# 2D Numerical Analysis of Sao Paulo Tunnel

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**Abstract**—Nonlinear finite element method and Serendipity eight nodes element are used for determining of ground surface settlement due to tunneling. Linear element with elastic behavior is used for modeling of lining. Modified Generalized plasticity model with nonassociated flow rule is applied for analysis of a tunnel in Sao Paulo – Brazil. The tunnel had analyzed by Lades' model with 16 parameters. In this work modified Generalized Plasticity is used with 10 parameters, also Mohr-Coulomb model is used to analysis the tunnel. The results show good agreement with observed results of field data by modified Generalized Plasticity model than other models. The obtained result by Mohr-Coulomb model shows less settlement than other model due to excavation.

*Keywords*—Non-associated flow rule, Generalized plasticity, tunnel excavation, Excavation method.

## I. INTRODUCTION

THE construction of urban tunnels at shallow depths requires determination of soil settlement at the ground surface. Such settlement may create unfavorable effects on buildings which were constructed at ground surface and are closed to the center of tunnel. Factors such as nonlinear behavior of soil, stress history, overburden depth and diameter of tunnel have major influence on the development of the ground deformation. In recent two decades, many researches have tried to simulate the tunneling process by using the finite element method. They have used Mohr-Coulomb and von-Mises criterions for 2D or 3D modeling [1]-[4]. Although such models are commonly used, they may not provide sufficient generality in terms of stress path dependency, nonlinearity, coupling of volumetric and shear response, and strain softening. Various often models have been used to determine settlements due to tunneling in the clay. Stallebrass et al. [5] applied the three surface kinematic hardening constitutive model for soil behavior, and the procedure was used to predict the associated movements with tunnel construction in a stiff clay. Karakus and Fowell [6] used modified Cam-Clay model to simulate London clay behavior and to consider settlement due to tunneling process. The finite element method was used to model the New Austrian Tunneling Method (NATM) in London Clay by use of a Strain Dependent Modified Cam-Clay model [7]. Mroueh and Shahrour [3] considered presence of structure during the construction of tunnel as interaction between tunneling in soft soil and adjacent structures. Elastic-perfectly plastic constitutive relation based on Mohr-coulomb criterion with a non-associative flow rule was assumed.

In this paper, two procedures for simulation of excavation in the tunneling process in finite element method are considered, and generalized plasticity model with non-associated flow rule is implemented to analyze the sand behavior. This model is able to simulate softening behavior and cyclic loading, and prediction of settlements at the ground surface due to tunneling excavation. Then interaction between tunnel and building on sand is considered for tunnel at different depths. In order to analysis of tunnel-soil-structure system due to excavation, first a nonlinear analysis for soil-structure system is implemented under applied loads on structure, weight of soil and in situ stresses. Then the system is analyzed for the excavation process (Therefore, used process is a cyclic loading because tunneling process decrease loads and weight of soil). Finally, predictions of the finite element are compared with field data.

## II. MODELING TUNNEL INSTALLATION

The conventional and general procedures are two main methods for numerical simulation of excavation. Conventional procedure was introduced by Clough and Woodward [8]. General method is based on the change of geometry of the system due to addition or removal of material [9].

The underlying cause of previously reported serious numerical errors in conventional procedure for simulation of excavation has been shown to be the inconsistent determination of equivalent nodal forces from element boundary tractions. The general method uses consistent equivalent nodal forces determined from loads and internal stresses, and is free from such errors [9]. Therefore, in present paper the general method is used for analysis of excavation processes.

## III. CONSTITUTIVE MODEL

Prediction of ground movements within the soil mass surrounding excavations is a major design issue, particularly in densely populated urban areas. Numerical modeling is used for evaluation of the behavior of excavation projects in big metropolises. However, the accuracy of the numerical modeling effort depends to a large extent on the adequacy of the stress-strain-strength relationships used to represent the behavior of the soils surrounding the excavation. Specifically, the constitutive model should be able to capture the soil behavior under stress paths typical in excavation projects. In other words, constitutive model must be able to predict behavior of soil in accordance with history of stress in soil mass and also cyclic loading because tunneling process involves unloading. Here, the generalized plasticity model is used for prediction of soil behavior. Brief description of the generalized plasticity is given bellow.

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Zienkiewicz et al. [10] applied the bounding surface theory as generalized plasticity model for analysis of static and transient soil loading. They used critical state model and modified plastic modulus which was obtained based on the critical state model. They took product of plastic modulus of critical state model and a nonlinear function of distance between current yield surface and bounding surface as a plastic modulus for the generalized plasticity theory. The same method for analysis of sand was used [11]. Chen and Baladi [12] expressed stress-strain relation in terms of the hydrostatic and deviatoric components of strain and stress; these relations can be used simply if there components of flow rule vector and plastic modulus are defined. Pastor et al. [13] proposed plastic modulus and components of vector normal to yield and potential surface dependent on dilatancy of soil without using of special yield and potential surface. They defined components of the unit vector in direction of volumetric and shear deformation. Liu and Ling [14] and Liu and Song [15] used some changes in plastic modulus. Akhaveissy et al. [16] proposed reformulated relations as general and unit vector normal to yield and potential surface are determined from yield and potential surfaces. Therefore, increment of stress in term of increment of strain is calculated by use of the classical plasticity theory. The yield (f) and potential surfaces (g) are as follow:

$$f = q - M_f \times p \times (1 + 1/\alpha) \times \left(1 - \left(\frac{p}{p_e}\right)^{\alpha}\right)$$

$$g = q - M_g \times p \times (1 + 1/\alpha) \times \left(1 - \left(\frac{p}{p_g}\right)^{\alpha}\right)$$
(1)

These surfaces are shown schematically in Fig. 1, where M can be used as M<sub>g</sub> and M<sub>f</sub> which are as the slopes defining zero dilatancy, Fig. 1, and  $\alpha$  is material parameter also  $p = I_1$  and  $q = \sqrt{3J_2}$ .  $p_e$  and  $p_g$  are mean of initial normal stresses.  $I_1$  and  $J_2$  are first invariant of the stress tensor and second invariant of deviatoric stress tensor, respectively.



The unit vector normal to yield (f) and potential surface (g) can be defined as

$$n = \frac{\frac{\partial f}{\partial \sigma}}{\left[\frac{\partial f}{\partial \sigma} : \frac{\partial f}{\partial \sigma}\right]^{\frac{1}{2}}}$$
(2)  
$$n_{gL} = \frac{\frac{\partial g}{\partial \sigma}}{\left[\frac{\partial g}{\partial \sigma} : \frac{\partial g}{\partial \sigma}\right]^{\frac{1}{2}}}$$

The derivatives in Eq. (2) can be written in as

$$\frac{\partial f}{\partial \sigma} = C_1 \frac{\partial I_1}{\partial \sigma} + C_2 \frac{\partial \sqrt{J_2}}{\partial \sigma} + C_3 \frac{\partial J_3}{\partial \sigma}$$
(3)

where  $J_3$  is third invariant of deviatoric stress tensor.  $M_f$  and  $M_g$  depend on Lode angle [13] but in here they are assumed as a constant; therefore, derivative of yield surface respect to  $J_3$ ,  $C_3$  is zero.

$$C_{1} = \frac{\partial f}{\partial I_{1}} = \left(1 + \alpha\right) \left(\frac{M_{f}}{3} - \frac{\sqrt{3J_{2}}}{I_{1}}\right)$$

$$C_{2} = \sqrt{3}$$
(4)

If Mg is substituted instead  $M_f$ , coefficient  $C_1$  relates to the potential surface. Increment of stress can be determined in finite element method as follow [13]:

$$d\sigma = \left(D_e - \frac{D_e n_g n^T D_e}{H + n^T D_e n_g}\right) d\varepsilon$$
(5)

where  $D_{\text{e}}$  is elastic constitutive matrix and H for loading or reloading :

$$H = H_0 p H_f (H_v + H_s)$$

$$H_f = \left(1 - \frac{\eta}{\eta_f}\right)^4$$

$$\eta_f = \left(1 + \frac{1}{\alpha}\right) M_f$$

$$H_v = \left(1 - \frac{\eta}{M_g}\right)$$

$$H_s = \beta_0 \beta_1 \exp(-\beta_0 \xi)$$
(6)

where  $\xi$  is the accumulated deviatoric plastic strain.  $\beta_0$  and  $\beta_1$  are material parameters. The expression of H for unloading [13]:

$$H_{U} = H_{U0} \left(\frac{M_{g}}{\eta_{U}}\right)^{\gamma_{U}} \qquad for \qquad \left|\frac{M_{g}}{\eta_{U}}\right| > 1$$

$$H_{U} = H_{U0} \qquad for \qquad \left|\frac{M_{g}}{\eta_{U}}\right| \le 1$$
(7)

where  $\eta_U$  is  $\eta$  for unloading,  $H_{U0}$  and  $\gamma_U$  are material parameters. Then, increment of stress can be found by use of Eq. (3) to Eq. (7). It must be noted sign of volumetric component of vector perpendicular on potential surface was altered in accordance to [13] as a constraint, but in present work, in accordance to Eq. (3) and Eq. (4), sign of vector is not changed as a constraint.

#### IV. NUMERICAL SIMULATION OF TUNNELING PROCESSES

For analysis of tunnel-soil-structure system, generalized plasticity theory is applied by using finite element program in SSINA2D [17] (Soil Structure Interaction Nonlinear Analysis of Two Dimensional) which includes a number of constitutive models. The program includes the Mohr-Coulomb criterion, von-Mises, Tresca, Drucker-Prager and generalized plasticity model. The predictions by both Mohr-Coulomb and generalized plasticity models are compared with filed data. The field data consider is the Sao Paulo tunnel [18].

Following processes are implemented for considered problems in present paper. In order to analysis of tunnel-soilstructure system due to excavation, first internal forces or stresses on excavation boundary are determined based on a nonlinear analysis of soil-structure system under applied loads on structure, weight of soil and in situ stress. In next stage, excavation stage, percentage of stresses on excavation boundary is released before installation of lining [3], [18]-[19]. Therefore, a nonlinear analysis as incremental is implemented to release stresses on excavation boundary in excavation stage and before installation of lining. In final stage, analysis is continued for remaining stresses with installation of lining. Therefore lining is tolerated remaining stresses.

## A. Sao Paulo Tunnel

First modified generalized plasticity model (GPM) and Mohr-Coulomb model are used to predict ground surface settlement induced by shallow underground excavation, in the tunnel in the underground transit system in Sao Paulo, Brazil [18]. Fig. 2 shows the characteristics of the tunnel.



Fig. 2 Tunnel cross section (Lengths in meters), Geotechnical profile, Tunnel construction phases and instrumentation [18]

The tunnel has a maximum height of 8.4 m and a maximum width of 11.4 m (82  $\text{m}^2$  net areas). The soil cover thickness at the instrumented section was 7.6 m. The support system consists of a 0.2 m-thick primary shotcrete and a 0.15 m-thick secondary shotcrete. Conventional triaxial Compression tests were used to determine parameters of the model, details of parameters are shown in Tables 1 and 2. Figs. 3 and 4 show comparison between prediction by the GPM and test data at two depths, 6.5 m and 12.5 m, respectively. The correlations are considered to be very good.

TABLE I FRICTIONAL ANGLE AND COHESION OF SOIL									
Depth	E (kPa)	υ	с	$\phi$	$K_{0} = 1 - \sin \phi$				
(m)			(kPa)	,	0				
6.5	135930	0.27	22.0	17.2	0.7				
12.5	3021480	0.17	53.0	14.2	0.75				

where E is elastic modulus, cohesion and friction angle of soil are c and  $\phi$ , respectively,  $\upsilon$  and K<sub>0</sub> are Poisson's ratio and lateral earth pressure ratio, respectively.



Fig. 3 Comparison predictions and laboratories data for samples at depth of 6.5 m (a) deviator stress curves and (b) Volumetric strain versus axial strain

## World Academy of Science, Engineering and Technology International Journal of Civil and Environmental Engineering Vol:4, No:3, 2010



(b)

Fig. 4. Comparison predictions and laboratories data for samples at depth of 12.5 m (a) deviator stress curves and (b) Volumetric strain versus axial strain

Cali	brated parame	eters for samp	Calibrated parameters for samples at				
depth				12.5m depth			
	Confining	Confining	Confining	Confining	Confining	Confining	
	pressure	pressure	pressure	pressure	pressure	pressure	
	49 kPa	98 kPa	196 kPa	98 kPa	196 kPa	294 kPa	
Е	135930	135930	135930	3021480	3021480	3021480	
υ	0.27	0.27	0.27	0.17	0.17	0.17	
M <sub>f</sub>	1	1	1	1	1	1	
Mg	1.4	1.2	1.1	1.3	1	0.95	
H <sub>0</sub>	100	80	50	1300	1250	850	
$eta_0$	0.2	0.2	0.2	0.2	0.2	0.2	
$\beta_1$	0.15	0.15	0.15	0.01	0.01	0.01	
α	0.57	0.8	0.9	0.45	1.1	2	
H <sub>u0</sub>	2500	2500	2500	34000	34000	34000	
$\gamma_u$	0.1	0.1	0.1	0.1	0.1	0.1	

TABLE II CALIBRATED PARAMETERS FOR MODIFIED GENERALIZE PLASTICITY MODEL

## B. Finite element analysis

The 2D plane strain analysis of the tunnel was performed using the SSINA2D [17] using plane strain idealization eight nodded isoparametric element, used to model soil, and two node bar element is used to simulate lining of tunnel. In this study, two models Mohr-coulomb and Generalized Plasticity are used. In accordance to table 1, lateral pressure coefficients equal to 0.7 and 0.75 are used for calculation of in situ stresses. The finite element mesh is given in Fig. 5 and the mesh consists 108 eight nodded isoparametric elements and 18 two nodded bar elements for lining. The lining was installed after 81% releasing of stress by Lades' model in [18]. In

present study, lining installs after 75% releasing of stress for analysis by use of generalized plasticity model and 37.5% releasing of stress by Mohr-coulomb model. Results of modified generalized plasticity model are compared with results of Mohr-coulomb, observed results and results of Lade model [18] in Fig. 6.



Fig. 5 Finite element mesh used in numerical simulation



Distance from center of tunnel (m)

Fig. 6 Comparisons of predicted surface settlement profile and observed results

The settlement of analysis by Mohr-Coulomb criterion is less than field data, Fig. 6; it expresses, lining tolerate total stresses on excavation boundary. Therefore, lining is designed for more forces. Fig. 6 shows induced settlement due to tunneling by Mohr-Coulomb model in distance between center of tunnel and 9m of that is more than induced settlement in out of this area while induced area by modified generalized plasticity and Lades' model [18] is about 16.5m. This subject is predictable because Mohr-Coulomb is elastic- perfectly plastic criterion. In other words, there is no plastic strain before reaching stress state to yield stress and after that, plastic strain and displacement are increased rapid, then lining may used to confine the settlement. Therefore, lining must be tolerated more force. In the other hand, Fig. 6 shows; if the analysis is implemented by use of Mohr-coulomb criterion, constructed buildings farther than the other ones respect to center of tunnel are induced less. Fig. 6 shows good agreements between obtained results of modified generalized plasticity model, field data and Lades' model. It is noticeable 16 parameters would be required to model the behavior of soil by Lades' model [18] but in present work, 10 parameters would be required to model behavior of soil by use of modified generalized plasticity. Also, modified generalized plasticity with less parameter than Lades' model gives good agreement with data field than other models.

## V. CONCLUSION

Nonlinear finite element method was used for tunneling process. Generalized plasticity model with non-associated flow rule was accepted. Serendipity eight nodded element was used in analysis. Obtained results for a case study of Sao Paulo tunnel in Brazil compared with field data. Predictions by generalized plasticity model show good agreement with field data while obtained results by Mohr-Coulomb model shows less settlement due to excavation.

## ACKNOWLEDGMENT

The author is grateful to Tehran Urban Subway Company (Metro) for providing research funding under project No. 84/200/12994.

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