Abstract—In this paper, a new method of image edge-detection and characterization is presented. "Parametric Filtering method" uses a judicious defined filter, which preserves the signal correlation structure as input in the autocorrelation of the output. This leads, showing the evolution of the image correlation structure as well as various distortion measures which quantify the deviation between two zones of the signal (the two Hamming signals) for the protection of an image edge.

Keywords—Edge detection, parametrable recursive filter, autocorrelation structure, distortion measurements.

I. INTRODUCTION

The edge detection is not an objective in itself, but it is the first step in a view system; for that end, various detectors have been proposed. They differ in their mathematical and algorithmic properties. This work presents a method which is called correlation structure analysis for the edge detection. This method is combining a parametrable recursive filter with the autocorrelation analysis of the filtered image signal. This analysis will produce a characteristic function for the image spectrum. We will propose then various measurements of the distortion as indicators of the spectral change; these indicators present good performances for the edge detection.

In this article, we suggest a new method to detecting image contours based on the technique called "Parametric Filtering" [8], [9]. To segment the image edge in relatively homogeneous sections, the suggested method combines a parametric filter with the analysis of the lag one autocorrelation of the filtered image signal, once done; it will produce a new characteristic function for the signal image spectrum, based on this new characteristic function. Various distortion measures are proposed as indicators of the spectral change. Initial research showed that these indicators show good performances for edge detection and segmentation and resistance to background noise and fluctuations of dominant spectral peaks.

The methods of parametric filtering use the technique of passage by zero, to detect the maxima points of edge and make edge detection more sophisticated. In each of these methods, one may finds more indicators, such as the number of passages by zero, the uncertainty of spectral distortion measure error, to detect significant variations.

II. AUTOCORRELATION STRUCTURE ANALYSIS

Suppose \( \{X_t\} \) is real-valued stationary signal with mean equal to zero and autocorrelation function equal to

\[
ρ_θ = E(X_{t+k}X_t)/E(X_t^2).
\]

Let's consider the recursive IIR (all-pole) filter \( H(z^{-1};α) \) as defined by

\[
Y_t(α) = \sum_{l=0}^∞ α^l X_{t-1} = ZH_{\alpha\eta\theta}(α) + X_t
\]

where \( α = ηe^{-jθ} \) is a complex number with \( |η| < 1 \) and \( θ \in [−π, π] \), and the overbar represents complex conjugate. Let \( ρ(α) \) be the lag-one (first-order) autocorrelation of \( \{Y_t(α)\} \), namely

\[
ρ(α) = \frac{EY_{t+1}(α)Y_t(α)}{EY_t^2(α)}
\]

For any fixed \( θ \), define the demodulated lag-one autocorrelation of \( \{Y_t(α)\} \) as

\[
γ_θ(η) = \Re(e^{-jθ}ρ(α))
\]

where \( \Re(\cdot) \) represents the real part of a complex number. In this paper, we use \( γ_θ(η) \) as a new characterization function, complementary to the Fourier spectrum, for representing the correlation structure of \( \{X_t\} \). We call this method of analyzing correlation (spectral) structure, the parametric filtering method.

To calculate \( γ_θ(η) \), we do not impose any parametric models or distributional assumptions on \( \{X_t\} \); therefore, the method belongs to the model-free category without explicitly using the spectral densities. For each evaluation of \( γ_θ(η) \), the number of required multiplications is proportional to the length of the signal, and usually a few evaluations are sufficient for edge detection. If necessary, the analysis can also be carried out with a variable sequence 0, using the prototype filter bank.
III. DISTORTION MEASURES

Given two fields of image signal, say \( X^{(1)}_t \) and \( X^{(2)}_t \), the characterization property of \( \gamma_\theta(\eta) \) can be exploited to derive “distortion measures” that quantify the deviation of \( X^{(1)}_t \) and \( X^{(2)}_t \) in their correlation structures.

The PF-based distortion measures that have been found effective in our pilot of image edge-detection include the \( L_p \) distance of \( \gamma_\theta(\eta) \), i.e.:

\[
\gamma^p_\Omega = \left( \int_{\Omega} \left| \gamma^{(1)}_\theta(\eta) - \gamma^{(2)}_\theta(\eta) \right|^p \, d\theta \, d\eta \right)^{1/p}
\]

for \( p \in (0, \infty) \), and the (symmetrized) KL-type divergence measures

\[
\hat{K}_\Omega = \int_{\Omega} \left( K \left[ p^{(1)}_\theta(\eta) \right] + K \left[ p^{(2)}_\theta(\eta) \right] \right) \, d\theta \, d\eta
\]

and

\[
\hat{K}_\Omega = \int_{\Omega} \left[ p^{(2)}_\theta(\eta) K \left[ p^{(1)}_\theta(\eta) \right] + p^{(1)}_\theta(\eta) K \left[ p^{(2)}_\theta(\eta) \right] \right] \, d\theta \, d\eta
\]

In these expression, \( \Omega \) is a subset of \((-\pi, \pi) \times [\eta_a, \eta_b] \), \( \gamma^{(1)}_\theta(\eta) \) and \( \gamma^{(2)}_\theta(\eta) \) are the characterization functions obtained from \( X^{(1)}_t \) and \( X^{(2)}_t \), respectively, and \( p^{(1)}_\theta(\cdot) \) and \( p^{(2)}_\theta(\cdot) \) are normalized “density functions” on \([\eta_a, \eta_b] = (-1, 1)\), taking the form

\[
p_\theta(\eta) = \frac{1}{2} \left( \frac{d \gamma_\theta(\eta)}{d \eta} + \delta(\eta - \eta_a) + 1 \delta(\eta - \eta_a) + \left[ 1 - \gamma_\theta(\eta_a) \right] \delta(\eta - \eta_b) \right) \]

Note that \( p_\theta(\eta) \) also possesses the characterization property because of its equivalence to \( \gamma_\theta(\eta) \). In fact, it is easy to see that

\[
\gamma_\theta(\eta) = 2 \int_{\eta_a}^{\eta_b} p_\theta(\lambda) \, d\lambda - 1
\]

for any \( \eta \in [\eta_a, \eta_b] \). In applications, these distortion measures are discretized using the output \( \gamma_\theta(\eta_k) \) from the filter bank.

IV. MAXIMUM POINTS IN EDGE-DETECTION

In our preliminary experiments of image edge-detection, the approach of the peak choice (ex: [3],[14],[15]) is used for detection [3]. A typical method of the peak choice takes windows of \( 2N \)-points of the original image signal, in each point \( t \), a window is centered in \( t \) and the other shifted in front of \( m \)-points.

For the continuation of all the experiments, we take \( m = N/2 \) (i.e. 50% of overlapping). The two windows are multiplied by N-point Hamming window before they are used in the evaluation of distortion measures. Let the resulting distortion measures be \( D \). Then, the locations of significant peaks in the trajectory of \( D \) are regarded as locations of spectral changes. These locations may be identified from the zero-crossings of difference \( D_t \). A peak in \( D_t \) is considered significant if its magnitude exceeds a threshold \( T \). Improved performance can be achieved in general by smoothing the trajectory of \( D_t \) before the peak-picking step [3],[14].

V. PARAMETRIC FILTERING ALGORITHM

We apply this technique (PF method) to an image \( I(x,y) \), therefore we will follow stages:

We will calculate the parameter \( \alpha \):

\[
\alpha_k = \eta_k e^{j\theta_k}
\]

with \( \eta_k = \eta_a + \frac{(k-1)(\eta_a - \eta_a)}{(m-1)} \) for \( k=1, \ldots, m \)

We will filter \( I(x,y) \) according to lines (X):

\[
Y_x^{(1)} = \overline{X}_x^{(1)} + X_x^{(1)}
\]

The autocorrelation according to line (X):

\[
\rho(\alpha) = \frac{E(Y_x^{(1)}(\alpha)Y_x^{(1)}(\alpha))}{E(Y_x^{(1)}(\alpha))^2}
\]

and then we take:

\[
Y_x^{(1)}(\eta_k) = \Re(e^{-j\theta_k} \rho(\alpha))
\]

We fix 0, \( N \) and \( m \) to measure the distortion by:

The \( D_{kk} \) method following lines (X):

\[
\hat{K}_{\theta,x} = \sqrt{N} \left( \frac{1}{m+1} + \frac{1}{m} \sum_{k=0}^{m} \left[ \frac{\rho^{(1)}_{\theta,x,k}}{\rho^{(2)}_{\theta,x,k}} \right] \right)
\]

where:

\[
p^{(1)}_{\theta,x,k} = \rho^{(1)}_{\theta,x}(\eta_k) + 1 \]

and:

\[
p^{(2)}_{\theta,x,m} = 1 - \rho^{(1)}_{\theta,x}(\eta_m)
\]

and:

\[
p^{(2)}_{\theta,x,k} = \rho^{(1)}_{\theta,x}(\eta_{k+1}) - \rho^{(1)}_{\theta,x}(\eta_k)
\]

for \( k=1, \ldots, m-l \)

the \( D_{LLL} \) method following lines (X):

\[
\gamma^2_{\theta,x} = \sqrt{N} \left( \frac{1}{m} \sum_{k=1}^{m} \left[ \rho^{(1)}_{\theta,x}(\eta_k) - \rho^{(2)}_{\theta,x}(\eta_k) \right]^2 \right)
\]

with: \( \gamma^{(1)}_{\theta,x}(\eta_k) \) and \( \gamma^{(2)}_{\theta,x}(\eta_k) \) are the two filtered signals.

We will filter \( I(x,y) \) according to columns (Y):

\[
Y_y^{(1)} = \overline{X}_y^{(1)} + X_y^{(1)}
\]

the autocorrelation according to columns (Y):
\[ \rho(\alpha) = \frac{E(Y_{\gamma+1} \cdot F_Y(\alpha))}{E[F_Y(\alpha)^2]} \]

and then we take:

\[ Y^{(1)}_\theta(\eta_k) = \Re[e^{-j\theta} \rho^{(1)}(\alpha)] \]

we fix \( \theta, N \) and \( m \) to measure the distortion with:

the \( DKL \) method following columns (Y):

\[ \check{K}_{\theta,y} = \sqrt{N} \frac{1}{m+1} \sum_{k=0}^{m+1} \left[ \frac{p_{\theta,y,k}^{(1)}}{p_{\theta,y,k}^{(2)}} \right] + \left[ \frac{p_{\theta,y,k}^{(2)}}{p_{\theta,y,k}^{(1)}} \right] \]

where: \( p_{\theta,y,k}^{(1)} = \gamma_{\theta,y}^{(1)}(\eta_k) + 1 \) for k=0.

and: \( p_{\theta,y,m}^{(1)} = 1 - \gamma_{\theta,y}^{(1)}(\eta_m) \) for k=m.

and \( p_{\theta,y,k}^{(2)} = \gamma_{\theta,y}^{(1)}(\eta_{k+1}) - \gamma_{\theta,y}^{(1)}(\eta_k) \) for: k=1,...,m-1.

the \( DLL2 \) method following columns (Y):

\[ \check{Y}_{\theta,y} = \sqrt{N} \frac{1}{m} \sum_{k=0}^{m} \left[ \gamma_{\theta,y}^{(1)}(\eta_k) - \gamma_{\theta,y}^{(2)}(\eta_k) \right]^2 \]

with: \( \gamma_{\theta,y}^{(1)}(\eta_k), \gamma_{\theta,y}^{(2)}(\eta_k) \) are the two filtered signals.

finally, we calculate the average of the two distortion measures according to X and Y:

the \( DKL \) method: \( \check{K}_\theta = (\check{K}_{\theta,y} + \check{K}_{\theta,x})/2 \)

the \( DLL2 \) method: \( \check{Y}_\theta = \sqrt{\check{Y}_{\theta,x}^2 + \check{Y}_{\theta,y}^2} \)

we get the following maxima (X) and (Y) of the image to have the desired contour.

VI. PRACTICAL RESULTS

Fig. 3 shows an image of 256*256 dimension, coded on 256 levels of grey, representing various objects and on which we tested the algorithm of the parametric filtering method.

Fig. 4 shows the image edges when \( DKL \) distortion measure is used with the following parameters:

\[ \eta_a = 0.0, \eta_b = 0.9, \phi = 0.00063\pi, m = 2 \]

Whereas Fig.5 illustrates the image edges when \( DLL2 \) distortion measure is used with the parameters:

\[ \eta_a = 0.0, \eta_b = 0.9, \phi = 0.00063\pi, m = 2 \]

Regarding the size of Hamming windows, our tests are carried out on windows of size \( N = d \).

The tests parameters are:

\[ \eta_a, \eta_b, \phi : \text{Parameters of the parametric filter} \]

m: size of the parametric filter.

On the practical level, the parameters which were carried out on the audio segmentation [19] are also adequate for edge detection. According to your Ta-Hsin [19], the adequate parameters are:

\[ \eta_a = 0.0, \eta_b = 0.9, \phi = 0.00063\pi, m = 2 \]

We note that the chains of edges are continuous (Fig. 4 and Fig. 5) this allows the statement of good results.

VII. COMPARISON WITH CANNY METHOD

We have done the comparison founding it on the following criteria.

Precision of the localization of the detected edging: Canny’s method has a good localization for the essential details are visible although some parasiting points still exist within the objects. Parametric filtering method has a good detecting and localization for the localization error is very weak.

The noise filtering performance is estimated by the calculation of the signal-noise ratio (SNR).


<table>
<thead>
<tr>
<th>Performance</th>
<th>The parametric method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canny</td>
<td>DKL</td>
</tr>
<tr>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Average</td>
<td>Strong</td>
</tr>
</tbody>
</table>

According to the practical results obtained, the parametric filtering method 5DKL) has better immunity to the noise. For the non noised pictures, Canny’s method shows a better detection.

**VIII. CONCLUSION**

In this paper, a new method of image edge-detection and characterization is presented. PF method uses a judicious defined filter, which preserves the signal correlation structure as input in the autocorrelation \( \gamma_\theta (\eta) \) of the output. This leads to the TCA plot showing the evolution of the image correlation structure as well as various distortion measures which quantify the deviation between two zones of the signal (the two Hamming signals) for the protection of an image edge.

Future research will include the experimentation in the other edge-detection algorithms such as the multistage dynamic programming [14]. With new distortion measures as well as the complementary exploration of the TCA plots response and distortion measures of the various spectral changes and parameter selection.

Our work opens the field to experimentation and tests on other types of images and with various input parameters.

As the results we obtained are encouraging, we may, consequently, consider some prospects for extension listed hereafter:

1. A study of performance of the parametric filtering method, with a theoretical evaluation as well as an automatic selection of entry parameters for each type of images;

2. The use other distortion measure algorithms.

**REFERENCES**


