands and  $k_i$  the number of blocks in the  $i$ th sub-band. Therefore, we can extract the best features and reduce the size of the data while keeping only the principal discriminant features. Figure 3 shows the overall diagram of the proposed face features extraction.

### B. Linear Discriminant Analysis LDA

LDA is a popular discriminant criterion that measures the between-class scatter normalized by the within-class scatter

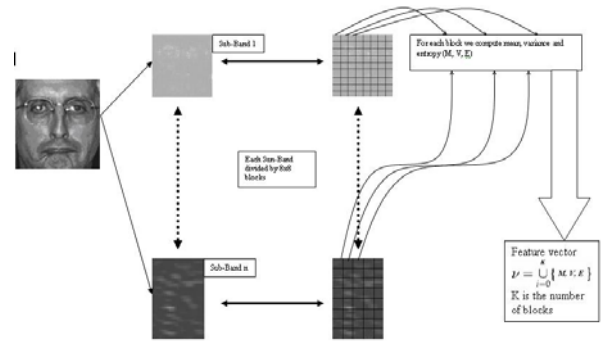


Fig. 3. Diagram of the Block-based S-P features extraction process

[5]. Let  $\omega_1, \omega_2, \dots, \omega_L$  and  $N_1, N_2, \dots, N_L$  denote the classes and the number of images within each class, respectively. Let  $M_1, M_2, \dots, M_L$  and  $M$  be the means of the classes and the grand mean. The within- and between-class scatter matrices,  $\Sigma_\omega$  and  $\Sigma_b$ , are defined as follows:

$$\Sigma_\omega = \sum_{i=1}^L P(\omega_i) \varepsilon\{(\gamma^p - M_i)(\gamma^p - M_i)^t | \omega_i\} \quad (8)$$

and

$$\Sigma_b = \sum_{i=1}^L P(\omega_i) (M_i - M)(M_i - M)^t \quad (9)$$

where  $P(\omega_i)$  is a priori probability,  $\Sigma_\omega, \Sigma_b \in R^{m \times m}$ , and  $L$  denotes the number of classes. LDA derives a projection matrix  $\Psi$  that maximizes the ratio  $|\Psi^t \Sigma_b \Psi| / |\Psi^t \Sigma_\omega \Psi|$  [1]. This ratio is maximized when  $\Psi$  consists of the eigenvectors of the matrix  $\Sigma_\omega^{-1} \Sigma_b$  [5]:

$$\Sigma_\omega^{-1} \Sigma_b \Psi = \Psi \Delta \quad (10)$$

where  $\Psi, \Delta \in R^{m \times m}$  are the eigenvector and eigenvalue matrices of  $\Sigma_\omega^{-1} \Sigma_b$ , respectively.

### C. Classification

Once Blocked-Curvelet database constructed, an LDA algorithm is carried out on the training vectors taken randomly from the Blocked-Curvelet database in order to reduce the size of the data while keeping only the principal discriminant features (principal components). Then, the test vectors are projected on the Eigenspace and the distances to the training vectors are computed using the euclidian distance for the purpose of classification as follows: For two face images  $p$  and  $q$ , let  $V_p$  and  $V_q$  representing the corresponding feature vectors, the euclidian distance  $d_{pq}$  between the two patterns in the feature space is defined as:

$$d_{pq} = \sqrt{\sum_{i=1}^k \sum_{j=1}^{k_i} (n u_{pij} - \nu_{qij})^2} \quad (11)$$

The classification performance evaluation is based on pairwise distance matrix. If there are  $m$  training and  $n$  test samples, then a distance matrix should be of size  $m \times n$ , with each column representing the distances from the corresponding test sample to all training samples (classes). The lower the distance, the closer the two samples.

#### IV. EXPERIMENTAL RESULTS

To validate the accuracy of the proposed algorithm, we have used three different databases: ORL<sup>1</sup>, Yale<sup>2</sup> and Feret [19]. The ORL database contains ten different images of 40 distinct subjects in up-right, frontal position with tolerance for some tilting and rotation of up to 20%. Moreover, the most variation of some image scale is close to 10%. Therefore, it is expected that this is a more difficult database to work with. 5 face images per person are chosen randomly as training images while the remaining 5 images are set as test images. Figure 4 depicts some sample images from the ORL database.



Fig. 4. Faces from the ORL Face Database

The Yale face database consists of 15 individuals, where for each individual, there are 11 face images containing variations in illumination and facial expression. From these 11 face images, we use 5 for training, chosen randomly. The remaining 6 images are used for testing. Figure 5 depicts some sample images from the Yale database.



Fig. 5. Faces from the YALE Face Database

A subset of FERET face database, *fafb* image set, containing images of 145 individuals is used in our experiments. In this subset, there are four frontal views of each individual: a neutral expression and a change of expression from one session, and a neutral expression and change of expression from a second session that occurred three weeks after the first. For each of the individual in the set, three of their images are used for training and the remaining is used for testing purposes. Figure 6 depicts some sample images from the Yale database.

All the images are aligned with respect to the manually detected eye coordinates, scaled to  $128 \times 128$  pixels resolution. The experiments were carried out in Matlab 7.5, on a 32-bit DUAL 2 CORE 2GHz processor, with 2Gb RAM. The

<sup>1</sup>[http://www.cl.cam.ac.uk/Research/DTG/attarchive:pub/data/att\\_faces.zip](http://www.cl.cam.ac.uk/Research/DTG/attarchive:pub/data/att_faces.zip)

<sup>2</sup><http://cvc.yale.edu/>



Fig. 6. Faces from the FERET Face Database

Curvelet transformation was done using the Curvelet 2.1.2 toolbox (available from <http://www.Curvelet.org>).

#### A. Block-based Curvelet transform vs PCA, LDA & ICA

In order to assess the efficiency of the proposed technique described above, we carried out a series of experiments using all databases separately. In this section, we aim to compare our proposed method (Block-based Curvelet) to the original PCA method, LDA and ICA. Table I reports the results obtained for all databases. It is clear that the proposed method outperforms the original PCA, LDA and ICA algorithms. On the ORL Database for instance, improvements of 9.5%, 10.5% and 17.5% have been obtained for the PCA, LDA and ICA methods, respectively. It is also worth mentioning that significant enhancements have been obtained for YALE database: 93.33% for the proposed method against 90% for PCA, 92.22% for LDA and 87.78% for ICA. Finally for the FERET database the improvement is about 15.86%, 18.62% and 17.24% compared with PCA, LDA and ICA respectively.

TABLE I  
 RECOGNITION ACCURACY OF BLOCK-BASED CURVELET TRANSFORM  
 COMPARED WITH HOLISTIC ALGORITHMS

Method	PCA	LDA	ICA	Curvelet-LDA
ORL	88.5%	87.5%	80.5%	<b>98%</b>
YALE	90%	92.22%	87.78%	<b>93.33%</b>
FERET	75.86%	73.1%	74.48%	<b>91.72%</b>

#### B. Comparison of different block based multi-resolution transforms

The Curvelet-based algorithm was also compared to two other multi-resolution algorithms, wavelet and Contourlet. Table II illustrates a comparison of accuracy results for the different algorithms.

The Curvelet-based descriptors had an even higher performance in comparison to both the wavelet and Contourlet, with accuracy rates approximately 2.22–19.5% and 15.55–46.89% higher, respectively. Curvelet-based features yield accuracy rates between 91.72% and 98%, which significantly improved accuracy ranges for Contourlet-based features and wavelet-based features. Accuracy rates for wavelet-based texture descriptors ranged between 78.64–91.11%, while Contourlet-based descriptors were 44.83–78.5%. This was also expected since the Curvelet transform is able to capture multi-directional features in wedges, as opposed to lines or points as

in the Contourlet or wavelet transform. The multidirectional features in Curvelets prove to be very effective in the face recognition figure 7.

TABLE II  
RECOGNITION ACCURACY OF BLOCK-BASED CURVELET COMPARED WITH WAVELET AND CONTOURLET

Block Based method	DWT-LDA	Contourlet-LDA	Curvelet-LDA
ORL	89.5%	78.5%	<b>98%</b>
YALE	91.11%	77.78%	<b>93.33%</b>
FERET	78.64%	44.83%	<b>91.72%</b>

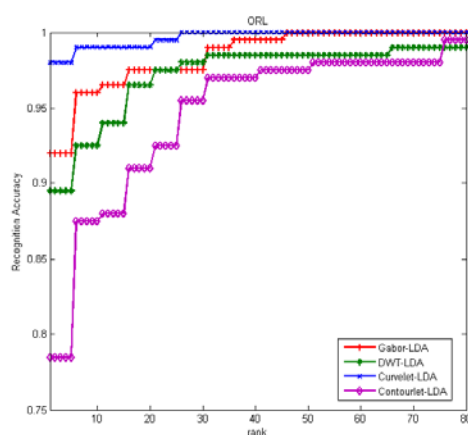


Fig. 7. Recognition accuracy of Block-based Curvelet Transform compared with multi-resolution algorithms on ORL database

### C. our approach against well established existing techniques

In these section we compare our proposed method against well established existing techniques like standard eigenface based methods, wavelet and Curvelet based methods. Table III reports the results obtained for ORL and YALE Database. It is clear from the Tables III that the proposed method outperforms the other algorithms.

TABLE III  
COMPARATIVE STUDY.

Method	ORL	YALE
Standard Eigenface [12]	92.2%	76%
Waveletface[13]	92.5%	83.3%
Waveletface + PCA[13]	94.5%	84%
Waveletface + LDA[14]	94.7%	84.6%
Waveletface + Weighted Modular PCA[15]	95%	83.6%
Waveletface + LDA + NFL[14]	95.2%	83.5%
Waveletface + KAM[16]	96.6%	84%
Curveletface + PCA[17]	96.6%	83.9%
Curveletface + PCA + LDA[17]	97.7%	92%
<b>Curvelet-LDA</b>	<b>98%</b>	<b>93.33%</b>

## V. CONCLUSIONS

This paper proposes a new approach for face recognition based on exploiting the features of the Block-based

Curvelet Transform when combined with LDA. Three different databases (ORL, Yale Face Database and FERET Database) have been used to evaluate the proposed method. The technique introduced in our paper appears to be robust to the changes in facial expression as it shows good results for the ORL database and to the lighting variation, good results for Yale database. Another advantage of the Curvelet stage is that it provides a very helpful solution when the number of face images in the database is insufficient (FERET databases), since the number of images will increase, thus providing more discriminant power for the classification phase. Our future work would include making some revisions on the Curvelet transform, and investigating other multi-resolution transform such as steerable pyramid.

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