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Exponential stability analysis for switched cellular neural networks with time-varying delays and impulsive effects

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Abstract—In this Letter, a class of impulsive switched cellular neural networks with time-varying delays is investigated. At the same time, parametric uncertainties assumed to be norm bounded are considered. By dividing the network state variables into subgroups according to the characters of the neural networks, some sufficient conditions guaranteeing exponential stability for all admissible parametric uncertainties are derived via constructing appropriate Lyapunov functional. One numerical example is provided to illustrate the validity of the main results obtained in this paper.

Keywords—Switched systems, exponential stability, cellular neural networks.

I. Introduction

SINCE cellular neural networks (CNNs) have been introduced by Chua and Yang [1], they attracted many researchers' interest because of their widely use in many science fields such as image processing, optimal computation, etc. However, because of the existence of time delays and impulsive effects, neural dynamical system may be unstable, even if it is stable without these effects. Thus, the qualitative properties in the mathematical theory of delayed impulsive cellular neural networks are very important. In recent years, a number of mathematicians have been interested in and developed such properties (see [2-6]), and their studies have attracted much attention. On the other hand, With the rapid development of intelligent control, hybrid systems have been investigated for their extensive applications, (see [7-13]). As a special class of hybrid systems, switched systems are regarded as a typical nonlinear system, which are composed of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching between the subsystems. In past years, considerable efforts have been focused on analysis and design of switched systems.

In this paper, we will study a class of impulsive switched cellular neural networks with time-varying delay via integrating the theory of switched systems with neural networks. The individual subsystems of the switched cellular neural networks are a set of cellular neural networks with time-varying delay. In addition, the parametric uncertainties are also considered, some criteria are obtained to guarantee the switched system to be globally exponentially stable for all admissible uncertainties.

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II. Preliminaries and assumptions

The model of cellular neural networks with variable delay can be expressed as

$$\frac{du(t)}{dt} = -u(t) + Af(u(t)) + Bf(u(t-\tau(t))) + J, \quad (1)$$

where $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in \mathscr{R}^n$ is the state vector associated with the neurons, $A \in \mathscr{R}^{n\times n}$ is the feedback matrix, $B \in \mathscr{R}^{n\times n}$ is the delayed feedback matrix, $f(u(t)) = (f_1(u_1(t)), f_2(u_2(t)), \ldots, f_n(u_n(t)))^T$ is the output function, $J = (J_1, J_2, \ldots, J_n)^T$ is a constant external input vector, and the output equations are given by

$$f_i(u_i(\cdot)) = \frac{1}{2}(|u_i(\cdot)+1| - |u_i(\cdot)-1|), \quad i = 1, 2, \ldots, n. \quad (2)$$

By lemma 2.1 in [2], there exists at least one equilibrium point for system (1). Let $u_* = (u_{*1}, u_{*2}, \ldots, u_{*n})^T$ be an equilibrium of system (1), and define $x(t) = u(t) - u_*$, then system (1) can be rewritten as

$$\dot{x}(t) = -x(t) + Ag(x(t)) + Bg(x(t-\tau(t))), \quad (3)$$

where $g(x(\cdot)) = (g_1(x_1(\cdot)), g_2(x_2(\cdot)), \ldots, g_n(x_n(\cdot)))^T$ and $g_i(x_i(\cdot)) = f_i(x_i(\cdot) + u_{*i}) - f_i(u_{*i})$. Then the origin $0 = (0, 0, \ldots, 0)^T$ is the equilibrium of system (3).

It is known that parametric uncertainty can enter into systems, as well as cellular neural networks, due to the modelling inaccuracies and/or changes in the environment of the model. The model of uncertain cellular neural network with time-varying delay can be formulated by:

$$\dot{x}(t) = -x(t) + (A + \Delta A(t))g(x(t)) + (B + \Delta B(t))g(x(t-\tau(t))). \quad (4)$$

Based on equation (4), now we consider the following impulsive switched cellular neural networks

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -x_i(t) + \displaystyle\sum_{j=1}^{n}(a_{ij}^{(\alpha(t))} + \Delta a_{ij}^{(\alpha(t))}(t))g(x_j(t)) \\ \qquad + \displaystyle\sum_{j=1}^{n}(b_{ij}^{(\alpha(t))} + \Delta b_{ij}^{(\alpha(t))}(t))g(x_j(t-\tau^{(\alpha(t))}(t))), \\ \qquad t \geq 0, t \neq t_k, \\ x_i(t^+) = F_i(\alpha(t^-), \alpha(t^+), x_i(t^-)), \\ \qquad t = t_k, i \in \{1, 2, \ldots, n\}, \alpha(t) \in \aleph, k = 1, 2, \ldots, \end{cases} \quad (5)$$

or

$$\begin{cases} \frac{dx(t)}{dt} = -x(t) + (A^{(\alpha(t))} + \Delta A^{(\alpha(t))}(t))g(x(t)) \\ \quad + (B^{(\alpha(t))} + \Delta B^{(\alpha(t))}(t))g(x(t-\tau^{(\alpha(t))}(t))), \\ \quad t \geq 0, t \neq t_k, \\ x(t^+) = F(\alpha(t^-), \alpha(t^+), x(t^-)), \\ \quad t = t_k, k = 1, 2, \ldots, \alpha(t) \in \aleph, \end{cases} \tag{6}$$

where $\Delta A(t)$ and $\Delta B(t)$ are parametric uncertainties, which are continuous and norm bounded matrix-valued functions of t, and satisfy $\|\Delta A(t)\| \leq \beta_1, \|\Delta B(t)\| \leq \beta_2$, where β_1, β_1 are given positive scalars, $\alpha(t)$ is a switching signal which takes its values in the finite set $\aleph = \{1, 2, \ldots, N\}$, $x(\cdot) = [x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot)]^T$ is the state vector, $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)), \ldots, g_n(x_n(\cdot))]^T$, $x(t - \tau^{(\alpha(t))}(t)) = [x_1(t - \tau^{(\alpha(t))}(t)), x_2(t - \tau^{(\alpha(t))}(t)), \ldots, x_n(t - \tau^{(\alpha(t))}(t))]^T$, $\tau^{(\alpha(t))}(t) \geq 0(\alpha(t) \in \aleph)$ is the delay parameter, $F(\alpha(t^-), \alpha(t^+), x(t^-)) \in \mathscr{R}^n$ satisfying $F(\alpha(t^-), \alpha(t^+), 0) = 0$, are the impulses at moments t_k and $t_1 < t_2 < \ldots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$. As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t \mapsto x_i(t)$, we assume that $x_i(t_k) \equiv x_i(t_k^-)$. It is clear that, in general, the derivatives $\dot{x}(t_k)$ do not exist. On the other hand, according to the first equality of (5) there exist the limits $\dot{x}(t_k^{\pm})$, so we assume that $\dot{x_i}(t_k) \equiv \dot{x_i}(t_k^-)$. It's easy to see that g_i is globally Lipschitz continuous with Lipschtiz constant $\mu_i = 1$ for $i = 1, 2, \ldots, n$, i.e $|g_i(u) - g_i(v)| \leq |u - v|, \forall u, v \in \mathscr{R}$, and we give the uniform initial conditions for system (5) as $x(t) = \phi(t), t \in [-\tau^*, 0]$, where $\phi(t)$ is continuous on $[-\tau^*, 0], \tau^* = \max\{\sup_t(\tau^{(\alpha(t))}(t))\}, \alpha(t) \in \aleph$.

Let $x \in \mathscr{R}^n, A \in \mathscr{R}^{n\times n}$, we use the following notation

$$\|x\| = \sqrt{\sum_{i=1}^n x_i^2}, \quad \|A\| = \sup_{x\neq 0} \frac{\|Ax\|}{\|x\|}.$$

In order to discuss the exponential stability properties of system (5), we define the concept of exponential stability as following.

Definition 2.1: The zero solution of system(5) is said to be exponentially stable if there exist $\varepsilon \geq 1, \beta > 0$, such that for any $t \geq 0$ and $\phi \in C([-\tau^*, 0], R^n)$

$$\|x(t)\| \leq \varepsilon\|\phi\|e^{-\beta t},$$

where $\|\phi\| = \sqrt{[\sum_{i=1}^n \sup_{-\tau^*\leq t\leq 0} \phi_i^2(t)]}$, $C([-\tau^*, 0], \mathscr{R}^n)$ is the Banach space of continuous functions, which map $[-\tau^*, 0]$ to $\mathscr{R}^n$ with the topology of uniform convergence. For further discussion, we introduce the following lemma.

Lemma 2.1: (Halanays inequality) Let $a > b > 0$ and $v(t)$ be a non-negative continuous function on $[t_0 - \tau, t_0]$, and satisfy the following inequality:

$$D^+v(t) \leq -av(t) + b\sup_{t-\tau\leq s\leq t} v(s), t \geq t_0,$$

where τ is a non-negative constant, then there exist constants $k, \delta > 0$ satisfy $v(t) \leq ke^{-\delta(t-t_0)}(t \geq t_0)$, where $k = \sup_{t_0-\tau\leq s\leq t_0} v(s)$, and δ is unique positive solution of the following equation: $\delta = a - be^{\delta\tau}$.

III. Main results

In this section, we consider the exponential stability for the switched neural networks.

Theorem 3.1: If

$$\max\{\sup_t[\|A_{22}^{(\alpha(t))} + \Delta A_{22}^{(\alpha(t))}(t)\| + \|B_{22}^{(\alpha(t))} + \Delta B_{22}^{(\alpha(t))}(t)\|]\} < 1,$$

then every subsystems of equation (5) are exponentially stable.

Proof. Consider the ith subsystem

$$\begin{aligned} \frac{dx(t)}{dt} &= -x(t) + (A^{(i)} + \Delta A^{(i)}(t))(g(x(t))) + (B^{(i)} \\ &\quad + \Delta B^{(i)}(t))(g(x(t - \tau^{(i)}(t))), i \in \aleph. \end{aligned} \tag{7}$$

Let us divide the set $H^{(i)} = \{1, 2, \ldots, n\}$ into subsets $H_1^{(i)}, H_2^{(i)}$ and $H_3^{(i)}$, such that: $H^{(i)} = H_1^{(i)} \cup H_2^{(i)} \cup H_3^{(i)}$ where $H_1^{(i)} = \{j \in H^{(i)}|u_{*j}^{(i)} > 1\}$, $H_2^{(i)} = \{j \in H^{(i)}| -1 \leq u_{*j}^{(i)} \leq 1\}$, $H_3^{(i)} = \{j \in H^{(i)}|u_{*j}^{(i)} < -1\}$. We may rearrange the order of $x_1, x_2, \ldots, x_n$ such that $H_1^{(i)} = \{1, 2, \ldots, r\}$, $H_2^{(i)} = \{r+1, r+2, \ldots, r+m\}$, $H_3^{(i)} = \{r+m+1, r+m+2, \ldots, n\}$, where $r, m, n-r-m$ are non-negative integers. The variables of system (7) are reordered, but for convenience, we may use the same symbols as those in system (7).

Let

$$x(t) = (x_{(1)}^T(t), x_{(2)}^T(t), x_{(3)}^T(t))^T$$

where

$$\begin{aligned} x_{(1)}(t) &= (x_1(t), x_2(t), \ldots, x_r(t))^T, \\ x_{(2)}(t) &= (x_{r+1}(t), x_{r+2}(t), \ldots, x_{r+m}(t))^T, \\ x_{(3)}(t) &= (x_{r+m+1}(t), x_{r+m+2}(t), \ldots, x_n(t))^T. \end{aligned}$$

So system (7) can be decomposed into

$$\begin{cases} \frac{dx_{(1)}(t)}{dt} = -x_{(1)}(t) + (A_{11}^{(i)} + \Delta A_{11}^{(i)}(t))g(x_{(1)}(t)) \\ \quad + (A_{12}^{(i)} + \Delta A_{12}^{(i)}(t))g(x_{(2)}(t)) \\ \quad + (A_{13}^{(i)} + \Delta A_{13}^{(i)}(t))g(x_{(3)}(t)) \\ \quad + (B_{11}^{(i)} + \Delta B_{11}^{(i)}(t))g(x_{(1)}(t - \tau^{(i)}(t))) \\ \quad + (B_{12}^{(i)} + \Delta B_{12}^{(i)}(t))g(x_{(2)}(t - \tau^{(i)}(t))) \\ \quad + (B_{13}^{(i)} + \Delta B_{13}^{(i)}(t))g(x_{(3)}(t - \tau^{(i)}(t))), \\ \frac{dx_{(2)}(t)}{dt} = -x_{(2)}(t) + (A_{21}^{(i)} + \Delta A_{21}^{(i)}(t))g(x_{(1)}(t)) \\ \quad + (A_{22}^{(i)} + \Delta A_{22}^{(i)}(t))g(x_{(2)}(t)) \\ \quad + (A_{23}^{(i)} + \Delta A_{23}^{(i)}(t))g(x_{(3)}(t)) + (B_{21}^{(i)} \\ \quad + \Delta B_{21}^{(i)}(t))g(x_{(1)}(t - \tau^{(i)}(t))) \\ \quad + (B_{22}^{(i)} + \Delta B_{22}^{(i)}(t))g(x_{(2)}(t - \tau^{(i)}(t))) \\ \quad + (B_{23}^{(i)} + \Delta B_{23}^{(i)}(t))g(x_{(3)}(t - \tau^{(i)}(t))), \\ \frac{dx_{(3)}(t)}{dt} = -x_{(3)}(t) + (A_{31}^{(i)} + \Delta A_{31}^{(i)}(t))g(x_{(1)}(t)) \\ \quad + (A_{32}^{(i)} + \Delta A_{32}^{(i)}(t))g(x_{(2)}(t)) \\ \quad + (A_{33}^{(i)} + \Delta A_{33}^{(i)}(t))g(x_{(3)}(t)) \\ \quad + (B_{31}^{(i)} + \Delta B_{31}^{(i)}(t))g(x_{(1)}(t - \tau^{(i)}(t))) \\ \quad + (B_{32}^{(i)} + \Delta B_{32}^{(i)}(t))g(x_{(2)}(t - \tau^{(i)}(t))) \\ \quad + (B_{33}^{(i)} + \Delta B_{33}^{(i)}(t))g(x_{(3)}(t - \tau^{(i)}(t))), \end{cases} \tag{8}$$

where

$$A^{(i)} = \begin{bmatrix} A_{11}^{(i)} & A_{12}^{(i)} & A_{13}^{(i)} \\ A_{21}^{(i)} & A_{22}^{(i)} & A_{23}^{(i)} \\ A_{31}^{(i)} & A_{32}^{(i)} & A_{33}^{(i)} \end{bmatrix},$$

$$B^{(i)} = \begin{bmatrix} B_{11}^{(i)} & B_{12}^{(i)} & B_{13}^{(i)} \\ B_{21}^{(i)} & B_{22}^{(i)} & B_{23}^{(i)} \\ B_{31}^{(i)} & B_{32}^{(i)} & B_{33}^{(i)} \end{bmatrix},$$

$$\Delta A^{(i)}(t) = \begin{bmatrix} \Delta A_{11}^{(i)}(t) & \Delta A_{12}^{(i)}(t) & \Delta A_{13}^{(i)}(t) \\ \Delta A_{21}^{(i)}(t) & \Delta A_{22}^{(i)}(t) & \Delta A_{23}^{(i)}(t) \\ \Delta A_{31}^{(i)}(t) & \Delta A_{32}^{(i)}(t) & \Delta A_{33}^{(i)}(t) \end{bmatrix},$$

$$\Delta B^{(i)}(t) = \begin{bmatrix} \Delta B_{11}^{(i)}(t) & \Delta B_{12}^{(i)}(t) & \Delta B_{13}^{(i)}(t) \\ \Delta B_{21}^{(i)}(t) & \Delta B_{22}^{(i)}(t) & \Delta B_{23}^{(i)}(t) \\ \Delta B_{31}^{(i)}(t) & \Delta B_{32}^{(i)}(t) & \Delta B_{33}^{(i)}(t) \end{bmatrix},$$

$$(g^T(x_{(1)}(\cdot)), g^T(x_{(2)}(\cdot)), g^T(x_{(3)}(\cdot))^T = g(x(\cdot)+u_*^{(i)}) - g(u_*^{(i)}).$$

Let $k = \min\{\min_{j\in H_1^{(i)}}(u_{*j}^{(i)} - 1), \min_{j\in H_3^{(i)}}(-1 - u_{*j}^{(i)})\}$, then $k > 0$. Assume that the initial function ϕ satisfied $\sup_{-\tau^*\le t\le 0} |\phi_j(t)| < k, (j = 1, 2, \ldots, n)$.

By continuity, there exists a constant $T > 0$, such that for any $t \in [-\tau^*, T), |x_j(t)| < k$. Therefore, for $\forall t \in [0, T)$, we have

$$g(x_j(t) + u_{*j}^{(i)}) - g(u_{*j}^{(i)}) = 0, \forall j \in H_1^{(i)} \cup H_3^{(i)},$$
$$g(x_j(t - \tau^{(i)}(t)) + u_{*j}^{(i)}) - g(u_{*j}^{(i)}) = 0, \forall j \in H_1^{(i)} \cup H_3^{(i)}.$$

Thus $g(x_{(1)}(t)) \equiv g(x_{(3)}(t)) \equiv 0, g(x_{(1)}(t - \tau^{(i)}(t))) \equiv g(x_{(3)}(t - \tau^{(i)}(t))) \equiv 0$.

It follows that, for any $t \in [0, T]$,

$$\begin{cases} \dfrac{dx_{(1)}(t)}{dt} = -x_{(1)}(t) + (A_{12}^{(i)} + \Delta A_{12}^{(i)}(t))g(x_{(2)}(t)) \\ \qquad + (B_{12}^{(i)} + \Delta B_{12}^{(i)}(t))g(x_{(2)}(t - \tau^{(i)}(t))), \\ \dfrac{dx_{(2)}(t)}{dt} = -x_{(2)}(t) + (A_{22}^{(i)} + \Delta A_{22}^{(i)}(t))g(x_{(2)}(t)) \\ \qquad + (B_{22}^{(i)} + \Delta B_{22}^{(i)}(t))g(x_{(2)}(t - \tau^{(i)}(t))), \\ \dfrac{dx_{(3)}(t)}{dt} = -x_{(3)}(t) + (A_{32}^{(i)} + \Delta A_{32}^{(i)}(t))g(x_{(2)}(t)) \\ \qquad + (B_{32}^{(i)} + \Delta B_{32}^{(i)}(t))g(x_{(2)}(t - \tau^{(i)}(t))). \end{cases} \quad (9)$$

By the condition of Theorem 3.1, there exist an $\varepsilon > 0$ such that:

$$2 - 2\sup_t[\|A_{22}^{(i)} + \Delta A_{22}^{(i)}(t)\|] - (1 + e^{\varepsilon\tau^*})\sup_t[\|B_{22}^{(i)}$$
$$+\Delta B_{22}^{(i)}(t)\|] - \varepsilon \ge 0.$$

We construct the Lyapunov functional for the second equation of system (9) as:

$$V(t, x_{(2)}(t)) = e^{\varepsilon t}\|x_{(2)}(t)\|^2.$$

Along the trajectories of the second equation of system (9), the derivative of $V(t, x_{(2)}(t))$ is

$$\begin{aligned} &\dot{V}(t, x_{(2)}(t)) \\ =\ & \varepsilon e^{\varepsilon t}\|x_{(2)}(t)\|^2 + 2e^{\varepsilon t}x_{(2)}^T(t)\dot{x}_{(2)}(t) \\ =\ & \varepsilon V(t, x_{(2)}(t)) + 2e^{\varepsilon t}x_{(2)}^T(t)[-x_{(2)}(t) \\ & +(A_{22}^{(i)} + \Delta A_{22}^{(i)}(t))g(x_{(2)}(t)) \\ & +(B_{22}^{(i)} + \Delta B_{22}^{(i)}(t))g(x_{(2)}(t - \tau^{(i)}(t)))] \\ \le\ & \varepsilon V(t, x_{(2)}(t)) - 2e^{\varepsilon t}x_{(2)}^T(t)x_{(2)}(t) \\ & +2e^{\varepsilon t}\|A_{22}^{(i)} + \Delta A_{22}^{(i)}(t)\| \cdot \|x_{(2)}^T(t)\| \cdot \|g(x_{(2)}(t))\| \\ & +2e^{\varepsilon t}\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\| \cdot \|x_{(2)}^T(t)\| \cdot \|g(x_{(2)}(t - \tau^{(i)}(t)))\| \\ \le\ & \varepsilon V(t, x_{(2)}(t)) - 2e^{\varepsilon t}x_{(2)}^T(t)x_{(2)}(t) \\ & +2e^{\varepsilon t}\|A_{22}^{(i)} + \Delta A_{22}^{(i)}(t)\| \cdot \|x_{(2)}^T(t)\| \cdot \|x_{(2)}(t)\| \\ & +2e^{\varepsilon t}\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\| \cdot \|x_{(2)}^T(t)\| \cdot \|x_{(2)}(t - \tau^{(i)}(t))\| \\ =\ & \varepsilon V(t, x_{(2)}(t)) - 2V(t, x_{(2)}(t)) \\ & +2\|A_{22}^{(i)} + \Delta A_{22}^{(i)}(t)\|V(t, x_{(2)}(t)) \\ & +2e^{\varepsilon t}\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\| \cdot \|x_{(2)}^T(t)\| \cdot \|x_{(2)}(t - \tau^{(i)}(t))\| \\ \le\ & \varepsilon V(t, x_{(2)}(t)) - 2V(t, x_{(2)}(t)) + 2\|A_{22}^{(i)} \\ & +\Delta A_{22}^{(i)}(t)\|V(t, x_{(2)}(t)) \\ & +e^{\varepsilon t}\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\| \cdot (\|x_{(2)}^T(t)\|^2 + \|x_{(2)}(t - \tau^{(i)}(t))\|^2) \\ =\ & -[2 - \varepsilon - 2\|A_{22}^{(i)} + \Delta A_{22}^{(i)}(t)\| - \\ & \|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\|]V(t, x_{(2)}(t)) \\ & +e^{\varepsilon t}\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\| \cdot \|x_{(2)}(t - \tau^{(i)}(t))\|^2 \\ \le\ & -\{2 - \varepsilon - 2\sup_t[\|A_{22}^{(i)} + \Delta A_{22}^{(i)}(t)\|] \\ & -\sup_t[\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\|]\}V(t, x_{(2)}(t)) \\ & +e^{\varepsilon\tau^*}\sup_t[\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\|] \cdot \overline{V(t, x_{(2)}(t))}, \end{aligned} \quad (10)$$

where $\overline{V(t, x_{(2)}(t))} = \sup_{t-\tau^*\le s\le t} V(s, x_{(2)}(s))$.
Let $a = 2 - \varepsilon - 2\sup_t[\|A_{22}^{(i)} + \Delta A_{22}^{(i)}(t)\|] - \sup_t[\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\|]$, $b = e^{\varepsilon\tau^*}\sup_t\|B_{22}^{(i)} + \Delta B_{22}^{(i)}(t)\|$, from lemma2.1 we obtain

$$V(t, x_{(2)}(t)) \le \overline{V(0)}e^{-\delta^{(i)}t} \le \|\phi\|^2 e^{-(\delta^{(i)}-\varepsilon)t},$$

where $\delta^{(i)}$ is the unique positive root of equation $\delta^{(i)} = a - be^{-\delta^{(i)}t}$, and it can be sufficient small. Then, we have

$$\|x_{(2)}(t)\| \le \|\phi\|e^{-\delta^{(i)}t}, \quad (11)$$

for all $t \in [0, T)$, and $\delta^{(i)} \in (0, 1)$. For the first and the third equations of system (9), by using the method of variation of parameters, we get

$$\begin{aligned} x_{(j)}(t) &= x_{(j)}(0)e^{-t} + \int_0^t e^{-(t-s)}\{[A_{j2}^{(i)} \\ &\quad +\Delta A_{j2}^{(i)}(s)]g(x_{(2)}(s)) + [B_{j2}^{(i)} + \Delta B_{j2}^{(i)}(s)] \\ &\quad \times g(x_{(2)}(s - \tau^{(i)}(s)))\}ds, (j = 1, 3). \end{aligned} \quad (12)$$

In views of (11), we obtain

$$\begin{aligned}
&\|x_{(j)}(t)\| \\
\leq\ & \|x_{(j)}(0)\|e^{-t} \\
&+\int_0^t e^{-(t-s)}\{\|A_{j2}^{(i)}+\Delta A_{j2}^{(i)}(s)\|\cdot\|g(x_{(2)}(s))\| \\
&+\|B_{j2}^{(i)}+\Delta B_{j2}^{(i)}(s)\|\cdot\|g(x_{(2)}(s-\tau^{(i)}(s)))\|\}ds \\
\leq\ & \|x_{(j)}(0)\|e^{-t} \\
&+\int_0^t e^{-(t-s)}\{\|A_{j2}^{(i)}+\Delta A_{j2}^{(i)}(s)\|\cdot\|x_{(2)}(s)\| \\
&+\|B_{j2}^{(i)}+\Delta B_{j2}^{(i)}(s)\|\cdot\|x_{(2)}(s-\tau^{(i)}(s))\|\}ds \\
\leq\ & \|\phi\|e^{-t}+\int_0^t e^{-(t-s)}\{\|A_{j2}^{(i)}+\Delta A_{j2}^{(i)}(s)\|\cdot\|\phi\|e^{-\delta^{(i)}s} \\
&+\|B_{j2}^{(i)}+\Delta B_{j2}^{(i)}(s)\|\cdot\|\phi\|e^{-\delta^{(i)}s}e^{\delta^{(i)}\tau^{(i)}(s)}\}ds \\
\leq\ & \|\phi\|e^{-t}+\int_0^t e^{-(t-s)}\{\|A_{j2}^{(i)}+\Delta A_{j2}^{(i)}(s)\|\cdot\|\phi\|e^{-\delta^{(i)}s} \\
&+\|B_{j2}^{(i)}+\Delta B_{j2}^{(i)}(s)\|\cdot\|\phi\|e^{-\delta^{(i)}s}e^{\delta^{(i)}\tau^*}\}ds \\
\leq\ & \|\phi\|\{e^{-t}+\sup_t[\|A_{j2}^{(i)}+\Delta A_{j2}^{(i)}(t)\| \\
&+e^{\delta^{(i)}\tau^*}\|B_{j2}^{(i)}+\Delta B_{j2}^{(i)}(t)\|\}\int_0^t e^{-(t-(1-\delta^{(i)})s)}ds\} \\
\leq\ & \{1+\frac{1}{1-\delta^{(i)}}\sup_t[\|A_{j2}^{(i)}+\Delta A_{j2}^{(i)}(t)\| \\
&+e^{\delta^{(i)}\tau^*}\|B_{j2}^{(i)}+\Delta B_{j2}^{(i)}(t)\|]\}\|\phi\|e^{-\delta^{(i)}t} \\
=\ & M_j^{(i)}(\delta^{(i)})\|\phi\|e^{-\delta^{(i)}t}, \qquad (13)
\end{aligned}$$

where $M_j^{(i)}(\delta^{(i)}) = 1+\frac{1}{1-\delta^{(i)}}\sup_t[\|A_{j2}^{(i)}+\Delta A_{j2}^{(i)}(t)\| + e^{\delta^{(i)}\tau^*}\|B_{j2}^{(i)}+\Delta B_{j2}^{(i)}(t)\|]$.

Let $M^{(i)} = \max(1, M_1^{(i)}(\delta^{(i)}), M_3^{(i)}(\delta^{(i)}))$, then we have $M_j^{(i)}(\delta^{(i)}) < M^{(i)}$ for $j = 1,2,3$. Since $0 < \delta^{(i)} < 1$, and if we choose the initial function ϕ such that $\|\phi\| \leq \frac{k}{M^{(i)}}$,then we obtain

$$\begin{aligned}
\|x_{(j)}(t)\| &\leq M_j^{(i)}(\delta^{(i)})\|\phi\|e^{-\delta^{(i)}t} \\
&\leq M^{(i)}\|\phi\|e^{-\delta^{(i)}t} < k, \forall t\in[0,T). \qquad (14)
\end{aligned}$$

By repeating these procedures, we can ensure that the same result holds for $t \in [T,T_1),\ldots,[T_{n-1},T_n)$ with $T_n \to \infty$ when $n \to \infty$. So under the condition of the theorem, the existing interval of solution of system (7) is $[0,+\infty)$ and zero solution of system (7) is exponential stable, because of the arbitrary of i, we can conclude that every subsystems of equation (5) are exponentially stable, which complete the proof.

Theorem 3.2: The switched system (5) is exponential stable, if the following conditions are satisfied:

(i) $\max\{\sup_t[\|A_{22}^{(\alpha(t))} + \Delta A_{22}^{(\alpha(t))}(t)\| + \|B_{22}^{(\alpha(t))} + \Delta B_{22}^{(\alpha(t))}(t)\|]\} < 1$;

(ii) $\|F(\alpha(t^-),\alpha(t^+),x(t))\| \leq \|F(\alpha(t^-),\alpha(t^+)\|\cdot\|x(t))\|$;

(iii) There exist positive scalars $\overline{\delta},\delta,T_d$ such that $0<\overline{\delta}<\delta<\min_{i\in\aleph}\{\delta^{(i)}\}$, $0<T_d=\inf\{T_k : T_k = t_k-t_{k-1}\}, k=1,2,\ldots,$ and

$$t_{k+1}-t_k \geq \frac{\ln(\|F(\alpha(t_k^-),\alpha(t_k^+))\|)M^{(\alpha(t_k))})}{\overline{\delta}}. \qquad (15)$$

Proof. For convenience, we let $x^{\alpha(t_k)}(t)$ denotes the state equation for system (5) when $t\in(t_k,t_{k+1}]$, and $M^{(\alpha(t))}$ denotes $M^{(\alpha(t))}(\delta^{(\alpha(t))})$. From Theorem 3.1, for all $t\in[0,t_1)$ and $t=t_1$ we have

$$\|x^{(\alpha(0))}(t)\| \leq M^{(\alpha(0))}\cdot\|\phi\|e^{-\delta^{(\alpha(0))}t},$$

$$\|x^{(\alpha(0))}(t_1)\| \leq M^{(\alpha(0))}\cdot\|\phi\|e^{-\delta^{(\alpha(0))}t_1}.$$

In views of condition (ii), we can obtain

$$\begin{aligned}
\|x^{(\alpha(t_1))}(t_1^+)\| &= \|F(\alpha(t_1^-),\alpha(t_1^+),x^{(\alpha(0))}(t_1))\| \\
&\leq \|F(\alpha(t_1^-),\alpha(t_1^+))\|\cdot\|x^{(\alpha(0))}(t_1)\|, \\
&\leq \|F(\alpha(t_1^-),\alpha(t_1^+))\| \\
&\quad\times M^{(\alpha(0))}\cdot e^{-\delta^{(\alpha(0))}t_1}\|\phi\|. \qquad (16)
\end{aligned}$$

Similarly, when $t\in(t_1,t_2]$, we can get

$$\begin{aligned}
\|x^{(\alpha(t_1))}(t)\| &\leq M^{(\alpha(t_1))}\cdot e^{-\delta^{(\alpha(t_1))}(t-t_1)}\|x^{(\alpha(t_1))}(t_1^+)\| \\
&\leq \|F(\alpha(t_1^-),\alpha(t_1^+))\|M^{(\alpha(0))} \\
&\quad\times e^{-\delta^{(\alpha(0))}t_1}M^{(\alpha(t_1))}\cdot e^{-\delta^{(\alpha(t_1))}(t-t_1)}\|\phi\|. \qquad (17)
\end{aligned}$$

By direct calculation, when $t_1 < t_k < t \leq t_{k+1}$, we have

$$\begin{aligned}
\|x^{(\alpha(t_k))}(t)\| &\leq \prod_{s=1}^k \|F(\alpha(t_s^-),\alpha(t_s^+))\|\cdot M^{(\alpha(0))} \\
&\quad\cdot M^{(\alpha(t_1))}\ldots M^{(\alpha(t_k))}e^{-\delta^{(\alpha(0))}t_1} \\
&\quad\times e^{-\delta^{(\alpha(t_1))}(t_2-t_1)-\ldots-\delta^{(\alpha(t_k))}(t-t_k)}\cdot\|\phi\| \\
&\leq \prod_{s=1}^k \|F(\alpha(t_s^-),\alpha(t_s^+))\|\cdot M^{(\alpha(0))}\cdot M^{(\alpha(t_1))} \\
&\quad\times M^{(\alpha(t_k))}\times e^{-\overline{\delta}t}\cdot e^{-(\delta-\overline{\delta})t}\|\phi\| \\
&\leq \prod_{s=1}^k \|F(\alpha(t_s^-),\alpha(t_s^+))\| \\
&\quad\times M^{(\alpha(t_s))}e^{-\overline{\delta}(t_s-t_{s-1})}\times Me^{-(\delta-\overline{\delta})t}\|\phi\|, \qquad (18)
\end{aligned}$$

where $M=\max M^{(\alpha(t_k))}, k=0,1,\ldots,$. From the condition (iii) and the arbitrary of k, we obtain

$$\|x(t)\| \leq Me^{-(\delta-\overline{\delta})t}\|\phi\|, \qquad (19)$$

which complete the proof.

Remark 1: For any switching signal $\alpha(t)$ and any $T_2 > T_1 \geq 0$, let $N_{\alpha(t)}(T_1,T_2)$ denotes the number of switching signal $\alpha(t)$ over the interval$[T_1,T_2]$, and for given $T_a > 0, N_0 \geq 0$, if $N_{\alpha(t)}(T_1,T_2) \leq N_0 + \frac{T_2-T_1}{T_a}$, the positive constant T_a is referred to as average dwell time. Utilizing the average dwell time, we can obtain the following Theorem.

Theorem 3.3: The switched system (5) is exponential stable, if the following conditions are satisfied.

(i) $\max\{\sup_t[\|A_{22}^{(\alpha(t))} + \Delta A_{22}^{(\alpha(t))}(t)\| + \|B_{22}^{(\alpha(t))} + \Delta B_{22}^{(\alpha(t))}(t)\|]\} < 1$;

(ii) There exist arbitrary scalars $\overline{\lambda},\mu$ satisfying $0<\overline{\lambda}<\delta, 0<\mu<\infty$ such that $\alpha(t)\in[\tau_a^*,N_0]$, where $N_0 = \frac{\mu}{a}$,$\tau_a^* = \frac{a}{\delta-\overline{\lambda}}$, $a = \inf_{\lambda>0}\{\lambda : \max(\|F(\alpha(t_k^-),\alpha(t_k^+)\|M) \leq e^{\lambda}\}$, τ_a^* denotes the average dwell time.

Proof. Similar to the proof of Theorem 3.2, we can get

$$\begin{aligned}\|x^{(\alpha(t_k))}(t)\| &\le \prod_{s=1}^{k}\|F(\alpha(t_s^-),\alpha(t_s^+))\|M^{(\alpha(t_k))}\times Me^{-\delta t}\|\phi\| \\ &\le (\max(\|F(\alpha(t_k^-),\alpha(t_k^+))\|M)^k\times Me^{-\delta t}\|\phi\|.\end{aligned}$$

In views of the define of average dwell time and condition (ii), when $\alpha(t)\in[\tau_a^*,N_0]$, we have $k\le N_0+\frac{t-t_0}{\tau_a^*}(t_0=0)$, so we can obtain

$$\begin{aligned}\|x^{(\alpha(t_k))}(t)\| &\le e^{a[N_0+\frac{t}{\tau_a^*}]}Me^{-\delta t}\|\phi\| = e^{aN_0+at/\tau_a^*-(\delta-\overline{\delta})t}\|\phi\| \\ &\le Me^{\mu}e^{-\overline{\lambda}t}\|\phi\|, \end{aligned}\tag{20}$$

which complete the proof.

Let $\Delta x(t) = x(t^+)-x(t^-) = F(\alpha(t^-),\alpha(t^+))\times x(t^-), t=t_k, k=1,2,\ldots,\alpha(t)\in\aleph$, then we can easily derive the following corollaries.

Corollary 3.1: The switched system (5) is exponential stable, if the following condition are satisfied:
(i) $\max\{\sup_t[\|A_{22}^{(\alpha(t))}+\Delta A_{22}^{(\alpha(t))}(t)\|+\|B_{22}^{(\alpha(t))}+\Delta B_{22}^{(\alpha(t))}(t)\|]\}<1$;
(ii) there exist positive scalars $\overline{\delta},\delta,T_d$ such that $0<\overline{\delta}<\delta<\min_{i\in\aleph}\{\delta^{(i)}\}$, $0<T_d=\inf\{T_k:T_k=t_k-t_{k-1}\}, k=1,2,\ldots$, and

$$t_{k+1}-t_k\ge\frac{\ln(1+\|F(\alpha(t_k^-),\alpha(t_k^+))\|)M^{(\alpha(t_k))})}{\overline{\delta}}.$$

Proof. If $\Delta x(t)=x(t^+)-x(t^-)=F(\alpha(t^-),\alpha(t^+))\cdot x(t^-)$, then we can get

$$\begin{aligned}\|x(t^+)\| &= \|(I+F(\alpha(t^-),\alpha(t^+)))x(t^-)\| \\ &\le (\|I\|+\|F(\alpha(t^-),\alpha(t^+))\|)\cdot\|x(t^-)\| \\ &= (1+\|F(\alpha(t^-),\alpha(t^+))\|)\cdot\|x(t^-)\|.\end{aligned}$$

Similar to the proof of Theorem 3.2 we obtain

$$\begin{aligned}\|x^{(\alpha(t_k))}(t)\| &\le \prod_{s=1}^{k}(1+\|F(\alpha(t_s^-),\alpha(t_s^+))\|) \\ &\times M^{(\alpha(0))}\cdot M^{(\alpha(t_1))}\ldots M^{(\alpha(t_k))}e^{-\delta^{(\alpha(t_0))}t_1} \\ &\times e^{-\delta^{(\alpha(t_1))}(t_2-t_1)-\ldots-\delta^{(\alpha(t_k))}(t-t_k)}\cdot\|\phi\| \\ &\le \prod_{s=1}^{k}(1+\|F(\alpha(t_s^-),\alpha(t_s^+))\|)M^{(\alpha(0))} \\ &\times M^{(\alpha(t_1))}\ldots M^{(\alpha(t_k))}e^{-\overline{\delta}t}\cdot e^{-(\delta-\overline{\delta})t}\|\phi\| \\ &\le \prod_{s=1}^{k}(1+\|F(\alpha(t_s^-),\alpha(t_s^+))\|) \\ &\times M^{(\alpha(t_s))}e^{-\overline{\delta}(t_s-t_{s-1})}\times Me^{-(\delta-\overline{\delta})t}\|\phi\|, \end{aligned}\tag{21}$$

where $M=\max M^{(\alpha(t_k))}$. From the condition (ii), we obtain

$$\|x(t)\|\le Me^{-(\delta-\overline{\delta})t}\|\phi\|, \tag{22}$$

which complete the proof.

Corollary 3.2: The switched system (5) is exponential stable, if the following conditions are satisfied.
(i) $\max\{\sup_t[\|A_{22}^{(\alpha(t))}+\Delta A_{22}^{(\alpha(t))}(t)\|+\|B_{22}^{(\alpha(t))}+\Delta B_{22}^{(\alpha(t))}(t)\|]\}<1$;
(ii) There exist arbitrary scalars $\overline{\lambda},\mu$ satisfying $0<\overline{\lambda}<\delta, 0<\mu<\infty$ such that $\alpha(t)\in[\tau_a^*,N_0]$, where $N_0=\frac{\mu}{a},\tau_a^*=\frac{a}{\delta-\overline{\lambda}}$, $a=\inf_{\lambda>0}\{\lambda:\max(1+\|F(\alpha(t_k^-),\alpha(t_k^+)\|M)\le e^{\lambda}\}$, τ_a^* denotes the average dwell time.

Remark 2: These impulsive effects are in commonly used in switched system. Obviously, this impulsive form can be seen as special cases of that in system (5), and ours are more general.

Let $\Delta x(t)=F(\alpha(t^-),\alpha(t^+),x(t^-))=-\gamma x(t^-), 0<\gamma<2$, then we can easily derive the following corollaries.

Corollary 3.3: The switched system (5) is exponential stable, if the following condition are satisfied:
(i) $\max\{\sup_t[\|A_{22}^{(\alpha(t))}+\Delta A_{22}^{(\alpha(t))}(t)\|+\|B_{22}^{(\alpha(t))}+\Delta B_{22}^{(\alpha(t))}(t)\|]\}<1$;
(ii) there exist positive scalars $\overline{\delta},\delta,T_d$ such that $0<\overline{\delta}<\delta<\min_{i\in\aleph}\{\delta^{(i)}\}$, $0<T_d=\inf\{T_k:T_k=t_k-t_{k-1}\}, k=1,2,\ldots$, and

$$t_{k+1}-t_k\ge\frac{\ln M_k^{(\alpha(t_k))})}{\overline{\delta}}.$$

Proof. If $x(t)=x(t^+)-x(t^-)=F(\alpha(t^-),\alpha(t^+), x(t^-))=-\gamma x(t^-)$, then we can get

$$\|x(t^+)\|=\|(I-\gamma I)x(t^-)\|=|1-\gamma|\cdot\|x(t^-)\|\le\|x(t^-)\|$$

Similar to the proof of Theorem 3.2 we obtain

$$\begin{aligned}\|x^{(\alpha(t_k))}(t)\| &\le \prod_{s=1}^{k}M^{(\alpha(0))}M^{(\alpha(t_s))}e^{-\delta^{(\alpha(t_0))}t_1} \\ &\quad\times e^{-\delta^{(\alpha(t_1))}(t_2-t_1)-\ldots-\delta^{(\alpha(t_k))}(t-t_k)}\cdot\|\phi\| \\ &\le \prod_{s=1}^{k}M^{(\alpha(0))}M^{(\alpha(t_s))}e^{-\overline{\delta}t}\cdot e^{-(\delta-\overline{\delta})t}\|\phi\| \\ &\le \prod_{s=1}^{k}M^{(\alpha(t_s))}e^{-\overline{\delta}(t_s-t_{s-1})}Me^{-(\delta-\overline{\delta})t}\|\phi\|, \end{aligned}\tag{23}$$

where $M=\max M^{(\alpha(t_k))}$. From the condition (ii), we obtain

$$\|x(t)\|\le Me^{-(\delta-\overline{\delta})t}\|\phi\|, \tag{24}$$

which complete the proof.

Corollary 3.4: The switched system (5) is exponential stable, if the following conditions are satisfied.
(i) $\max\{\sup_t[\|A_{22}^{(\alpha(t))}+\Delta A_{22}^{(\alpha(t))}(t)\|+\|B_{22}^{(\alpha(t))}+\Delta B_{22}^{(\alpha(t))}(t)\|]\}<1$;
(ii) There exist arbitrary scalars $\overline{\lambda},\mu$ satisfying $0<\overline{\lambda}<\delta, 0<\mu<\infty$ such that $\alpha(t)\in[\tau_a^*,N_0]$, where $N_0=\frac{\mu}{a},\tau_a^*=\frac{a}{\delta-\overline{\lambda}}$, $a=\inf_{\lambda>0}\{\lambda:\max(M^{(\alpha_{t_k})})\le e^{\lambda}\}$.

Remark 3: These impulsive effects are in commonly used in neural networks. Obviously, this impulsive form can be also seen as special cases of those in system (5), and ours are more general.

Remark 4: If every subsystems are the same, then the switched system (5) become an ordinary impulsive cellular neural networks

$$\begin{cases} \frac{dx(t)}{dt} = -x(t) + (A + \Delta A(t))g(x(t)) \\ \qquad + (B + \Delta B(t))g(x(t-\tau(t)), t \geq 0, t \neq t_k, \\ x(t^+) = F(\alpha(t^-), \alpha(t^+), x(t^-)), t = t_k, \end{cases} \tag{25}$$

then we can obtain the following corollaries.

Corollary 3.5: The system (25) is exponential stable, if the following condition are satisfied:

(i) $\sup_t\{\|A_{22} + \Delta A_{22}(t)\| + \|B_{22} + \Delta B_{22}(t)\|\} < 1$;

(ii) there exist positive scalars $\overline{\delta}, \delta, T_d$ such that $0 < \overline{\delta} < \delta$, $0 < T_d = \inf\{T_k : T_k = t_k - t_{k-1}\}, k = 1, 2, \ldots$, and

$$t_{k+1} - t_k \geq \frac{\ln(1 + \|F(\alpha(t_k^-), \alpha(t_k^+))\|)M)}{\overline{\delta}}.$$

Corollary 3.6: The system (25) is exponential stable, if the following conditions are satisfied.

(i) $\sup_t[\|A_{22}^{(\alpha(t))} + \Delta A_{22}^{(\alpha(t))}(t)\| + \|B_{22}^{(\alpha(t))} + \Delta B_{22}^{(\alpha(t))}(t)\|] < 1$;

(ii) There exist arbitrary scalars $\overline{\lambda}, \mu$ satisfying $0 < \overline{\lambda} < \delta, 0 < \mu < \infty$ such that $\alpha(t) \in [\tau_a^*, N_0]$, where $N_0 = \frac{\mu}{a}, \tau_a^* = \frac{a}{\delta - \overline{\lambda}}$, $a = \inf_{\lambda > 0}\{\lambda : \max(1 + \|F(\alpha(t_k^-), \alpha(t_k^+)\|M) \leq e^{\lambda}\}$.

IV. Numerical examples

In this section, we will present a simple example to demonstrate the results developed above. Let N=2, and consider the following cellular neural networks:

$$\begin{aligned} \begin{bmatrix} \dot{u_1}(t) \\ \dot{u_2}(t) \end{bmatrix} = & \begin{bmatrix} -u_1(t) \\ -u_2(t) \end{bmatrix} \\ & + \begin{bmatrix} \frac{1}{8}(1+\sin(t)) & \frac{1}{7}(1+\cos(t)) \\ \frac{1}{2}(1+\sin(t)) & \frac{1}{5}(1+\cos(t)) \end{bmatrix} \cdot \begin{bmatrix} f(u_1(t)) \\ f(u_2(t)) \end{bmatrix} \\ & + \begin{bmatrix} -\frac{1}{8}(1+\sin(t)) & -\frac{1}{7}(1+\cos(t)) \\ -\frac{1}{2}(1+\sin(t)) & -\frac{1}{5}(1+\cos(t)) \end{bmatrix} \\ & \times \begin{bmatrix} f(u_1(t-\sin(t))) \\ f(u_2(t-\cos(t))) \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \end{aligned} \tag{26}$$

$$\begin{aligned} \begin{bmatrix} \dot{u_1}(t) \\ \dot{u_2}(t) \end{bmatrix} = & \begin{bmatrix} -u_1(t) \\ -u_2(t) \end{bmatrix} \\ & + \begin{bmatrix} \frac{1}{9}(1+\cos(t)) & \frac{1}{6}(1+\sin(t)) \\ \frac{1}{5}(1+\cos(t)) & \frac{1}{5}(1+\sin(t)) \end{bmatrix} \cdot \begin{bmatrix} f(u_1(t)) \\ f(u_2(t)) \end{bmatrix} \\ & + \begin{bmatrix} -\frac{1}{9}(1+\cos(t)) & -\frac{1}{6}(1+\sin(t)) \\ -\frac{1}{5}(1+\cos(t)) & -\frac{1}{5}(1+\sin(t)) \end{bmatrix} \\ & \times \begin{bmatrix} f(u_1(t-\cos(t))) \\ f(u_2(t-\sin(t))) \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}. \end{aligned} \tag{27}$$

Direct computation shows that $u^* = (3, -1)$ is an equilibrium of system (26) and (27). Let $x(t) = u(t) - u^*$, and $|x_i(t)| \leq 1$, then system (26) and (27) can be rewritten as the follows respectively

$$\begin{cases} \dot{x_1}(t) = -x_1(t) + \frac{1}{7}(1+\cos(t))g(x_2(t)) \\ \qquad - \frac{1}{7}(1+\cos(t))g(x_2(t-\cos(t))), \\ \dot{x_2}(t) = -x_2(t) + \frac{1}{5}(1+\cos(t))g(x_2(t)) \\ \qquad - \frac{1}{5}(1+\cos(t))g(x_2(t-\cos(t))), \end{cases} \tag{28}$$

$$\begin{cases} \dot{x_1}(t) = -x_1(t) + \frac{1}{6}(1+\sin(t))g(x_2(t)) \\ \qquad - \frac{1}{6}(1+\cos(t))g(x_2(t-\sin(t))), \\ \dot{x_2}(t) = -x_2(t) + \frac{1}{5}(1+\sin(t))g(x_2(t)) \\ \qquad - \frac{1}{5}(1+\cos(t))g(x_2(t-\sin(t))). \end{cases} \tag{29}$$

Now, consider the following switched system with impulsive effects

$$\begin{cases} \frac{dx(t)}{dt} = -x(t) + (A^{(\alpha(t))} + \Delta A^{(\alpha(t))}(t))g(x(t)) + (B^{(\alpha(t))} \\ \qquad + \Delta B^{(\alpha(t))}(t))g(x(t-\tau^{(\alpha(t))}(t))), t \geq 0, t \neq t_k, \\ x(t^+) = F(\alpha(t^-), \alpha(t^+)) \cdot x(t^-), t = t_k, k \in \aleph, \alpha(t) \in \{1, 2\}, \end{cases} \tag{30}$$

where

$$A^{(1)} = \begin{bmatrix} \frac{1}{8} & \frac{1}{7} \\ \frac{1}{2} & \frac{1}{5} \end{bmatrix}, \Delta A^{(1)} = \begin{bmatrix} \frac{1}{8}\sin(t) & \frac{1}{7}\cos(t) \\ \frac{1}{2}\sin(t) & \frac{1}{5}\cos(t) \end{bmatrix},$$

$$F(2,1) = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}, F(1,2) = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix},$$

$$B^{(1)} = \begin{bmatrix} -\frac{1}{8} & -\frac{1}{7} \\ -\frac{1}{2} & -\frac{1}{5} \end{bmatrix}, \Delta B^{(1)} = \begin{bmatrix} -\frac{1}{8}\sin(t) & -\frac{1}{7}\cos(t) \\ -\frac{1}{2}\sin(t) & -\frac{1}{5}\cos(t) \end{bmatrix},$$

$$A^{(2)} = \begin{bmatrix} \frac{1}{9} & \frac{1}{6} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}, \Delta A^{(2)} = \begin{bmatrix} \frac{1}{9}\cos(t) & \frac{1}{6}\sin(t) \\ \frac{1}{5}\cos(t) & \frac{1}{5}\sin(t) \end{bmatrix},$$

$$B^{(2)} = \begin{bmatrix} -\frac{1}{9} & -\frac{1}{6} \\ -\frac{1}{5} & -\frac{1}{5} \end{bmatrix}, \Delta B^{(2)} = \begin{bmatrix} -\frac{1}{9}\cos(t) & -\frac{1}{6}\sin(t) \\ -\frac{1}{5}\cos(t) & -\frac{1}{5}\sin(t) \end{bmatrix}.$$

Set $\delta^{(i)} = 0.5, (1 = 1, 2), \overline{\delta} = 0.4$, then we can obtain that $M = \max\{M^{(1)}, M^{(2)}\} = \sup_t\{1 + \frac{1}{2}|1 + \cos(t)|(1 + \sqrt{e})\} = 2 + \sqrt{e}$, $\|F\| \triangleq \max\{\|F_{12}\|, \|F_{21}\|\} = 1$, Since $\max\{\sup_t[\|A_{22}^{(\alpha(t))} + \Delta A_{22}^{(\alpha(t))}(t)\| + \|B_{22}^{(\alpha(t))} + \Delta B_{22}^{(\alpha(t))}(t)\|]\} = \frac{4}{5} < 1$, according to Theorem 3.2, if $t_k - t_{k-1} > \ln(5 + \frac{5}{2}\sqrt{e})$, then the switched system (30) is exponential stable.

V. Conclusion

This paper investigates the problem of exponential stabilization for a class of nonlinear switched systems with cellular neural networks as subsystems. At the same time, parametric uncertainty and a more generally impulsive effects are considered, some stable criterions are derived, which are associated with dwell time and some blocks of the feedback matrices. Our criterion generalizes some known results. Illustrated numerical example shows that the criteria obtained in this paper are valid.

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