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# Effect of a magnetic field on the onset of Marangoni convection in a micropolar fluid

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*Abstract*—With the presence of a uniform vertical magnetic field and suspended particles, thermocapillary instability in a horizontal liquid layer is investigated. The resulting eigenvalue is solved by the Galerkin technique for various basic temperature gradients. It is found that the presence of magnetic field always has a stability effect of increasing the critical Marangoni number.

*Keywords*—Marangoni convection, Magnetic field, Micropolar fluid, Non-uniform thermal gradient, Thermocapillary



## I. Introduction

The analysis of Marangoni convection in a thin fluid layer induced by thermocapillary has many important applications in a number of engineering problems, such as the production of paints, colloids and detergents in chemical engineering. The study of the onset of steady Marangoni convection in an electrically conducting fluid layer with a non-deformable free surface in a magnetic field was initiated by [1]. Later, [2] showed numerically that oscillatory convection cannot occur if the free surface is non-deformable. Subsequently, [3] extended the work of [4] on steady convection to take the effect of the surface deflection into account.

Most of the previous studies were concerned with a uniform vertical temperature gradient in a fluid layer. The analysis of the combined effect of magnetic field and non-uniform basic temperature gradient on steady Marangoni convection in the absence of rotation have been presented by [5]. Also, [6] concluded that the inverted parabolic temperature gradient distribution could be the most stabilizing. The analysis of [6] was extended by [7] to solve the problem of stationary Rayleigh-Bénard convection in a micropolar fluid layer with a non-uniform basic temperature gradient. The fourth order Runge-Kutta-Gill's method and the linear stability theory was used by [8] to attack the problem of oscillatory Bénard-Marangoni convection of an electrically conducting liquid in a magnetic field with a non-uniform temperature gradient. Several authors [9], [10], [11] discussed the effect of feedback control on the onset of convection.

This paper is concerned with the presence of a uniform vertical magnetic field and the effect of a cubic basic temperature distribution in micropolar fluid.

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## II. Mathematical Formulation

The aim of the present work is to examine the stability of a horizontal layer of quiescent micropolar fluid of thickness $d$ in the presence of a magnetic field. Following [12], the linearized and dimensionless governing equations can be written as (cf. [6]),

$$\begin{aligned}&(1+N_1)(D^2-a^2)^2W+N_1(D^2-a^2)\Omega\\&\quad -QD^2W=0, &(1)\\&N_1(D^2-a^2)W-N_3(D^2-a^2)\Omega+\\&\quad 2N_1\Omega=0, &(2)\\&(D^2-a^2)T+f(z)(W-N_5\Omega)=0, &(3)\end{aligned}$$

where $W$, $T$ and $\Omega$ are respectively the amplitudes of the infinitesimal perturbations of velocity, temperature and spin, $N_1=\zeta/(\zeta+\eta)$ is the coupling parameter ($0\le N_1\le 1$), $N_3=\eta'/(\zeta+\eta)$ is the couple stress parameter ($0\le N_3\le m$), $N_5=\beta/(\rho_0 C_v d^2)$ is the micropolar heat conduction parameter ($0\le N_5\le n$). $Q=\mu_m H_o^2 d^2/[(\zeta+\eta)\gamma_m]$ is the Chandrasekhar number and $M_a=\sigma_T\Delta T d/\mu\chi$ is the Marangoni number. Here, $\eta'$ is the shear spin viscosity coefficient, $\zeta$ is the coupling viscosity coefficient or vortex viscosity, $\eta$ is the shear kinematic viscosity coefficient, $\rho_0$ is the reference density, $\beta$ is the micropolar heat conduction coefficient, $\chi$ is the thermal conductivity, $H$ is the magnetic field, $\mu$ is the viscosity, $C_v$ is the specific heat, $\sigma_T$ is the coefficient of surface tension, $g$ is the acceleration due to gravity, $\Delta T$ is the temperature difference between two boundaries ($T_H-T_L$), $m$ and $n$ are real numbers. The differentiation with respect to the vertical coordinate $z$ is denoted by an operator $D=\mathrm{d}/\mathrm{d}z$ and $a$ is the total horizontal wave number.

The layer is assumed to be bounded below by a rigid boundary, which is kept at a constant temperature, and above by a perfectly insulated, flat free surface. Moreover, the spin-vanishing boundary condition is assumed at the boundaries. The boundary conditions, lower and upper, for the amplitudes of the normal mode are then given by

$$W=DW=T=\Omega=0 \quad \text{at} \quad z=0, \qquad (4)$$
$$W=D^2W+a^2M_aT=DT=\Omega=0 \quad \text{at} \quad z=1 \qquad (5)$$

Following [13] and [14], the steady state temperature profile given by

$$\bar{T}_b=\bar{T}_{OS}-a_1(\bar{z}-d)-a_2(\bar{z}-d)^2-a_3(\bar{z}-d)^3, \qquad (6)$$

is considered which precisely represents an experimental data [14], where $(\bar{\ })$ denotes dimensional quantities, $\bar{T}_{OS}$ is the temperature at the upper free surface and $a_i$, $i=1,2,3$ are constants. In non-dimensional form, the $f(z)$ in (3) is given by

$$f(z) = a_1^* + 2a_2^*(z-1) + 3a_3^*(z-1)^2. \tag{7}$$

The special case $a_1^* = 1$, $a_2^* = 0$ and $a_3^* = 0$ recovers the classical linear basic state temperature distribution. The different temperature gradients studied in this paper are listed in Table I. Model 4 (Cubic 2) represents the experimental conditions of [14].

## III. Method of Solution

Eqns. (1) – (3) are solved subject to the boundary conditions (4) – (5). The condition on $\Omega$ is the spin-vanishing boundary condition. The single term Galerkin technique is used to find the critical eigen value. Multiplying equations (1), (2), and (3) by $W$, $\Omega$, and $T$, respectively. Then performing the integration by parts with respect to $z$ from 0 to 1 for the resulting equations. By using the boundary conditions (4) – (5) and taking $W = AW_1(z)$, $\Omega = B\Omega_1(z)$ and $T = CT_1(z)$ in which $A$, $B$ and $C$ are constants and $W_1(z) = z^2(1-z^2)$, $\Omega_1(z) = z(1-z)$ and $T_1(z) = z(2-z)$ are trial functions, yields the following equation for the eigen value:

$$M_a = \frac{f_4[f_2(315(1+N_1)f_3 + 132Q) - 315f_1^2]}{630(1+N_1)[f_2 f_6 - N_5 f_1 f_5]}, \tag{8}$$

where

$$f_1 = \frac{1}{15}N_1\left(4 + \frac{11}{28}a^2\right) \tag{9}$$

$$f_2 = \frac{1}{3}\left(N_3 + \frac{1}{10}N_3 a^2 + \frac{1}{5}N_1\right) \tag{10}$$

$$f_3 = \frac{4}{5}\left(21 + \frac{22}{21}a^2 + \frac{2}{63}a^4\right) \tag{11}$$

$$f_4 = \frac{4}{3}\left(1 + \frac{2}{5}a^2\right) \tag{12}$$

$$f_5 = \frac{1}{10}\left(\frac{11}{14}a_3^* - a_2^* + \frac{7}{6}a_1^*\right)a^2 \tag{13}$$

$$f_6 = \frac{1}{21}\left(a_3^* - \frac{31}{20}a_2^* + \frac{23}{10}a_1^*\right)a^2. \tag{14}$$

The expression (8) is a generalization of the corresponding result obtained by [7] for the polynomial-type basic temperature distributions. The critical Marangoni number -$M_c$ for the onset of convection is the global minimum of $M_a$ over $a \geq 0$.

## IV. Results and Discussion

Fig. 1 displays result for the oscillatory neutral curves in the $(M_a, a)$-plane for different non-uniform basic temperature gradients. The coordinates of the minimum point on these curves correspond to the critical values of $M_c$ and $a_c$. The increase of $Q$ leads to a shift of the minimum point towards the region of larger wave numbers at lower $M_c$. The critical wave number -$a_c$, is in general, insensitive to the changes in the micropolar parameters but is influenced by the magnetic field as well as the changes of the basic temperature profiles. From the Table II it can be seen that the increase in $N_1$, $M_c$ becomes higher. This table illustrated that as the Chandrasekhar

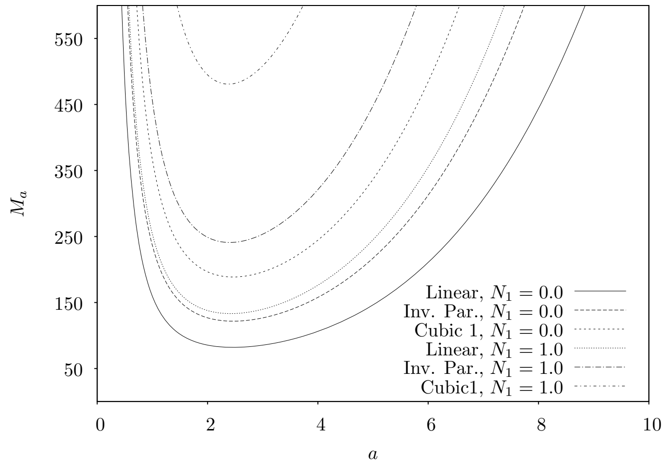


Fig. 1. Plot of $M_a$ versus $a$ for various value of $N_1$ in the case $Q = 300$, $N_3 = 2.0$, $N_5 = 1.0$.

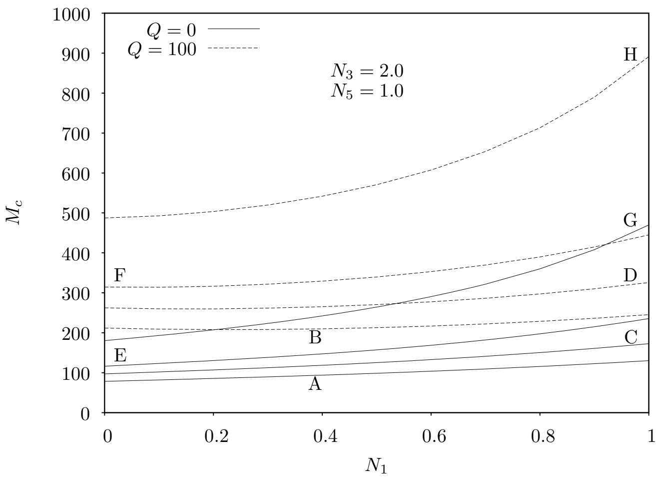


Fig. 2. Plot of $M_c$ versus $N_1$ with $N_3 = 2$ and $N_5 = 1$, A: Linear, $Q = 0$, B: Linear, $Q = 100$, C: Cubic 2, $Q = 0$, D: Cubic 2, $Q = 100$, E: Inv. Parabolic, $Q = 0$, F: Inv. Parabolic, $Q = 100$, G: Cubic 1, $Q = 0$, H: Cubic 1, $Q = 100$.

number $Q$ increases, the critical Marangoni number -$M_c$ also increases. It is clear that for the critical Marangoni number -$M_c$, the following inequality holds: $M_{c1} < M_{c2} < M_{c3}$. It is the linear model which is the most destabilizing, while the Cubic 1 is the most stabilizing $f(z)$.

Fig. 2 shows the variation of the critical Marangoni number -$M_c$ with the coupling parameter $N_1$ for assigned values of the Chandrasekhar number $Q$. The result indicates that the critical Marangoni number is generally an exponential increasing function of $N_1$. Further inspection of Fig. 2 reveals that the Linear temperature profile is the most destabilizing while the Cubic 1 profile is the most stabilizing one among these four types of non-uniform basic temperature profiles. Also it is observed that the increase in the concentration of the microelements, the critical Marangoni number -$M_c$ increases showing that the magnetic field has the stabilizing effect and is in compliance of the Newtonian results.

TABLE I
REFERENCE STEADY-STATE TEMPERATURE GRADIENTS.

| Model | Ref. steady-state temp. gradient | $f(z)$ | $a_1^*$ | $a_2^*$ | $a_3^*$ |
|---|---|---|---|---|---|
| 1 | Linear | 1 | 1 | 0 | 0 |
| 2 | Inverted parabolic | $2(1-z)$ | 0 | $-1$ | 0 |
| 3 | Cubic 1 | $3(z-1)^2$ | 0 | 0 | 1 |
| 4 | Cubic 2 | $0.6+1.02(z-1)^2$ | 0.6 | 0 | 0.34 |

TABLE II
CRITICAL MARANGONI NUMBER $(M_c)_j$ ($j$=1 TO 3) FOR DIFFERENT VALUES OF $Q$ AND $N_1$ ($N_3=2.0, N_5=1.0$).

| $N_1$ | $Q$ | $M_{c1}$ (Linear) | $M_{c2}$ (Inverted parabolic) | $M_{c3}$ (Cubic 1) |
|---|---|---|---|---|
| 0.0 | 100 | 213 | 313 | 487 |
| | 300 | 457 | 674 | 1047.9 |
| 1.0 | 100 | 243 | 444 | 887 |
| | 300 | 457 | 835 | 1675.4 |

## V. CONCLUSION

The problem of Marangoni convection in a micropolar fluid by a cubic basic state temperature profile and vertical magnetic field has been studied theoretically. Of interests are the influences of non-uniform basic temperature gradients with imposed magnetic field on the onset of Marangoni instability. The above result indicates that it is possible to delay the onset of convection by the application of a cubic basic state temperature profile. In addition, the presence of a magnetic field for a viscous, conducting fluid is to reduce the intensity of Marangoni convection and hence leads to a more stable system. As expected, the presence of the micron-sized suspended particles add to the stabilizing effect of magnetic field.

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