

Short Time Identification of Feed Drive Systems using Nonlinear Least Squares Method

M.G.A. Nassef, Linghan Li, C. Schenck, and B. Kuhfuss

Abstract—Design and modeling of nonlinear systems require the knowledge of all inside acting parameters and effects. An empirical alternative is to identify the system's transfer function from input and output data as a black box model. This paper presents a procedure using least squares algorithm for the identification of a feed drive system coefficients in time domain using a reduced model based on windowed input and output data. The command and response of the axis are first measured in the first 4 ms, and then least squares are applied to predict the transfer function coefficients for this displacement segment. From the identified coefficients, the next command response segments are estimated. The obtained results reveal a considerable potential of least squares method to identify the system's time-based coefficients and predict accurately the command response as compared to measurements.

Keywords—feed drive systems, least squares algorithm, online parameter identification, short time window

I. INTRODUCTION

IMPROVING the performance of high speed high precision feed drive systems in terms of productivity and tracking accuracy remains an actual challenge for machine tool designers. Therefore, a significant amount of research has been dedicated to design integrated system models combining mechanical and control subsystems together in order to optimize the whole system parameters and predict precisely the system dynamic response with time [1]-[4].

As feed drive systems comprise nonlinear components in high speed machine tools, algorithms and techniques such as least squares methods, fuzzy logic, and neural networks [5] are applied to model their behavior either by differential equations or in s-domain transfer function form [6].

Recursive Least Squares (RLS) method was used to identify DC motor actual parameters in terms of mechanical time constant and gain using input voltage and output rotation speed of the motor [7].

The same technique was applied for online identification of a permanent magnet DC motor in open loop conditions [8]. However, for induction motors the problem is more complex

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because the rotor state variables cannot be obtained from measurements and simplifying the model by assuming the parameters to be linearly varying results in high residual error.

Nonlinear least squares approach was applied to induction motors for the online estimation of the motor parameters effective values in terms of inductance, resistance, load torque and inertia [9]. With this method, the parameters values can be corrected continuously during operation.

Compact modeling using least-squares algorithm in the orthogonal form was efficiently applied in the identification of actual parameter identification for nonlinear complex system models such as magnetosphere dynamic system problem [10]. In the same work, Error Reduction Ratio (ERR) and mutual information criteria were used as two competitive methods to define and select the significant parameters of the reduced model.

Other techniques such as Neural Networks (NN) and fuzzy logic are used in the field of high speed machine tools as a parameter estimator for nonlinear systems [11]-[12]. Online nonlinear system parameters identification has been achieved using recurrent radial basis function NN with the aid of a multi stage training algorithm [13]. To overcome the problem of network instability in case of un-modeled dynamics and disturbances, training of a recurrent neural network is being modified by the use of an input-to-state stability approach [14].

A calibration method has been used to identify the actual geometric and kinematic parameters of a single degree of freedom mechanism with the aid of radial basis function neuron algorithm in order to minimize the differences between measurement and simulation results [15]. For such mechanism, NN is trained with simulated scenarios to identify gray box model parameters from sensor based acquired data.

For uncoupled closed loop DC motors, feed forward with back propagation neural networks were applied [16] to estimate the actual system parameters. Using a voltage step input to the system, the network was trained by several current values as inputs and the correlated transfer function parameters (resistance, inductance, friction, and rotor inertia) as outputs of the network.

Most of previous researches focused on identifying the nonlinear systems parameters and, hence, transfer functions in offline phase only. Furthermore, many measurements and/or simulations are necessary to estimate the coefficients of a system's transfer function at a single operating point. As system parameters may change with time during operation due to effects such as position dependant stiffness, mechanical friction, and variable preloading, it is difficult to minimize the tracking error.

There is a need for online identification algorithm to adjust the controller parameters values continuously to yield better tracking performance. This may be done with an adaptive controller approach or by a model based correction of the set point values.

In this work, nonlinear Least Squares Method (LSM) is proposed as a promising tool to predict the transfer function of feed drive system in short time periods based on windowed input/output inverse modeling. A short time window of 4 ms is applied to input/output signals at the start of motion. Using inverse Laplace transform, the transfer function of the system is converted into a set of system equations in time domain and LSM is then conducted on the windowed signals to get knowledge about the system coefficients.

These identified coefficients are then used to predict the system output for the next 2 ms and compared to measurements. With shifting forward the time window, the dynamic response of the system is estimated accurately online.

The tested feed drive system and its virtual model are described and the principle of nonlinear LSM and parameter identification procedure is introduced. The algorithm is illustrated and discussed with measurements.

II. SYSTEM DESCRIPTION

A. Physical system

In this investigation, feed drive system is chosen as an example for a nonlinear system. Its development is described in [17] and [18]. The components consist of a ball screw, a flexible coupling, nut, and thrust bearings -as shown in fig.1. The ball screw has a 10 mm pitch, 10 mm diameter, and 320 mm length. The nut supports the tool center point during feed motion.

A closed loop servo system consisting of DC permanent magnet motor and a servo drive is used to drive the assembly and perform the tracking commands. The controller scheme used to control the motion is P/PI cascaded structure.

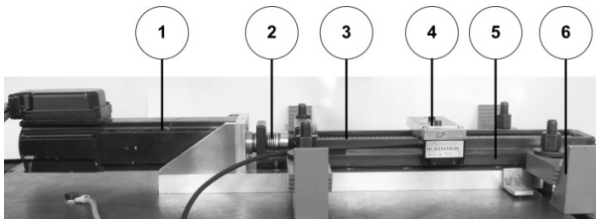


Fig. 1 A typical ball screw feed drive system: 1) DC motor, 2) flexible coupling, 3) ball screw, 4) tool center point, 5) fixation jig, and 6) linear bearing guideways

The controller structure is closed with a dual position measuring system [19] using rotary encoders (built in the servo motor) and linear encoders (mounted on the table). In addition, the linear encoder signals are interpreted and stored by a digital signal processing and acquisition device.

B. Simulation Model

A physical model as shown in fig. 2 is constructed for the original feed drive system; where J_m , $J_{bearings}$, $J_{coupling}$ and J_{bs} are the servo motor rotor, bearings, coupling and ball screw

inertias. C is the viscous damping coefficient. K_{bs} , K_{nut} , $K_{coupling}$ and $K_{bearing}$ are the ball screw, nut, coupling, and bearing stiffness. The equivalent axial stiffness K_l and the total inertia J_{total} for the axis can be calculated as follows:

$$\frac{1}{K_l} = \left(\frac{1}{K_{bs}} + \frac{1}{2K_{bearing}} + \frac{1}{K_{nut}} \right) \quad (1)$$

$$J_{total} = J_{bs} + J_m + J_{bearings} \quad (2)$$

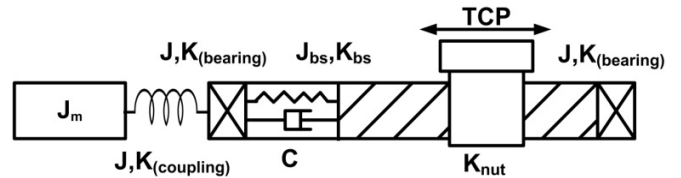


Fig. 2 Physical model of the feed drive system

Inertia and axial stiffness of the coupling element are assumed to be insignificant for the simulation; they are not considered in the physical model. The effect of linear mass during motion is modeled separately. From the physical model in fig. 2, a block diagram of the system is constructed in Matlab/Simulink software, as shown in fig. A1(Appendix). The mechanical and control parameters values in the original system are placed in the compact model to investigate the differences between simulation and measurement results. The s-domain transfer function of the control system between the command position and actual position and its coefficients are expressed in the equations (A3)-(A13) (Appendix).

However, undesired differences between the analytical and measurement results are presented always due to the fact that the model includes only basic parameters of the gray box real system. Furthermore, nonlinearities presented in the real feed drive system such as position dependant stiffness, mechanical friction, and variable preloading change the values of system parameters significantly during motion. Therefore, an identification process of the effective values of system parameters in the model was successfully conducted using feed forward NN with back propagation learning algorithm [20].

III. IDENTIFICATION ALGORITHM

As described in the previous section, the Transfer function (Tf) of the feed drive system is expressed as follows:

$$Tf = X_a(s) / X_s(s) = \frac{\sum_{i=2}^0 N_i s^i}{\sum_{i=7}^0 D_i s^i} \quad (3)$$

where X_a and X_s are output and input trajectory. N and D indicate the number of numerator and denominator coefficients in descending powers of s .

The work aims at determining the coefficients N_i and D_i , and hence, the transfer function of system with time. First, the transfer function is divided by s^8 :

$$X_a(s) / X_s(s) = \frac{\sum_{i=2}^0 N_i s^{i-8}}{\sum_{i=7}^0 D_i s^{i-8}} \quad (4)$$

In this work, a rough but accepted simplification is made by assuming that the input and output signals from system are equal to the state description of numerator and denominator. By

applying the inverse Laplace transform to (5) they are found in time domain in (6).

$$\begin{cases} X_a(s) = \sum_{i=2}^0 N_i s^{i-8} \\ X_s(s) = \sum_{i=7}^0 D_i s^{i-8} \\ X_a(t) = \sum_{i=7}^5 P_i t^i \\ X_s(t) = \sum_{i=7}^0 Q_i t^i \end{cases} \quad (5)$$

$$\begin{cases} X_a(t) = \sum_{i=7}^5 P_i t^i \\ X_s(t) = \sum_{i=7}^0 Q_i t^i \end{cases} \quad (6)$$

where $P_7=N_0/5040$, $P_6=N_1/720$, $P_5=N_2/120$, $Q_7=D_0/5040$, $Q_6=D_1/720$, $Q_5=D_2/120$, $Q_4=D_3/24$, $Q_3=D_4/6$, $Q_2=D_5/2$, $Q_1=D_6$, $Q_0=D_7$.

Using (6), curve fitting of the windowed output and input trajectories is conducted to compute the local coefficients.

Equation (6) can be rewritten as a following general fitting model, e.g. $X_a(t)$ equation:

$$Y_i = \hat{Y}_i(t; N_2, N_1, N_0) \quad (7)$$

where Y_i is measured output, \hat{Y}_i is estimated function, and t is the time base.

The principle of the least squares is to find the coefficients N that minimize the sum of squares of the residuals δ as shown in (8) [21].

$$\delta = \sum_{i=1}^n (Y_i - \hat{Y}_i(t; N_2, N_1, N_0))^2 \quad (8)$$

To obtain regression coefficients N of the function (8), the basic simplex method is applied. Simplex method is a technique in local frame that works on the processed current data set to predict the future values of data with minimum error [22], [23].

The coefficients obtained from least squares are substituted in (3) in order to find the estimated next trajectory for the next short period. In the following section, the procedure is illustrated in detail.

IV. PROCEDURE OF WORK

The work starts experimentally and analytically by commanding the feed drive system to run with a ramp function of 30 mm amplitude and set velocity of 50 mm/s. Then, the measured input and output signals are acquired with a sampling rate of 5 KHz. Curve fitting function in (6) is applied then with a window of size 4 ms (20 points) to both signals at the start of motion and the coefficients of the function are estimated using (7) and (8).

The computed actual coefficients values are substituted in the transfer function in (3) to predict the next 2 ms (10 points) of trajectory. The same procedure is done for the next window with 20 samples and the window function is shifted over the whole signal to obtain the predicted system command response for the ramp function. The procedure of work is illustrated in Fig.3. In order to validate the proposed algorithm, the same steps are conducted with a step function of 1 mm amplitude in 0.2 ms applied to the feed drive system. The calculated results

are compared with the measurements again.

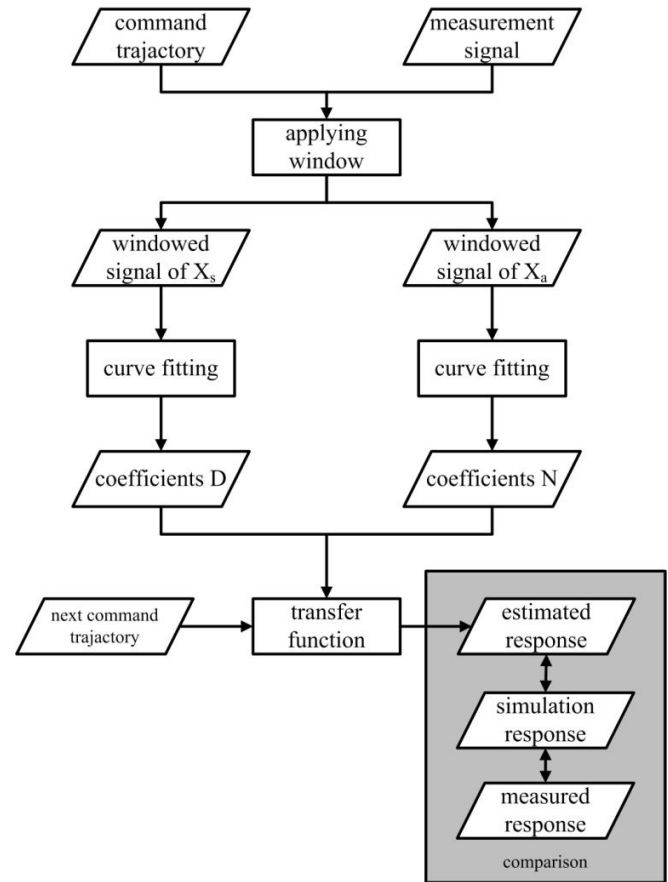


Fig. 3 The general procedure of work for model identification using nonlinear least squares method

V. RESULTS AND DISCUSSION

In order to investigate the effectiveness of the proposed algorithm to estimate the next trajectory data in short time, three kinds of results are compared in this section; the measured data from the axis motion, the simulation response from the simulation model (Appendix), and the short time identified model using least squares curve fitting technique.

In Fig.5, a zoomed view of the applied ramp function in first 60 ms of the motion is chosen for comparison. As compared to the simulation model, least squares estimated response shows more accuracy in tracking the measurements than the simulation model. However, periodic ripples are presented in the identified model response. These ripples are due to the high order of curve fitting functions. Reducing the order of the windowed transfer function may improve the accuracy of curve fitting. This overfitting problem and the suitable window size are to be studied in future work.

In fig.6, it can be observed that the calculated error as the difference between LSM model response and measurements is in range of 40 μ m whereas the error between the simulation model and measurements is equal to 80 μ m. Fig. 6 proves that the LSM model is more accurate in estimating system response than the simulation model.

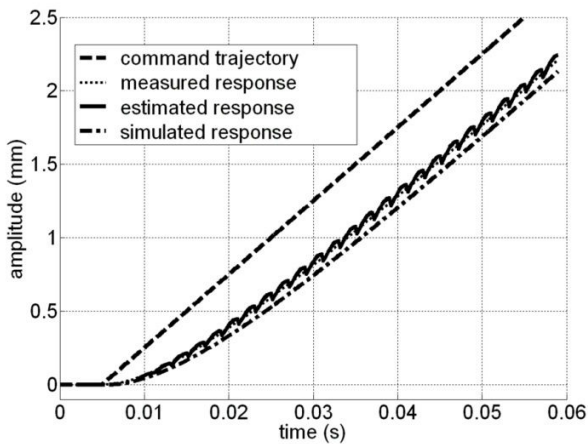


Fig. 5 Identified model results against measurement and simulation response results in the first 0.06 s for a ramp function

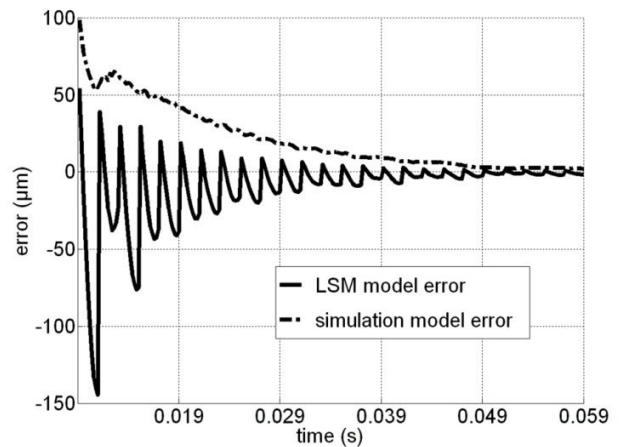


Fig. 8 LSM model error and simulation model error as compared to real system measurements

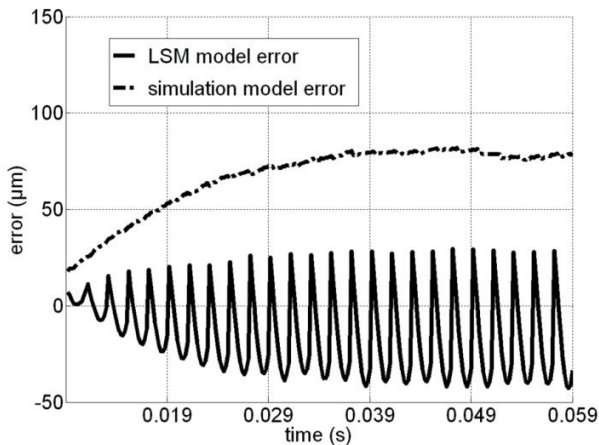


Fig. 6 LSM model error and simulation model error as compared to real system measurements

Fig. 7 presents the results for the step function. It can be seen that same periodic ripples are presented in the estimated model response within the time window and the LSM model results follow the measurements.

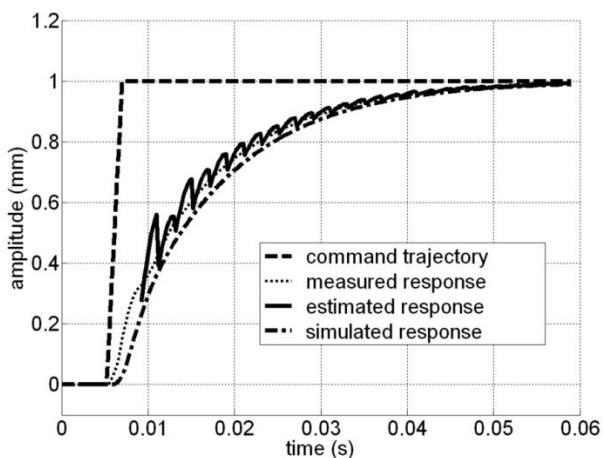


Fig. 7 Identified model results against measurement and simulation response results in the first 0.06 s for a step function

The calculated error, as the difference between both model results and measurements, is found to be higher in amplitude than the error in case of the ramp function - as shown in fig. 8. This is due to the higher slope of the trajectory in the acceleration phase presented in step function. The maximum error of LSM model is found around 0.15 mm at the start of motion, i.e. during acceleration phase. The error value is higher than the simulation model and decreases with time for both methods. The experimental results show that the proposed least squares model can identify the dynamic behavior in the short time window. The accuracy is reduced with increasing of the severity of curve slope.

VI. CONCLUSION AND FUTURE WORKS

A method to identify the dynamic behavior of a nonlinear feed drive system in short time is presented in this work. Nonlinear least squares function is applied to windowed input and output signals from the system and the time-based transfer function coefficients are identified in time domain.

The system parameters calculated from the windowed signals are then used to estimate the system response for the next time steps. By shifting the constant windowed function with time, the system response with time is estimated. As compared to the simulation model response and measurement results, the short time windowed least squares model is found to be more accurate than the simulation model and proves to predict the command response of the system with time in case of a ramp function. Similar results are found when a step function is applied.

For future work, the performance of the proposed method is to be examined under various window sizes. Alternatively, other techniques such as neural networks are to be investigated to estimate the system response in short time and compared with the current method.

Moreover, reduction of the transfer function order is to be investigated and according to the fitted curve, the order will be selected automatically to obtain more accuracy of the fitting process and minimize the response ripples.

We have identified successfully the system transfer function in short time. With this proposed method, model based compensation can improve the position accuracy of feed drive system by a model based adjustment of the command trajectory. On the other hand, if the parameters in the control system, like K_v , K_p and K_i , can be calculated though the identified coefficients N_i and D_i by the local transfer function, the compensation of the real trajectory in online is also achievable.

APPENDIX

Fig. 1 presents a block diagram of the feed drive system simulation model. The model is constructed in Matlab/Simulink software where; K_T (N/m) is the torque constant, C_1 and C_2 (Ns/m) are the damping coefficients of motor and table, K (N/m) is the axial equivalent stiffness, CL is the current loop, J (Kgm^2) is the system total inertia, and $R = L/2\pi$ where L (m) is the lead.

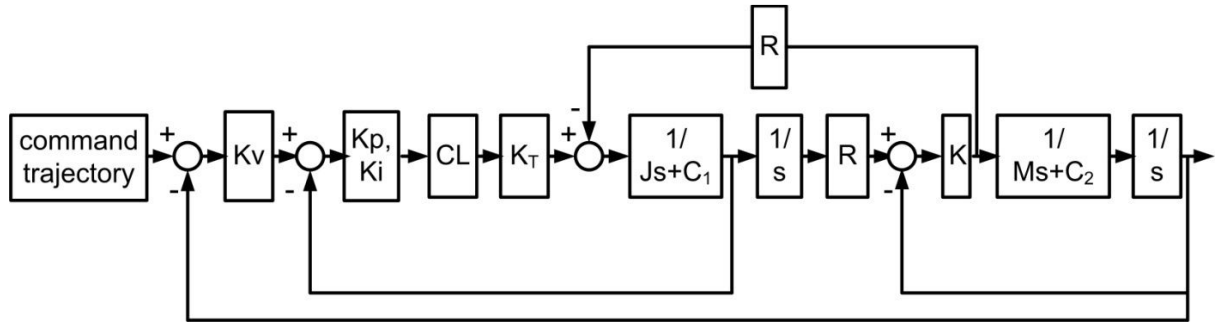


Fig. 1 Block diagram of the feed drive system model

The Transfer function of the control system in the s-domain between the command position and actual position is expressed as in the following equation:

$$Tf = \frac{X_a(s)}{X_s(s)} = \frac{N_2s^2 + N_1s + N_0}{D_7s^7 + D_6s^6 + D_5s^5 + D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0} \quad (1)$$

N and D are the numerator and denominator coefficients of the system with descending order of s where the coefficients of numerator are:

$$N_2 = K_v \cdot K_p \cdot K_{pi} \cdot K_t \cdot K_1^2 \cdot K \quad (2)$$

$$N_1 = K_{pi} \cdot K_t \cdot K_1^2 \cdot K \cdot K_v \cdot K_p (K_{ii} + K_i) \quad (3)$$

$$N_0 = K_{pi} \cdot K_t \cdot K_1^2 \cdot K \cdot K_{ii} \cdot K_p \cdot K_i \cdot K_v \quad (4)$$

Whereas the denominator coefficients are:

$$D_7 = K_1 \cdot M \cdot L \cdot J \quad (5)$$

$$D_6 = C_2 \cdot L \cdot K_1 \cdot J + L \cdot K_1 \cdot C_1 \cdot M + K_1 \cdot J \cdot R \cdot M + K_1 \cdot J \cdot K_{pi} \cdot M \quad (6)$$

$$D_5 = L \cdot K \cdot K_1^2 \cdot J + L \cdot K_1 \cdot C_1 \cdot C_2 + K_1 \cdot J \cdot R \cdot C_2 + K_1 \cdot C_1 \cdot R \cdot M + K_1 \cdot J \cdot K_{pi} \cdot C_2 + K_1 \cdot K_{pi} \cdot C_1 \cdot M + K_1 \cdot J \cdot K_p \cdot K_{ii} \cdot M + K_1 \cdot K_t^2 \cdot M + K_p \cdot K_1 \cdot K_t \cdot K_{pi} \cdot M + r^2 \cdot K \cdot M \cdot L \quad (7)$$

$$D_4 = K_1^2 \cdot K \cdot L \cdot C_1 + K_1^2 \cdot J \cdot R \cdot K + K_1 \cdot C_1 \cdot R \cdot C_2 + K_1^2 \cdot K \cdot J \cdot K_{pi} + K_1 \cdot C_1 \cdot K_{pi} \cdot C_2 + K_1 \cdot J \cdot K_p \cdot K_{ii} \cdot C_2 + K_1 \cdot C_1 \cdot K_p \cdot K_{ii} \cdot M + K_1 \cdot K_t^2 \cdot C_2 + K_p \cdot K_1 \cdot K_t \cdot K_{pi} \cdot C_2 + K_1 \cdot K_t \cdot K_{pi} \cdot K_p \cdot K_i \cdot M + K_1 \cdot K_{pi} \cdot K_t \cdot K_p \cdot M \cdot K_{ii} + r^2 \cdot K \cdot C \cdot L + r^2 \cdot K \cdot M \cdot R + r^2 \cdot K \cdot M \cdot K_{pi} \quad (8)$$

$$D_3 = K_1^2 \cdot K \cdot C_1 \cdot R + K_1^2 \cdot C_1 \cdot K_{pi} \cdot K + K_1^2 \cdot J \cdot K_p \cdot K_{ii} \cdot K + K_1 \cdot C_1 \cdot K_p \cdot K_{ii} \cdot C_2 + K_t^2 \cdot K_1^2 \cdot K + K_p \cdot K_1^2 \cdot K_{pi} \cdot K_t \cdot K + K_1 \cdot K_t \cdot K_{pi} \cdot K_p \cdot K_i \cdot C_2 + K_t \cdot K_{pi} \cdot K_1 \cdot K_t \cdot K_p \cdot C_2 + K_{pi} \cdot K_t \cdot K_p \cdot K_{ii} \cdot K_t \cdot M \cdot K_1 + r^2 \cdot K \cdot C_2 \cdot R + r^2 \cdot K \cdot C_2 \cdot K_{pi} + r^2 \cdot K \cdot M \cdot K_p \cdot K_{ii} \quad (9)$$

$$D_2 = K_1^2 \cdot K \cdot C_1 \cdot K_p \cdot K_{ii} + K_1^2 \cdot K_t \cdot K_{pi} \cdot K_p \cdot K_i \cdot K + K_1^2 \cdot K_t \cdot K_p \cdot K_{pi} \cdot K \cdot K_{ii} + K_p \cdot K_1 \cdot K_{ii} \cdot K_i \cdot C_2 \cdot K_t + r^2 \cdot K \cdot C_2 \cdot K_p \cdot K_{ii} \quad (10)$$

$$D_1 = K_1^2 \cdot K_t \cdot K_p \cdot K_{ii} \cdot K_i \cdot K \cdot K_{pi} \quad (11)$$

$$D_0 = N_0 \quad (12)$$

Where K_v (1000/min) is position loop gain, K_p (As /rad) and K_i (1/ms) are velocity loop proportional and integral gain, and K_{p_i} (V/A) and K_{i_i} (1/ms) are current loop proportional and integral gain.

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