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Radiation Effect on Unsteady MHD Flow over a Stretching Surface

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Abstract**—**Unsteady magnetohydrodynamics (MHD) boundary layer flow and heat transfer over a continuously stretching surface in the presence of radiation is examined. By similarity transformation, the governing partial differential equations are transformed to a set of ordinary differential equations. Numerical solutions are obtained by employing the Runge-Kutta-Fehlberg method scheme with shooting technique in Maple software environment. The effects of unsteadiness parameter, radiation parameter, magnetic parameter and Prandtl number on the heat transfer characteristics are obtained and discussed. It is found that the heat transfer rate at the surface increases as the Prandtl number and unsteadiness parameter increase but decreases with magnetic and radiation parameter.

Keywords**—**Heat transfer, magnetohydrodynamics, radiation, unsteadiness.

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I. Introduction

FLOW and heat transfer due to a stretching surface are important in the industrial manufacturing of metal and polymer sheets, paper production, glass blowing, metal spinning, and drawing plastic film. The quality of the final product usually depends on the rate of heat transfer at the stretching surface during the manufacturing processes. The flow and heat transfer problems for stretching surfaces have received considerable attention by numerous researchers [1]–[10]. Most of the investigations are motivated by Sakiadis [11], the pioneer on steady laminar boundary layer flow over a moving surface with constant velocity.

The flow and heat transfer can be unsteady due to a sudden stretching of the flat sheet or by a step change of the temperature of the sheet [12]. The stream velocity is dependent on time and causes the unsteadiness in the flow and temperature fields. However, the unsteady problems due to a stretching surface received less attention. Elbashbeshy and Bazid [13] examined the effects of unsteadiness parameter and Prandtl number on the flow and heat transfer characteristics. The results show that the unsteadiness parameter and Prandtl number increases the heat transfer rate at the surface. These results were supported by the investigations from other researchers, for example, the effect of chemical reaction which includes unsteadiness parameter and Prandtl number on heat transfer due to unsteady stretching sheet by El-Aziz [14]. Ishak et al. [15] obtain the exact solution of unsteady mixed convection boundary layer flow and heat transfer and concluded that the buoyancy parameter increases the heat transfer rate at the surface. Bachok et al. [16] studied the effect of material parameter (dimensionless viscosity ratio) of the unsteady laminar flow of an incompressible micropolar fluid. They found that the skin friction coefficient decreases as the material parameter increases and the micropolar fluid reduces drag compared to viscous fluid. Liu and Andersson [17] observed the thermal characteristics of a viscous film on an unsteady stretching surface. The effect of increasing the unsteadiness is to decrease the rate of heat transfer but the increasing Prandtl number increases the rate of heat transfer.

The present paper extends Ishak's work [18] to include the presence of radiation for the MHD flow and heat transfer characteristics over an unsteady stretching surface. Radiation is considered in this study due to the fact that thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry [12].

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II. Mathematical Formulation

Consider the unsteady two-dimensional laminar boundary layer flow on a continuously stretching surface immersed in an incompressible electrically conducting fluid. It is assumed that the unsteady fluid and heat flows start at time $t = 0$. Keeping the origin fixed, the surface is stretched with the velocity $U_w(x,t)$ along the x-axis. The stretching velocity $U_w(x,t)$ and the surface temperature $T_w(x,t)$ are assumed to be in the form of

$$U_w(x,t) = \frac{ax}{1-ct}, \quad T_w(x,t) = T_\infty + \frac{bx}{1-ct}, \tag{1}$$

respectively, where a, b and c are constants with dimension time^{-1} [19].

The velocity and temperature fields in the boundary layer are governed by the two-dimensional boundary layer equations for mass, momentum and energy,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}, \tag{4}$$

subject to the boundary conditions:

$$u = U_w, v = 0, T = T_w \quad \text{at} \quad y = 0,$$
$$u \to 0, T \to T_\infty \quad \text{as} \quad y \to \infty, \tag{5}$$

where u and v are the velocity components along the x axes and y axes, respectively, T is the fluid temperature in the boundary layer, t is the time, υ is the kinematic viscosity, ρ is the fluid density, α is the thermal diffusivity, C_p is the specific heat at constant pressure and q_r is the radiative heat flux. To obtain similarity solution for Eqns. (2)-(5), the unsteady magnetic field B is assumed to be of the form $B = B_0 / \sqrt{1-ct}$ where B_0 is a constant [18].

The radiative heat flux q_r, under Rosseland approximation [12], has the form:

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ is the Stefan-Boltzman constant and k^* is the absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. Expanding T^4 about T_∞ in Taylor's series and neglecting higher orders yields:

$$T^4 = 4T_\infty^{\ 3}T - 3T_\infty^{\ 4}. \tag{7}$$

Substituting (6) and (7) into (4) gives:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha(1+N)\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where $N = 16\sigma T_\infty^{\ 3} / (3k^* k)$ is the radiation parameter where k is the thermal conductivity.

The mathematical problem is simplified by introducing the following dimensionless functions f and θ, and the similarity variable η [18], [19].

$$\eta = \left[\frac{U_w}{\upsilon x}\right]^{\frac{1}{2}} y, \quad \psi = [U_w \upsilon x]^{\frac{1}{2}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{9}$$

The equation of continuity is satisfied for the stream function $\psi(x, y)$ with the relations,

$$u = \frac{\partial \psi}{\partial y} = \frac{ax}{1-ct} f'(\eta), \; v = -\frac{\partial \psi}{\partial x} = -\left[\frac{va}{1-ct}\right]^{\frac{1}{2}} f(\eta). \tag{10}$$

The mathematical problem defined by (2), (3), (8) and boundary conditions (5) are then transformed into a set of ordinary differential equations,

$$f''' + ff'' - f'^2 - Mf' - A\left[f' + \frac{1}{2}\eta f''\right] = 0, \tag{11}$$

$$\frac{1}{\Pr}(1+N)\theta'' + f\theta' - f'\theta - A\left[\theta + \frac{1}{2}\eta\theta'\right] = 0, \tag{12}$$

and the new boundary conditions are:

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \quad \text{at } \eta = 0,$$
$$f'(\eta) \to 0, \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty, \tag{13}$$

where prime denotes differentiation with respect to η, $A = c / a$ is the unsteadiness parameter, $M = \sigma B_0^2 / \rho a$ is the magnetic parameter and $\Pr = \upsilon / \alpha$ is the Prandtl number.

The physical quantities of interest are the skin friction coefficient,

$$C_f = \frac{\tau_w}{\rho U_w^{\ 2} / 2}, \tag{14}$$

and the local Nusselt number:

$$Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \tag{15}$$

where the surface shear stress τ_w and the surface heat flux q_w are:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=o}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{16}$$

with μ being the dynamic viscosity. Using non-dimensional variable (9) in (16), we obtain:

$$\frac{1}{2}C_f Re_x^{1/2} = f''(0), \qquad Nu_x / Re_x^{1/2} = -\theta'(0), \tag{17}$$

where $\text{Re}_x = U_w x / \upsilon$ is the local Reynolds number.

III. Results and Discussion

The nonlinear ordinary differential equations (11) and (12) subject to the boundary conditions (13) are solved numerically by using the Runge-Kutta-Fehlberg method scheme with shooting technique in Maple software. To verify the validity and accuracy of the result obtained, numerical results for the value of heat transfer rate at the surface are compared with Ishak [18] and Liu [20] for the case $N = 0$ (without radiation) and steady-state flow case. The numerical values as shown in

Table I are in excellent agreement with the values of Ishak [18] and Liu [20].

TABLE I
VALUES OF $-\theta'(0)$ FOR VARIOUS VALUES OF $A.M$, N and Pr

A	M	N	Pr	$-\theta'(0)$ Liu[20]	Ishak[18]	Present
0	0	0	0.72		0.8086	0.808631
			1		1.0000	1.000000
			3		1.9237	1.923683
			6.7	3.00027	3.0003	3.000272
0	1	0	0.7	0.689699	0.6897	0.689712
			1	0.892147	0.8921	0.892147
			10	3.61699	3.617	3.616992
1	0	0	0.7		1.0834	1.083386
			7		3.7682	3.768235
1	1	0	0.7		1.0500	1.049986
			7		3.7164	3.716467
1	1	1	0.7			0.708645
			1			0.867918
			3			1.608920
			7			2.561119

The temperature profiles for various values of Prandtl number Pr and unsteadiness parameter A are presented in Figs. 1 and 2. Both figures show that the thermal boundary layer thickness decreases as Pr or A increases with increasing temperature gradient at the surface. Thus, the heat transfer rate at the surface increases with increasing Pr or A.

Figs. 3 and 4 depict the temperature profiles for various values of magnetic parameter M and radiation parameter N. It can be seen that the absolute value of the temperature gradient at the surface decreases with an increase in M or N. So, the heat transfer rate at the surface decreases as M or N increases.

The effects of magnetic parameter M and unsteadiness parameter A on the velocity profiles are observed in Figs. 5 and 6. The momentum boundary layer thickness decreases with M or A, hence, induces an increase in the absolute value of the velocity gradient at the surface. Thus, the rate of the heat transfer at the surface decreases with the presence of magnetic parameter and unsteadiness parameter.

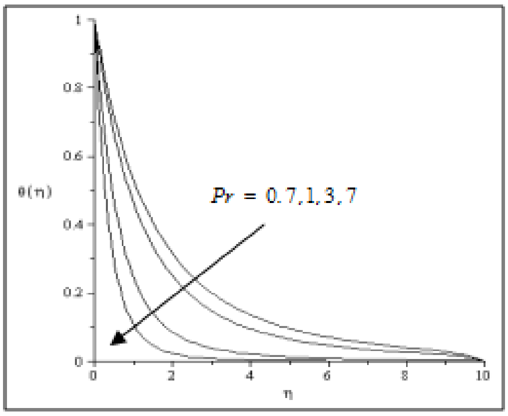

Fig. 1 Temperature profiles for various values of Pr when $M=1$, $N=1$ and $A=1$

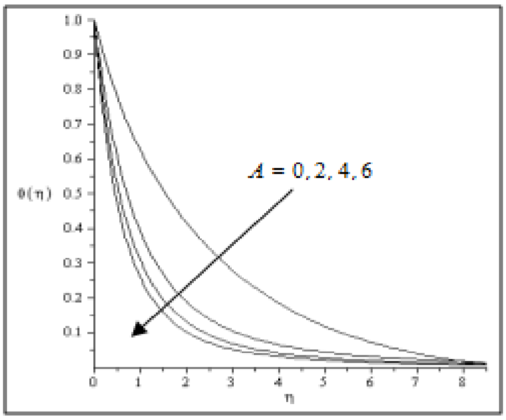

Fig. 2 Temperature profiles for various values of A when $M=1$, $\Pr=1$ and $N=1$

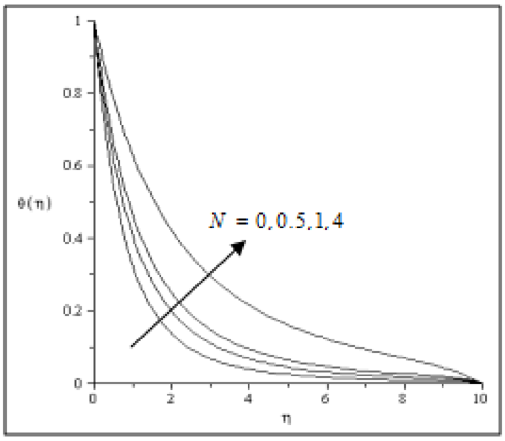

Fig. 3 Temperature profiles for various values of N when $M=1$, $\Pr=1$ and $A=1$

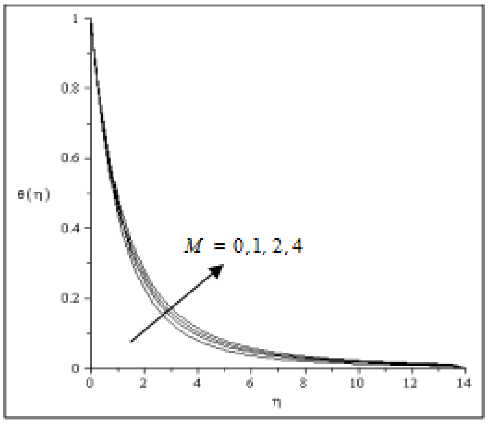

Fig. 4 Temperature profiles for various values of M when $A=1$, $\Pr=1$ and $N=1$

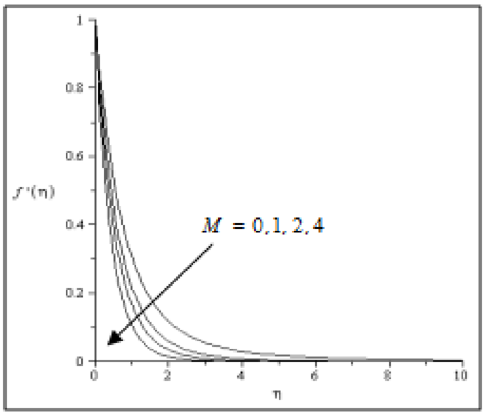

Fig. 5 Velocity profiles for various values of M when $A=1, Pr=1$ and $N=1$

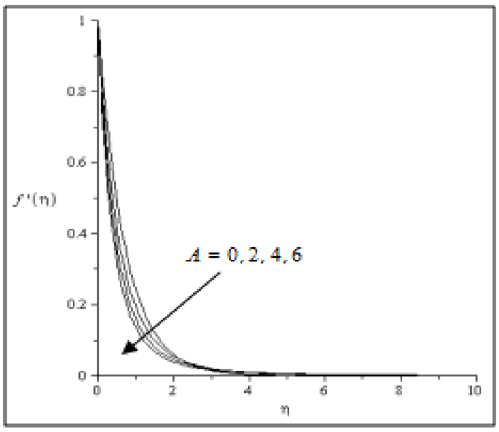

Fig. 6 Velocity profiles for various values of A when $M=1, \Pr=1$ and $N=1$

IV. Conclusion

The unsteady MHD boundary layer flow over a stretching surface with the presence of radiation has been studied in this paper. The effects of radiation parameter, magnetic parameter, unsteadiness parameter and Prandtl number on the heat transfer characteristics were observed. It can be concluded that the heat transfer rate at the surface increases as unsteadiness parameter and Prandtl number increases. Contrarily, the effects of increasing radiation parameter and magnetic parameter are to decrease the rate of heat transfer at the surface.

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