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Synchronization for impulsive fuzzy Cohen-Grossberg neural networks with time delays under noise perturbation

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Abstract—In this paper, we investigate a class of fuzzy Cohen-Grossberg neural networks with time delays and impulsive effects. By virtue of stochastic analysis, Halanay inequality for stochastic differential equations, we find sufficient conditions for the global exponential square-mean synchronization of the FCGNNs under noise perturbation. In particular, the traditional assumption on the differentiability of the time-varying delays is no longer needed. Finally, a numerical example is given to show the effectiveness of the results in this paper.

Keywords—Fuzzy Cohen-Grossberg neural networks(FCGNNs), Complete synchronization, Time delays, Impulsive, Noise perturbation.

I. Introduction

In recent years, the well-known Cohen-Grossberg neural networks [1] has been extensively studied due to their extensive applications in many fields such as pattern recognition, computing associative memory, signal and image processing and so on, see [2-5] for examples. In these applications, stability of the model is prerequisite.

In reality, the uncertainty or vagueness is unavoidable. In order to take vagueness into consideration, fuzzy theory is considered as a suitable method. Fuzzy cellular neural network (FCNN) was first introduced by Yang et al. in 1996 ([6]), it combines fuzzy logic with traditional CNN. Studies have shown the potential of FCNN in image producing and pattern recognition. In such applications, it is very important to ensure that the designed FCNN be stable. Some results on stability have been derived for the FCNN (see [7-9] for more details).

In 1990, Pecora and Carrol [10] introduced a new concept-synchronization of coupled chaotic systems to force the response of the slave system to synchronize the master system. The control and synchronization problems of chaotic systems have been intensively investigated due to their potential applications in various fields [11-16]. There are several different master-slaver synchronization schemes which have been described theoretically and observed experimentally. These include complete synchronization (CS) [17], generalized synchronization (GS) [18], lag synchronization (LS) [9,19,20], anticipating synchronization (AS) [21], and so on.

Meantime, time delays occur so often in many processes, even in almost every situation, that to ignore them is to ignore reality ([22]). On the other hand, many evolution processes are characterized by the fact that at certain moments of time, they experience a change of state abruptly, that is, in the form of impulses. There are lots of results about synchronization of impulsive delayed dynamic systems, one can see [7,9,12,23,24] for more details.

Besides, noise is omnipresent in nature and in man-made systems. And in the processes of applications, synchronizing effect is influenced by noise unavoidably. Recently, the synchronization of systems under noise perturbation has become a field of great interests. In [25], by virtue of stochastic analysis, Halanay inequality, complete synchronization is investigated for impulsive delayed Cohen-Grossberg neural networks under noise perturbation. In [26], based on the Lyapunov stability theory, sufficient conditions on the exponential synchronization are obtained for a class of stochastic perturbed chaotic delayed neural networks with constant delay.

To the best of our knowledge, however, there are few results for synchronization of impulsive fuzzy neural networks under noise perturbation. So, in this paper, complete synchronization of impulsive fuzzy Cohen-Grossberg neural networks with delay under noise perturbation will be studied.

Motivated by the above discussion, in this paper we investigate a class of fuzzy Cohen-Grossberg neural networks with time-varying delays and impulsive effects described by the following system:

$$\begin{cases} \frac{\mathrm{d}\,x_i(t)}{\mathrm{d}\,t} = \alpha_i(x_i(t))\Big[-\beta_i(x_i(t)) + \sum_{j=1}^{n}\delta_{ij}\mu_j + I_i \\ \qquad + \bigwedge_{j=1}^{n} a_{ij}f_j(x_j(t)) + \bigwedge_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j(t))) \\ \qquad + \bigwedge_{j=1}^{n} T_{ij}\mu_j + \bigvee_{j=1}^{n} c_{ij}f_j(x_j(t)) \\ \qquad + \bigvee_{j=1}^{n} d_{ij}f_j(x_j(t-\tau_j(t))) + \bigvee_{j=1}^{n} H_{ij}\mu_j\Big],\ t\neq t_k, \\ x_i(t_k) = x_i(t_k^-) + I_{ik}(x_i(t_k^-)),\ t=t_k, \end{cases} \tag{1}$$

for $i = 1, 2, \ldots, n$; $k = 1, 2, \ldots$, where $x_i(t)$ is the ith neuron state, $\alpha_i(x_i(t))$ represents an amplification function, $\beta_i(x_i(t))$ is an appropriately behaved function, f_j denote the activation function, $\tau_j(t)$ is time delay of jth neuron and corresponds to finite speed of axonal signal transmission at time t, δ_{ij} are elements of fuzzy feed-forward template, a_{ij}, b_{ij} are elements of fuzzy feedback MIN template, c_{ij}, d_{ij} are elements of fuzzy feedback MAX template, T_{ij} and H_{ij} are elements of fuzzy feed-forward MIN template and fuzzy feed-

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forward MAX template, respectively. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. μ_i and I_i denote input and bias of the ith neuron, respectively. t_k is called impulsive moment and satisfies $0 < t_1 < t_2 < \ldots, \lim_{k\to+\infty} t_k = +\infty$; $x_i(t_k^-)$ denotes the left limit at t_k; $I_k(x(t_k)) = (I_{1k}(x_1(t_k)), I_{2k}(x_2(t_k)), \ldots, I_{nk}(x_n(t_k)))^T$, $I_{ik}(x_i(t_k))$ shows impulsive perturbation of the ith neuron at t_k.

Remark 1.1: In system (1), if $I_{ik}(x_i(t_k)) \equiv 0 (i = 1, 2, \ldots, n; k = 1, 2, \ldots)$, then system (1) turns to continuous FCGNN

$$\begin{aligned}\frac{\mathrm{d}\, x_i(t)}{\mathrm{d}\, t} &= \alpha_i(x_i(t))\Big[-\beta_i(x_i(t)) + \sum_{j=1}^{n}\delta_{ij}\mu_j + I_i \\ &+ \bigwedge_{j=1}^{n} a_{ij} f_j(x_j(t)) + \bigwedge_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) \\ &+ \bigwedge_{j=1}^{n} T_{ij}\mu_j + \bigvee_{j=1}^{n} c_{ij} f_j(x_j(t)) \\ &+ \bigvee_{j=1}^{n} d_{ij} f_j(x_j(t-\tau_j(t))) + \bigvee_{j=1}^{n} H_{ij}\mu_j\Big]\end{aligned}$$

Throughout this paper, we assume that

(H_1) $\alpha_i(u)$ is a continuous function, $0 \le \alpha_i(u) \le \overline{\alpha}_i$ ($\overline{\alpha}_i$ is a constant.) and there exists $L_i^{\alpha} > 0$ such that

$$|\alpha_i(u) - \alpha_i(v)| \le L_i^{\alpha}|u - v|$$

for all $u, v \in R$, $i = 1, 2, \ldots, n$.

(H_2) $\frac{\alpha_i(u)\beta_i(u) - \alpha_i(v)\beta_i(v)}{u-v} \ge \gamma_i > 0$, for all $u, v \in R$, $u \neq v$, $i = 1, 2, \ldots, n$.

(H_3) f_j is bounded and Lipschitzian, that is, there exists constants $M_j, L_j^f > 0$ such that

$$|f_j(u)| \le M_j,\ L_j^f = \sup_{u\neq v}|\frac{f_j(u) - f_j(v)}{u - v}|,\ u \neq v,$$

$i = 1, 2, \ldots, n$.

(H_4) There exist constant $\lambda_{ik} \ge 0$ such that $|I_{ik}(u) - I_{ik}(v)| \le \lambda_{ik}|u - v|$ for all $u, v \in R, i = 1, 2, \ldots, n, k = 1, 2, \ldots, n$.

The initial value conditions of system 1 are given by

$$x_i(s) = \varphi_i(s),\ s \in [-\tau, 0], i = 1, 2, \ldots, n,$$

where $\varphi \in PC([-\tau, 0], R^n)$.

Let

$$\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2},\ \|\varphi\| = \sup_{s\in[-\tau,0]} \|\varphi(s)\|.$$

The response system with noise perturbation is defined as follows:

$$\begin{cases}\mathrm{d}\, y_i(t) = \Big\{\alpha_i(y_i(t))\Big[-\beta_i(y_i(t)) + \sum\limits_{j=1}^{n}\delta_{ij}\mu_j + I_i \\ \qquad + \bigwedge\limits_{j=1}^{n} a_{ij} f_j(y_j(t)) + \bigwedge\limits_{j=1}^{n} b_{ij} f_j(y_j(t-\tau_j(t))) \\ \qquad + \bigwedge\limits_{j=1}^{n} T_{ij}\mu_j + \bigvee\limits_{j=1}^{n} c_{ij} f_j(y_j(t)) \\ \qquad + \bigvee\limits_{j=1}^{n} d_{ij} f_j(y_j(t-\tau_j(t))) + \bigvee\limits_{j=1}^{n} H_{ij}\mu_j\Big] \\ \qquad + \varepsilon_i(y_i(t) - x_i(t))\Big\}\mathrm{d}\, t \\ \qquad + \sum\limits_{j=1}^{n} \sigma_{ij}(t, y(t) - x(t), y(t-\tau(t)) \\ \qquad - x(t-\tau(t)))\mathrm{d}\, W_j(t),\ t \neq t_k, \\ y_i(t_k) = y_i(t_k^-) + I_{ik}(y_i(t_k^-)),\ t = t_k, \\ y_i(s) = \psi_i(s),\ \ s \in [-\tau, 0],\ \ i = 1, 2, \ldots, n.\end{cases} \tag{2}$$

where $\sigma^T(t, u, v) = (\sigma_1^T(t, u, v), \ldots, \sigma_n^T(t, u, v)) : R_+ \times R^n \times R^n \to R^n$ is called the noise intensity matrix, $W = (W_1, \ldots, W_n)^T \in R^n$ is a n-dimensional Brownian motion defined on a complete probability space (Ω, F, P) with a natural filtration $\{F_t\}_{t\ge0}$, $y(t) - x(t) = (y_1(t) - x_1(t), \ldots, y_n(t) - x_n(t))^T$, $y(t-\tau(t)) - x(t-\tau(t)) = (y_1(t-\tau(t)) - x_1(t-\tau(t)), \ldots, y_n(t-\tau(t)) - x_n(t-\tau(t)))^T$, $\psi \in C^b_{F_0}[[-\tau, 0], R^n]$ denoted the family of all bounded F_0-measurable and $C[[-\tau, 0], R^n]$-valued random variables with norm $\|\psi\|_F^2 = \sup\limits_{s\in[-\tau,0]} E\|\psi\|^2$, $\tau = \max\limits_{j=1,2,\ldots,n} \sup\limits_{t\in R}\{\tau_j(t)\}$, and $E\{\cdot\}$ stands for the mathematical expectation operator.

Throughout this paper, we always make the following assumption:

(H_5) Assume that $\sigma(t, u, v)$ satisfies the Lipschitz condition and the linear growth condition, and there exist $p_{ij}, q_{ij} \ge 0$ such that

$$\sigma_i(t, u, v)\sigma_i^T(t, u, v) \le \sum_{j=1}^{n}(p_{ij}u_j^2 + q_{ij}v_j^2). \tag{3}$$

The condition (H_5) guarantees the global existence and uniqueness of the solution of system (2)(see [27] for more details).

Let $e_i(t) = y_i(t) - x_i(t)$, then the error dynamical system between the drive system (1) and the response system (2) is given as follows:

$$
\begin{cases}
\mathrm{d}\, e_i(t) = \Big\{ -\Big[\alpha_i(y_i(t))\beta_i(y_i(t)) - \alpha_i(x_i(t))\beta_i(x_i(t))\Big] \\
\quad +\Big(\alpha_i(y_i(t)) - \alpha_i(x_i(t))\Big)\widetilde{I_i} \\
\quad +\alpha_i(y_i(t))\Big[\bigwedge_{j=1}^{n} a_{ij} f_j(y_j(t)) - \bigwedge_{j=1}^{n} a_{ij} f_j(x_j(t)) \\
\quad + \bigwedge_{j=1}^{n} b_{ij} f_j(y_j(t-\tau_j(t))) \\
\quad - \bigwedge_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) + \bigvee_{j=1}^{n} c_{ij} f_j(y_j(t)) \\
\quad - \bigvee_{j=1}^{n} c_{ij} f_j(x_j(t)) + \bigvee_{j=1}^{n} d_{ij} f_j(y_j(t-\tau_j(t))) \\
\quad - \bigvee_{j=1}^{n} d_{ij} f_j(x_j(t-\tau_j(t)))\Big] \\
\quad +\Big(\alpha_i(y_i(t)) - \alpha_i(x_i(t))\Big)\Big[\bigwedge_{j=1}^{n} a_{ij} f_j(x_j(t)) \\
\quad + \bigwedge_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) + \bigvee_{j=1}^{n} c_{ij} f_j(x_j(t)) \\
\quad + \bigvee_{j=1}^{n} d_{ij} f_j(x_j(t-\tau_j(t)))\Big] - \varepsilon_i e_i(t)\Big\}\mathrm{d}\, t \\
\quad + \sum_{j=1}^{n} \sigma_{ij}(t, e(t), e(t-\tau(t)))\mathrm{d}\, W_j(t),\ t \neq t_k, \\
e_i(t_k) = e_i(t_k^-) + I_{ik}(y_i(t_k^-)) - I_{ik}(x_i(t_k^-)),\ t = t_k, \\
e_i(s) = \phi_i(s),\ \ s \in [-\tau, 0],\ \ i = 1, 2, \ldots, n.
\end{cases}
\tag{4}
$$

where $\widetilde{I_i} = \sum_{j=1}^{n} \delta_{ij}\mu_j + I_i + \bigwedge_{j=1}^{n} T_{ij}\mu_j + \bigvee_{j=1}^{n} H_{ij}\mu_j$

The organization of the rest of this paper is as follows: In Section 2, we introduce some notations and definitions, and state some preliminary results needed in later sections. In Section 3, we establish our main results by constructing a proper Lyapunov functional. In section 4, we give an example to illustrate our results.

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II. Preliminaries

In order to obtain our results, we need the following definition:

Definition 2.1: The drive system (1) and the response system (2) are said to be globally exponentially square-mean synchronized if, for a suitably designed feedback controller, there exist constant $\lambda > 0$ such that for any $t \geq 0$,

$$E\|y(t) - x(t)\|^2 \leq E\|y(0) - x(0)\|_F^2 e^{-\lambda t},$$

where $e(t)$ is any solution of system (4) and the constant λ is defined as the exponential synchronization rate.

Before stating our main results, we need a few more notations. Let $C^{1,2}(R_+ \times R^n, R_+)$ denote the family of all nonnegative functions $V(t,x)$ on $R_+ \times R^n$ which are continuously twice differentiable in x and once differentiable in t. For SDE

$$\mathrm{d}\, x = f(t, x(t), x_t) + \sigma(t, x(t), x_t)\mathrm{d}\, W(t), \tag{5}$$

where $x_t = x(t-s), s \in [-\tau, 0]$. For each $V \in C^{1,2}(R_+ \times R^n, R_+)$ define an operator $\mathcal{L}$ associated with system (5) acting on V by

$$\mathcal{L}V(t,x) = V_t(t,x) + V_x(t,x) f(t,x,y) + \frac{1}{2} trace[\sigma^T(t,x,y) V_{xx}(t,x)\sigma(t,x,y)],$$

where

$$V_t(t,x) = \frac{\partial V(t,x)}{\partial t}, V_x(t,x) = \left(\frac{\partial V(t,x)}{\partial x_1}, \ldots, \frac{\partial V(t,x)}{\partial x_n}\right),$$

$V_{xx}(t,x) = \left(\frac{\partial^2 V(t,x)}{\partial x_i \partial x_j}\right)_{n\times n}$.

The following lemmas are useful for the proof of our main results of this paper.

Lemma 2.1: [12] For any $i \in \{1, 2, \ldots, n\}$, suppose u and v are two states of system (1). Then we have

$$\left|\bigwedge_{j=1}^{n} \alpha_{ij} f_j(u_j) - \bigwedge_{j=1}^{n} \alpha_{ij} f_j(v_j)\right| \leq \sum_{j=1}^{n} |\alpha_{ij}||f_j(u_j) - f_j(v_j)|,$$

$$\left|\bigvee_{j=1}^{n} \beta_{ij} f_j(u_j) - \bigvee_{j=1}^{n} \beta_{ij} f_j(v_j)\right| \leq \sum_{j=1}^{n} |\beta_{ij}||f_j(u_j) - f_j(v_j)|.$$

Lemma 2.2: $(Halanay\ inequality)$[25] Assume that there exist $k_1 > k_2 > 0, y(t) \in C[t_0 - \tau, t_0]$ is nonnegative, and

$$D^+ y(t) \leq -k_1 y(t) + k_2 \overline{y}(t),$$

where $\overline{y}(t) = \sup_{t-\tau \leq s \leq t}\{y(s)\}, \tau > 0$, then

$$y(t) \leq \overline{y}(t_0) e^{-\lambda(t-t_0)}, t \geq t_0,$$

where λ is unique solution of the equation

$$\lambda = k_1 - k_2 e^{\lambda\tau}.$$

Lemma 2.3: [25] Assume that there exist $k_1 > k_2 > 0, V \in C^{1,2}(R_+ \times R^n, R_+)$, and

$$\mathcal{L}V(x,y) \leq -k_1 V(x) + k_2 V(y),$$

then

$$EV(x(t)) \leq \overline{EV}(x(0)) e^{-\lambda t}, t \geq 0,$$

where $\overline{EV}(x(t)) = \sup_{t-\tau \leq s \leq t} EV(x(s)), \lambda$ is unique solution of the equation $\lambda = k_1 - k_2 e^{\lambda\tau}$, $x(t)$ is any solution of system (5).

III. Main Result

Our main result of this paper is as follows:

Theorem 3.1: Assume that $(H_1) - (H_5)$ hold, furthermore, suppose that

(i) $k_1 > k_2$, where

$$\begin{aligned} k_1 &= \min_{i=1,\dots,n} \Big\{ 2\gamma_i + 2\varepsilon_i - 2L_i^{\alpha}(\widetilde{I}_i + N_i) \\ &\quad - \sum_{j=1}^{n} \big[\overline{\alpha}_i L_j^f(|a_{ij}| + |b_{ij}| + |c_{ij}| + |d_{ij}|) \\ &\quad + \overline{\alpha}_j L_i^f(|a_{ji}| + |c_{ji}|) + 2p_{ji}\big] \Big\}, \\ k_2 &= \max_{i=1,\dots,n} \Big\{ \sum_{j=1}^{n} \big[\overline{\alpha}_j L_i^f(|b_{ji}| + |d_{ji}|) + 2q_{ji}\big] \Big\}, \\ N_i &= \bigwedge_{j=1}^{n} |a_{ij}|M_j + \bigwedge_{j=1}^{n} |b_{ij}|M_j \\ &\quad + \bigvee_{j=1}^{n} |c_{ij}|M_j + \bigvee_{j=1}^{n} |d_{ij}|M_j \end{aligned}$$

(ii) $\Lambda_k \le e^{\mu \Delta t_k - \lambda\tau}$, where $\Lambda_k = \max_{i=1,\dots,n}\{(1+\lambda_{ik})^2\}, \Delta t_k = t_k - t_{k-1}, k = 1, \dots, n, t_0 = 0, 0 < \mu < \lambda, \lambda$ is the unique solution of the following equation $\lambda = k_1 - k_2 e^{\lambda\tau}$.

Then system (1) and system (2) is globally exponentially square-mean synchronized.

Proof: Let $V(e(t)) = \frac{1}{2}\sum_{i=1}^{n} e_i^2(t)$. Then, by (4), we obtain

$$\begin{aligned} \mathcal{L}V(t) &= \sum_{i=1}^{n} e_i(t) \Big\{ - \Big[\alpha_i(y_i(t))\beta_i(y_i(t)) \\ &\quad - \alpha_i(x_i(t))\beta_i(x_i(t))\Big] + \Big(\alpha_i(y_i(t)) - \alpha_i(x_i(t))\Big)\widetilde{I}_i \\ &\quad + \alpha_i(y_i(t))\Big[\bigwedge_{j=1}^{n} a_{ij} f_j(y_j(t)) - \bigwedge_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ &\quad + \bigwedge_{j=1}^{n} b_{ij} f_j(y_j(t-\tau_j(t))) - \bigwedge_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) \\ &\quad + \bigvee_{j=1}^{n} c_{ij} f_j(y_j(t)) - \bigvee_{j=1}^{n} c_{ij} f_j(x_j(t)) \\ &\quad + \bigvee_{j=1}^{n} d_{ij} f_j(y_j(t-\tau_j(t))) - \bigvee_{j=1}^{n} d_{ij} f_j(x_j(t-\tau_j(t)))\Big] \\ &\quad + \Big(\alpha_i(y_i(t)) - \alpha_i(x_i(t))\Big)\Big[\bigwedge_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ &\quad + \bigwedge_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j(t))) + \bigvee_{j=1}^{n} c_{ij} f_j(x_j(t)) \\ &\quad + \bigvee_{j=1}^{n} d_{ij} f_j(x_j(t-\tau_j(t)))\Big] - \varepsilon_i e_i(t) \Big\} \\ &\quad + \frac{1}{2} trace[\sigma^T(t, e(t), e(t-\tau(t)))\sigma(t, e(t), e(t-\tau(t)))] \\ &\le \sum_{i=1}^{n} e_i(t) \Big\{ - \gamma_i e_i(t) + L_i^{\alpha}(\widetilde{I}_i + N_i)|e_i(t)| \\ &\quad + \overline{\alpha}_i\big[\sum_{j=1}^{n}(|a_{ij}| + |c_{ij}|)L_j^f |e_j(t)| \\ &\quad + \sum_{j=1}^{n}(|b_{ij}| + |d_{ij}|)L_j^f |e_j(t-\tau_j(t))|\big] - \varepsilon_i e_i(t) \Big\} \\ &\quad + \frac{1}{2} trace[\sigma^T(t, e(t), e(t-\tau(t)))\sigma(t, e(t), e(t-\tau(t)))] \\ &\le \sum_{i=1}^{n} \Big\{ \big[- \gamma_i - \varepsilon_i + L_i^{\alpha}(\widetilde{I}_i + N_i)\big] e_i^2(t) \\ &\quad + \overline{\alpha}_i\big[\sum_{j=1}^{n}(|a_{ij}| + |c_{ij}|)L_j^f \frac{e_j^2(t) + e_i^2(t)}{2} \\ &\quad + \sum_{j=1}^{n}(|b_{ij}| + |d_{ij}|)L_j^f \frac{e_j^2(t-\tau_j(t)) + e_i^2(t)}{2}\big] \\ &\quad + \sum_{j=1}^{n}(p_{ij} e_j^2(t) + q_{ij}) e_j^2(t-\tau_j(t)) \Big\} \\ &\le \sum_{i=1}^{n} \big[- \gamma_i - \varepsilon_i + L_i^{\alpha}(\widetilde{I}_i + N_i) \\ &\quad + \sum_{j=1}^{n} \big(\frac{\overline{\alpha}_i L_j^f(|a_{ij}| + |b_{ij}| + |c_{ij}| + |d_{ij}|)}{2} \\ &\quad + \frac{\overline{\alpha}_j L_i^f(|a_{ji}| + |c_{ji}|)}{2}\big) + p_{ji}\big] e_i^2(t) \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n} \big(\frac{\overline{\alpha}_j L_i^f(|b_{ji}| + |d_{ji}|)}{2} + q_{ji}\big) e_i^2(t-\tau_i(t)) \\ &\le -k_1 V(t) + k_2 \overline{V}(t). \end{aligned}$$

Since $k_1 > k_2$, by Lemma 2.3, it follows that

$$EV(t) \le \overline{EV}(t_k) e^{-\lambda(t-t_k)}, t \in [t_k, t_{k+1}).$$

Because of

$$\begin{aligned} EV(t_k) &= E\sum_{i=1}^{n} \frac{e_i^2(t_k)}{2} \\ &= E\sum_{i=1}^{n} \frac{1}{2}(e_i(t_k^-) + I_{ik}(y_i(t_k^-)) - I_{ik}(x_i(t_k^-)))^2 \\ &= E\sum_{i=1}^{n} \frac{(1+\lambda_{ik})^2}{2} e_i^2(t_k^-) \\ &\le \max_{i=1,\dots,n}\{(1+\lambda_{ik})^2\} E\sum_{i=1}^{n} \frac{e_i^2(t_k^-)}{2} \le \Lambda_k EV(t_k^-). \end{aligned}$$

Then for $t \in [0, t_1)$,

$$EV(t) \le \overline{EV}(0) e^{-\lambda t},$$

hence,

$$EV(t_1) \le \Lambda_1 EV(t_1^-) \le \Lambda_1 \overline{EV}(0) e^{-\lambda t_1},$$

then for $t \in [t_1, t_2)$,

$$EV(t) \le \Lambda_1 \overline{EV}(0) e^{-\lambda(t-\tau)}.$$

Repeating the above process, when $t \in [t_k, t_{k+1})$,

$$\begin{aligned} EV(t) &\le \Lambda_1 \cdots \Lambda_k \overline{EV}(0) e^{-\lambda(t-k\tau)} \\ &\le e^{\mu\Delta t_1} \cdots e^{\mu\Delta t_k} \overline{EV}(0) e^{-\lambda t} \\ &= \overline{EV}(0) e^{\mu(t_k - t_0)} e^{-\lambda t} \\ &\le \overline{EV}(0) e^{-(\lambda-\mu)t}. \end{aligned}$$

Then

$$E\|e(t)\|^2 = E\Big(\sum_{i=1}^{n} e_i^2(t)\Big) \le \|\phi\|_F^2 e^{-(\lambda-\mu)t}, t \ge 0.$$

Therefore, system (1) and system (2) is globally exponentially square-mean synchronized.

This completes the proof. ■

From Theorem 3.1, we can easily obtain the following result about the continuous FCGNN described in Remark 1.1.

Corollary 3.1: Assume that $(H_1)-(H_5)$ hold, furthermore, suppose that the condition (i) in Theorem 3.1 is true, then the system described in Remark 1.1 is globally exponentially square-mean synchronized.

Remark 3.1: For our model (1), the time delays $\tau_j(t)$ can be constants τ_{ij}, then system (1) turns to be system (1) in [23]. By Theorem 3.1, we can obtain the similar results about system (1) in [23]. Meantime, if the amplification function $\alpha_i(x_i(t)) = 1$, behaved function $\beta_i(x_i(t)) = \beta_i x_i(t)$ where β_i are constants, we can have some special cases for our model, and they have been studied by many papers.

Remark 3.2: In Theorem 3.1, the traditional assumption on the differentiability of the time-varying delays is no longer needed. Therefore, our results are more general and easy to be verified.

Remark 3.3: In this paper, the amplification function and the activation function are required to be bounded, which is a strict condition, to make fuzzy Cohen-Grossberg neural networks synchronized. Therefore, the synchronization of impulsive fuzzy Cohen-Grossberg neural networks without the boundedness condition of the two functions remains to be further research.

IV. An Example

In this section, an example is given to demonstrate the results yielded above.

Example 4.1: Consider the following two-dimensional impulsive fuzzy Cohen-Grossberg neural networks with time-varying delays $(i = 1, 2)$:

$$\begin{cases} \dfrac{\mathrm{d}\, x_i(t)}{\mathrm{d}\, t} &= \alpha_i(x_i(t))\Big[-\beta_i(x_i(t)) + \sum\limits_{j=1}^{2}\delta_{ij}\mu_j + I_i \\ &\quad + \bigwedge\limits_{j=1}^{2} a_{ij} f_j(x_j(t)) + \bigwedge\limits_{j=1}^{2} b_{ij} f_j(x_j(t-\tau_j(t))) \\ &\quad + \bigwedge\limits_{j=1}^{2} T_{ij}\mu_j + \bigvee\limits_{j=1}^{2} c_{ij} f_j(x_j(t)) \\ &\quad + \bigvee\limits_{j=1}^{2} d_{ij} f_j(x_j(t-\tau_j(t))) + \bigvee\limits_{j=1}^{2} H_{ij}\mu_j \Big], \ t \ne t_k, \\ x_i(t_k) &= x_i(t_k^-) + k_0 \sin(x_i(t_k^-)), \ t = t_k, \end{cases} \tag{6}$$

where $\alpha_i(x_i) = 7 + \frac{1}{1+x_i^2}, \beta_1(x_1(t)) = 1.4x_1(t), \beta_2(x_2(t)) = 1.6x_2(t), f_j(x) = \tanh x, a_{11} = 1.8$, $a_{12} = -0.1, a_{21} = -2$, $a_{22} = 0.4$, $b_{11} = -1.7$, $b_{12} = -0.6$, $b_{21} = 0.5$, $b_{22} = -2.5$, $c_{11} = 0.2$, $c_{12} = 0.6$, $c_{21} = 0.2$, $c_{22} = 0.4$, $d_{11} = 0.3$, $d_{12} = 0.2$, $d_{21} = 0.5$, $d_{22} = 0.2$, $\delta_{ij} = T_{ij} = H_{ij} = \mu_j = 1$, $I_1 = 1, I_2 = -0.5$, $\tau_1(t) = \tau_2(t) = \frac{e^t}{1+e^t}$.

The response system with noise perturbation is defined as follows

$$\begin{cases} \mathrm{d}\, y_i(t) &= \Big\{\alpha_i(y_i(t))\Big[-\beta_i(y_i(t)) + \sum\limits_{j=1}^{2}\delta_{ij}\mu_j + I_i \\ &\quad + \bigwedge\limits_{j=1}^{2} a_{ij} f_j(y_j(t)) + \bigwedge\limits_{j=1}^{2} b_{ij} f_j(y_j(t-\tau_j(t))) \\ &\quad + \bigwedge\limits_{j=1}^{2} T_{ij}\mu_j + \bigvee\limits_{j=1}^{2} c_{ij} f_j(y_j(t)) \\ &\quad + \bigvee\limits_{j=1}^{2} d_{ij} f_j(y_j(t-\tau_j(t))) + \bigvee\limits_{j=1}^{2} H_{ij}\mu_j \Big] \\ &\quad + \varepsilon_i(y_i(t) - x_i(t))\Big\}\mathrm{d}\, t + \Big[\sum\limits_{j=1}^{2}(1 - e^{-y_j(t)-x_j(t)}) \\ &\quad + \sum\limits_{j=1}^{2}(1 - e^{-y_j(t-\tau_j(t))-x_j(t-\tau_j(t))})\Big]\mathrm{d}\, W_j(t), \ t \ne t_k, \\ y_i(t_k) &= y_i(t_k^-) + k_0 \sin(y_i(t_k^-)), \ t = t_k, \\ y_i(s) &= \psi_i(s), \ \ s \in [-\tau, 0], \ \ i = 1, 2. \end{cases} \tag{7}$$

where $\varepsilon_1 = 1, \varepsilon_2 = 3$.

By simply calculation, $7 \le \alpha_i(x_i) \le 8, M_j = L_j^f = 1, L_i^\alpha = 0.5, \gamma_1 = 9.1, \gamma_2 = 10.4, \tau = 1, \lambda_{ik} = k_0, p_{ij} = q_{ij} = 4, i, j = 1, 2, k = 1, 2, \ldots$. Then condition (i) is satisfied in Theorem 3.1. So there exists $\lambda \in (0, 1)$ such that $\lambda = k_1 - k_2 e^{\lambda\tau}$. Choose $\mu = \frac{\lambda}{3}$ and as $\Delta_k \ge \frac{6\ln(1+k_0)}{\lambda} + 1$, condition (ii) is satisfied in Theorem 3.1. Therefore, system (6) and (7) are globally exponentially square-mean synchronized.

V. Conclusion

This paper considers the global exponential square-mean synchronization of the FCGNNs, which is one of the most popular and typical network models. To the best of our knowledge, there are few results for synchronization of impulsive fuzzy neural networks under noise perturbation. Some sufficient conditions are obtained about the global exponential square-mean synchronization of the FCGNNs under noise perturbation. In particular, the traditional assumption on the differentiability of the time-varying delays is no longer needed. However, there are also some work worth further studying, such as the impulsive effects on the FCGNNs.

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