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The Effect of the Initial Stresses on the Reflection and Transmission of Plane Quasi-Vertical Transverse Waves in Piezoelectric Materials

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***Abstract*—**This study deals with the phenomena of reflection and transmission (refraction) of *qSV*-waves, for an incident of quasi transverse vertically waves, at a plane interface of two semi-infinite piezoelectric elastic media under the influence of the initial stresses. The relations governing the reflection and transmission coefficients of these reflected waves for various suitable boundary conditions are derived. We have shown analytically that reflection and transmission coefficients of (*qP*) and (*qSV*) waves depend upon the angle of incidence, the parameters of electric potential, the material constants of the medium as will as the initial stresses presented in the media. The numerical calculations of the reflection and transmission amplitude ratios for different values of initial stresses have been carried out by computer for different materials as examples and the results are given in the form of graphs. Finally, some of particular cases are considered.

***Keywords*—**Quasi plane vertical transverse waves; reflection and transmission coefficients; initial stresses; PZT-5H Ceramic; Aluminum Nitride (AlN), Piezoelectricity.

I. Introduction

UNDERSTANDING wave reflection and transmission at interfaces between two similar or dissimilar materials is of interest in many fields, e.g., composites engineering, geology, seismology, seismic exploration, control systems and acoustics. But, comparatively little investigation has been made in an anisotropic mixed media, such as piezoelectric materials. So, The present work is supposed to be useful in further studies of wave propagation in the more realistic models of piezoelectric and elastic solids which have been extensively used in many engineering and industrial applications such as computer technology, actuators, sensors, radio, intelligent structures and ultrasonic etc.

Previous work in this area includes that of Chattopadhyay et al [4] who illustrated the problem reflection of quasi-*P* waves and quasi-*SV* waves at free and rigid boundaries of a fibre-reinforced medium, and Dey et al [5] who investigated the phenomena of reflection and refraction of plane elastic waves at a plane interface between two semi-infinite elastic solid media in contact, when both the media are initially stressed. Moreover, Hussain et al [6] illustrated the influence of pure homogeneous strain on the reflection and transmission of plane waves at the boundary between two half-spaces of incompressible isotropic elastic material. Recently, some researchers have attempted some problems of reflection and refraction of elastic waves from anisotropic materials interfaces using different techniques. Abd-Alla et al [1] investigated Reflection and refraction of plane quasi-longitudinal waves at an interface of two piezoelectric media under initial stresses. Pang et al [11] discussed the reflection and refraction of plane waves at the interface between two transversely isotropic piezoelectric and piezomagnetic media. Kumar et al [8] studied a problem concerned with the reflection and transmission of plane waves at an imperfectly bonded interface of two orthotropic generalized thermoelastic half-spaces with different elastic and thermal properties. Othman and Song [9] have obtained the reflection coefficients of magneto-thermo-elastic waves from a rotating elastic half-space under generalized thermo-elasticity theory. They [10] also investigated the reflection amplitude ratios of the reflected waves for the incidence of P- and SV-waves from an insulated and isothermal stress-free surface under hydrostatic initial stress without energy dissipation. Singh and Khurana [12] also investigated the reflection and transmission of P- and SV-waves at the interface between two monoclinic elastic half-spaces. Singh J. et al [13] discussed the reflection and transmission of plane transverse waves through a plane interface between two different elastic half-spaces containing pores. Singh [14] also considered the reflection phenomena of SV wave from the free surface of an elastic solid with generalized thermoelastic diffusion and obtained the closed-form expressions for the reflection coefficients for various reflected waves. Moreover, Singh et al [15] discussed the reflection coefficients of various reflected waves of plane wave propagation in an orthotropic micropolar elastic solid from a stress-free boundary. Khurana [7] presented the reflection phenomenon of plane elastic waves from a stress free plane boundary of an electro-microelastic solid half-space. They obtained the amplitude ratios and energy ratios of

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various reflected waves when an elastic wave is made incident obliquely at the stress free plane boundary of an electro-microelastic solid half-space. Tomar et al [16] investigated the wave propagation and their reflection and transmission from a plane interface between two different microstretch elastic solid half-spaces in perfect contact. Tomar et al [17] analyzed a problem of reflection and transmission of plane longitudinal wave at a plane interface between two different elastic solid half-spaces with voids. For discussion of reflection and transmission in the context of the isotropic elastic bodies the reader is referred to [2] and the context of the anisotropic theory without pre-strain (see, [3]).

In this paper, we have attempted a problem of reflection and transmission of a quasi-*SV* wave (called *qSV*-wave) incident obliquely at interface between two dissimilar anisotropic pre-stressed elastic half-spaces. The reflection and transmission coefficients are obtained. The numerical calculations of the reflection and transmission amplitude ratios for different values of initial stresses have been carried out by computer for different materials as an example and the results are given in the form of graphs. Finally, some of particular cases are considered. This investigation is relevant to acoustic device application of piezoelectric composite structures.

II. Basic Equations and Formulation of the Problem

We will consider a transversely isotropic piezoelectric half-space (hexagonal crystal structure, class 6mm) occupying region $z \le 0$ and adjoining the vacuum $z > 0$. Let the wave motion in this medium be characterized by: the displacement vector $\vec{u}(u,0,w)$, the electric potential function φ, all these quantities being dependent only on the variables $x,z,t.$ (see fig. 1).

The general forms of the dynamical equations of motion with homogeneous initial stresses and the equation for electric field can be expressed as follows:

$$\sigma_{ij,j} + (u_{i,k}\sigma^{\circ}_{kj})_{,j} = \rho \ddot{u}_i \tag{1}$$

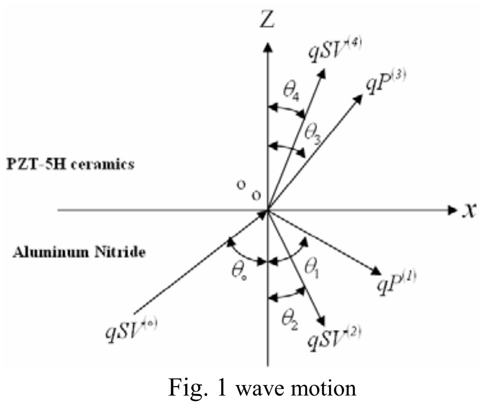

Fig. 1 wave motion

Completed by the constitutive equations:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \tag{2}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \in_{ik} E_k \tag{3}$$

where σ_{ij}, ε_{kl}, E_k and D_i indicate the stress tensor, strain tensor, electric field vector, and electric displacement vector, respectively. c_{ijkl}, e_{kij} and $\in_{ik}$ are elastic, piezoelectric and dielectric constants for piezoelectric medium, with $i, j, k, l = 1, 2, 3$ and ρ is the mass density, u_i denote the mechanical displacements in the ith direction, σ°_{kj} is the initial stress tensor. The dot denotes time differentiation, the comma denotes space-coordinate differentiation, the repeated index in the subscript implies summation.

In connection with Gauss's divergence equation:

$$D_{i,i} = 0.$$

(4) The relationship between the displacement components and strain components are given by

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

The Maxwell equations in the quasi-static approximation are

$$E_k = -\varphi_{,k} \tag{6}$$

The constitutive equations are different depending on the types of piezoelectric materials considered. In this paper, the piezoelectric material of hexagonal crystal structure, class 6 mm, is employed. For other types of piezoelectric material, the constitutive equations should be changed accordingly. Assuming the six-fold axes of the piezoelectric material parallel to the z-direction, its constitutive equations (2) can be expressed in the form

$$\begin{aligned}
\sigma_{xx} &= C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + C_{13}\varepsilon_{zz} - e_{31}E_z,\\
\sigma_{yy} &= C_{21}\varepsilon_{xx} + C_{11}\varepsilon_{yy} + C_{13}\varepsilon_{zz} - e_{31}E_z,\\
\sigma_{zz} &= C_{13}\varepsilon_{xx} + C_{13}\varepsilon_{yy} + C_{33}\varepsilon_{zz} - e_{33}E_z,\\
\sigma_{yz} &= 2C_{44}\varepsilon_{yz} - e_{15}E_y,\\
\sigma_{zx} &= 2C_{44}\varepsilon_{zx} - e_{15}E_x,\\
\sigma_{xy} &= (C_{11} - C_{12})\varepsilon_{xy}
\end{aligned} \tag{7}$$

$$\begin{aligned}
&D_x = 2e_{15}\varepsilon_{zx} + \in_{11} E_x,\ D_y = 2e_{15}\varepsilon_{yz} + \in_{11} E_y,\\
&D_z = 2(e_{31}\varepsilon_{xx} + e_{31}\varepsilon_{yy} + e_{33}\varepsilon_{zz}) + \in_{33} E_z.
\end{aligned} \tag{8}$$

where $C_{11}, C_{12}, C_{13}, C_{33}$ and C_{44} are the elastic constants, e_{31}, e_{15} and e_{33} are the piezoelectric constants, and $\in_{11}$ and $\in_{33}$ are the dielectric constants for the piezoelectric material $z \le 0$.

It is assumed that a state of plane strain parallel to the $x - z$ plane exists and the propagation of waves in the x-direction is considered. Substituting Eqs. (3) and (4) in Eqs. (1), (for the simplicity, we consider the component of initial stress affect only is σ°_{zz}). The electromechanical-coupling equations of motion in terms of mechanical displacements and electric potential may be simplified as

$$
\begin{aligned}
&(C_{11}+\sigma^{\circ}_{xx})u_{,xx}+(C_{44}+\sigma^{\circ}_{zz})u_{,zz}+(C_{13}+C_{44})w_{,xz}+\\
&\qquad (e_{31}+e_{15})\varphi_{,xz}=\rho\ddot{u},\\
&(C_{13}+C_{44})u_{,xz}+(C_{44}+\sigma^{\circ}_{xx})w_{,xx}+(C_{33}+\sigma^{\circ}_{zz})w_{,zz}+\\
&\qquad e_{15}\varphi_{,xx}+e_{33}\varphi_{,zz}=\rho\ddot{w},\\
&(e_{15}+e_{31})u_{,xz}+e_{15}w_{,xx}+e_{33}w_{,zz}-\in_{11}\varphi_{,xx}\\
&\qquad -\in_{33}\varphi_{,zz}=0.
\end{aligned}
\tag{9}
$$

III. Solution of the Problem for Incident qSV -Waves.

Let us consider the solution of Eqs. $(9)_{1,2}$ as:

$$(\vec{u}^{(n)},\varphi^{(n)})=(\vec{A},\vec{B})\,\vec{d}_n\,\exp(i\eta_n). \tag{10}$$

where different values of the index n serve to label the various types of waves that occur, $\vec{d}$ is the unit displacement vector and η_n is defined as:

$$\eta_n=K_n(\vec{x}\cdot\vec{p}-c_n t). \tag{11}$$

where $\vec{p}$ being the unit propagation vector, $\vec{x}$ is position vector, with $\vec{x}\cdot\vec{p}=const.$ is plane of constant phase, $\vec{A},\vec{B}$ are the vibration amplitudes, c_n the velocity of propagation and K_n the corresponding wave number.

In two dimensions, we have

$$
\begin{aligned}
&\vec{u}^{(n)}=(u_1^{(n)},u_3^{(n)}),\quad \vec{d}_n=(d_1^{(n)},d_3^{(n)}),\\
&\vec{p}^{(n)}=(p_1^{(n)},p_3^{(n)}).
\end{aligned}
\tag{12}
$$

such that $p_1^{(n)2}+p_3^{(n)2}=1$. Assigning $n=0$ for the incident (qSV) quasi-transverse wave which makes an angle $\theta_\circ$. The displacement components and the electric potential Eq. (10), with taking into consideration Eqs. (11)-(12), may be expressed as:

$$
\begin{aligned}
&u^{(\circ)}=-A_\circ\cos\theta_\circ\exp[ik_\circ(x\sin\theta_\circ+z\cos\theta_\circ-c_{T\circ}t)],\\
&w^{(\circ)}=A_\circ\sin\theta_\circ\exp[ik_\circ(x\sin\theta_\circ+z\cos\theta_\circ-c_{T\circ}t)],\\
&\varphi^{(\circ)}=B_\circ\exp[ik_\circ(x\sin\theta_\circ+z\cos\theta_\circ-c_{T\circ}t)].
\end{aligned}
\tag{13}
$$

where $c_\circ=c_{T\circ}=\omega/K_\circ$ is the velocity of incident (qSV) wave and $d^{(\circ)}\neq p^{(\circ)}$ but, in this case $\vec{d}^{(\circ)}=\vec{j}_2\wedge\vec{p}^{(\circ)}$, where $\vec{j}_2$ is the unit vector of the y axis of the Cartesian system of rectangular coordinates.

*For a reflected (qP) quasi-longitudinal wave which makes an angle θ_1, we have

$$
\begin{aligned}
&u^{(1)}=A_1\sin\theta_1\exp[ik_1(x\sin\theta_1-z\cos\theta_1-c_{L1}t)],\\
&w^{(1)}=-A_1\cos\theta_1\exp[ik_1(x\sin\theta_1-z\cos\theta_1-c_{L1}t)],\\
&\varphi^{(1)}=B_1\exp[ik_1(x\sin\theta_1-z\cos\theta_1-c_{L1}t)].
\end{aligned}
\tag{14}
$$

where $c_1=c_{L1}=\omega/K_1$ is the velocity of reflected (qP) wave and $d^{(1)}=p^{(1)}$.

*For a reflected (qSV) quasi- transverse wave which makes an angle θ_2, we have

$$
\begin{aligned}
&u_1^{(2)}=A_2\cos\theta_2\exp[ik_2(x\sin\theta_2-z\cos\theta_2-c_{T2}t)],\\
&u_3^{(2)}=A_2\sin\theta_2\exp[ik_2(x\sin\theta_2-z\cos\theta_2-c_{T2}t)],\\
&\varphi^{(2)}=B_2\exp[ik_2(x\sin\theta_2-z\cos\theta_2-c_{T2}t)].
\end{aligned}
\tag{15}
$$

where $c_2=c_{T2}=\omega/K_2$ is the velocity of reflected (qSV) wave and $d^{(2)}\neq p^{(2)}$ but, in this case $\vec{d}^{(2)}=\vec{i}_3\wedge\vec{p}^{(2)}$, where $\vec{j}_2$ is the unit vector of the y axis of the Cartesian system of rectangular coordinates.

*For a refracted (qP) quasi-longitudinal wave which makes an angle θ_3, we have

$$
\begin{aligned}
&u^{(3)}=A_3\sin\theta_3\exp[ik_3(x\sin\theta_3+z\cos\theta_3-c_{L3}t)],\\
&w^{(3)}=A_3\cos\theta_3\exp[ik_3(x\sin\theta_3+z\cos\theta_3-c_{L3}t)],\\
&\varphi^{(3)}=B_3\exp[ik_3(x\sin\theta_3+z\cos\theta_3-c_{L3}t)].
\end{aligned}
\tag{16}
$$

where $c_3=c_{L3}=\omega/K_3$ is the velocity of refracted (qP) wave and $d^{(3)}=p^{(3)}$.

*For a refracted (qSV) quasi- transverse wave which makes an angle θ_4, we have

$$
\begin{aligned}
&u^{(4)}=-A_4\cos\theta_4\exp[ik_4(x\sin\theta_4+z\cos\theta_4-c_{T4}t)],\\
&w^{(4)}=A_4\sin\theta_4\exp[ik_4(x\sin\theta_4+z\cos\theta_4-c_{T4}t)],\\
&\varphi^{(4)}=B_4\exp[ik_4(x\sin\theta_4+z\cos\theta_4-c_{T4}t)].
\end{aligned}
\tag{17}
$$

where $c_4=c_{T4}=\omega/K_4$ is the velocity of refracted (qSV) wave and $d^{(4)}\neq p^{(4)}$ but, in this case $\vec{d}^{(4)}=\vec{i}_2\wedge\vec{p}^{(4)}$.

$$c_{T1}=\frac{\omega}{k_1},\ c_{L2}=\frac{\omega}{k_2},\ c_{L3}=\frac{\omega}{k_3},\ c_{T4}=\frac{\omega}{k_4}.$$

IV. Boundary Conditions

When the plane $z=0$ is free of stresses and electric potential, the sum of the three tractions and three electric potential must vanish at $z=0$, So, we may write the boundary conditions as:

$$
\left.
\begin{aligned}
&u^{(\circ)}+u^{(1)}+u^{(2)}=u^{(3)}+u^{(4)},\\
&\sigma^{(\circ)}_{zx}+\sigma^{(1)}_{zx}+\sigma^{(2)}_{zx}=\sigma^{(3)}_{zx}+\sigma^{(4)}_{zx},\\
&\sigma^{(\circ)}_{zz}+\sigma^{(1)}_{zz}+\sigma^{(2)}_{zz}=\sigma^{(3)}_{zz}+\sigma^{(4)}_{zz},\\
&\varphi^{(\circ)}+\varphi^{(1)}+\varphi^{(2)}=\varphi^{(3)}+\varphi^{(4)}.
\end{aligned}
\right\}\quad for\quad z=0
\tag{18}
$$

Substituting in (18), the values of $u^{(n)},\sigma^{(n)}_{zx},\sigma^{(n)}_{zz}$ and $\varphi^{(n)}$ (for $n=0,1,2,3,4$) from (13)-(17) using Eqs. (5), (7), one may have

$$-A_\circ\cos\theta_\circ\exp(\overline{\eta}_\circ)+A_1\sin\theta_1\exp(\overline{\eta}_1)+A_2\cos\theta_2\exp(\overline{\eta}_2)=A_3\sin\theta_3\exp(\overline{\eta}_3)-A_4\cos\theta_4\exp(\overline{\eta}_4), \quad (19)$$

$$\begin{aligned}&k_\circ\left[-A_\circ C_{44}\cos 2\theta_\circ+B_\circ e_{15}\sin\theta_\circ\right]\exp(\overline{\eta}_\circ)\\&+k_1\left[-A_1C_{44}\sin 2\theta_1+e_{15}B_1\sin\theta_1\right]\exp(\overline{\eta}_1)\\&+k_2\left[-A_2c_{44}\cos 2\theta_2+e_{15}B_2\sin\theta_2\right]\exp(\overline{\eta}_2)\\&=k_3\left[A_3C'_{44}\sin 2\theta_3+e'_{15}B_3\sin\theta_3\right]\exp(\overline{\eta}_3)\\&+k_4\left[-A_4C'_{44}\cos 2\theta_4+e'_{15}B_4\sin\theta_4\right]\exp(\overline{\eta}_4),\end{aligned}$$

(20)

$$\begin{aligned}&k_\circ\left[A_\circ(C_{33}-C_{13})\sin\theta_\circ\cos\theta_\circ+e_{33}B_\circ\cos\theta_\circ\right]\exp(\overline{\eta}_\circ)\\&+k_1\left[A_1\left(C_{13}\sin^2\theta_1+C_{33}\cos^2\theta_1\right)-e_{33}B_1\cos\theta_1\right]\exp(\overline{\eta}_1)\\&+k_2\left[A_2(C_{13}-C_{33})\sin\theta_2\cos\theta_2-e_{33}B_2\cos\theta_2\right]\exp(\overline{\eta}_2)\\&=k_3\left[A_3\left(C'_{13}\sin^2\theta_3+C'_{33}\cos^2\theta_3\right)+e'_{33}B_3\cos\theta_3\right]\exp(\overline{\eta}_3)\\&+k_4\left[A_4(C'_{33}-C'_{13})\sin\theta_4\cos\theta_4+e'_{33}B_4\cos\theta_4\right]\exp(\overline{\eta}_4),\end{aligned} \quad (21)$$

$$B_\circ\exp(\overline{\eta}_\circ)+B_1\exp(\overline{\eta}_1)+B_2\exp(\overline{\eta}_2)=B_3\exp(\overline{\eta}_3)+B_4\exp(\overline{\eta}_4). \quad (22)$$

where

$$\overline{\eta}_\circ=\left[ik_\circ(x\sin\theta_\circ-c_{T\circ}t)\right],\qquad \overline{\eta}_1=\left[ik_1(x\sin\theta_1-c_{L1}t)\right],$$
$$\overline{\eta}_2=\left[ik_2(x\sin\theta_2-c_{T2}t)\right],\qquad \overline{\eta}_3=\left[ik_3(x\sin\theta_3-c_{L3}t)\right],$$
$$\overline{\eta}_4=\left[ik_4(x\sin\theta_4-c_{T4}t)\right].$$

Eqs. (19)-(22) must be valid for all values of t and x, hence $\overline{\eta}_\circ=\overline{\eta}_1=\overline{\eta}_2=\overline{\eta}_3=\overline{\eta}_4$.

$$k_\circ\sin\theta_\circ=k_1\sin\theta_1=k_2\sin\theta_2=k_3\sin\theta_3=k_4\sin\theta_4,$$
$$k_\circ c_{T\circ}=k_1c_{L1}=k_2c_{T2}=k_3c_{L3}=k_4c_{T4}=\omega. \quad (23)$$

From the above relations, we get

$$\left.\begin{aligned}&k_\circ=k_2,\quad \theta_\circ=\theta_2,\quad c_{T\circ}=c_{T2},\\&\sin\theta_1=\frac{1}{\overline{\tau}_1}\sin\theta_\circ,\ \ \sin\theta_3=\frac{1}{\overline{\tau}_2}\sin\theta_\circ,\ \ \sin\theta_4=\frac{1}{\overline{\tau}_3}\sin\theta_\circ.\end{aligned}\right\} \quad (24)$$

$$\overline{\tau}_1=\frac{k_1}{k_\circ},\quad \overline{\tau}_2=\frac{k_3}{k_\circ},\quad \overline{\tau}_3=\frac{k_4}{k_\circ}.$$

Furthermore, we should now use the equations of motion of the medium, i.e., Eqs. (1) which will give us additional relations between amplitudes. So, substituting from Eqs. (13)-(17) (when $z=0$) into Eq. $(9)_1$ for the incident qSV waves and the reflected qSV waves $(i.e., u=u^{(n)},\ w=w^{(n)},\ \varphi=\varphi^{(n)},\ n=0,1,2,3,4)$ and using the obtained relations (24), we get

$$B_\circ=\kappa_\circ A_\circ,\ B_1=\kappa_1A_1,\ B_2=\kappa_2A_2,\ B_3=\kappa_3A_3,\ B_4=\kappa_4A_4. \quad (25)$$

where

$$\begin{aligned}\kappa_\circ&=\frac{-1}{(e_{31}+e_{15})\sin\theta_\circ}\left[\rho c_{T\circ}^2-\left(C_{44}+\sigma_{zz}^\circ\right)\cos^2\theta_\circ-\left(C_{11}-C_{44}-C_{13}+\sigma_{xx}^\circ\right)\sin^2\theta_\circ\right],\\ \kappa_1&=\frac{1}{(e_{31}+e_{15})\cos\theta_1}\left[\left(2C_{44}+C_{13}+\sigma_{zz}^\circ\right)\cos^2\theta_1+\left(C_{11}+\sigma_{xx}^\circ\right)\sin^2\theta_1-\rho c_{L1}^2\right],\\ \kappa_2&=\frac{-1}{(e_{31}+e_{15})\sin\theta_2}\left[\rho c_{T2}^2-\left(C_{44}+\sigma_{zz}^\circ\right)\cos^2\theta_2-\left(C_{11}-C_{44}-C_{13}+\sigma_{xx}^\circ\right)\sin^2\theta_2\right],\\ \kappa_3&=\frac{-1}{(e'_{31}+e'_{15})\cos\theta_3}\left[\left(C'_{13}+2C'_{44}+\sigma_{zz}^\circ\right)\cos^2\theta_3+\left(C'_{11}+\sigma_{xx}^\circ\right)\sin^2\theta_3-\rho' c_{L3}^2\right],\\ \kappa_4&=\frac{1}{(e'_{31}+e'_{15})\sin\theta_4}\left[\left(C'_{44}+\sigma_{zz}^\circ\right)\cos^2\theta_4-\rho' c_{T4}^2+\left(C'_{11}-C'_{44}-C'_{13}+\sigma_{xx}^\circ\right)\sin^2\theta_4\right].\end{aligned} \quad (26)$$

From Eqs. $(26)_{1,3}$, It is easy to see that

$$\kappa_2=\kappa_\circ. \quad (27)$$

$$\left.\begin{aligned}a_{11}\frac{A_1}{A_\circ}+a_{12}\frac{A_2}{A_\circ}+a_{13}\frac{A_3}{A_\circ}+a_{14}\frac{A_4}{A_\circ}&=m_1,\\ a_{21}\frac{A_1}{A_\circ}+a_{22}\frac{A_2}{A_\circ}+a_{23}\frac{A_3}{A_\circ}+a_{24}\frac{A_4}{A_\circ}&=m_2,\\ a_{31}\frac{A_1}{A_\circ}+a_{32}\frac{A_2}{A_\circ}+a_{33}\frac{A_3}{A_\circ}+a_{34}\frac{A_4}{A_\circ}&=m_3,\\ a_{41}\frac{A_1}{A_\circ}+a_{42}\frac{A_2}{A_\circ}+a_{43}\frac{A_3}{A_\circ}+a_{44}\frac{A_4}{A_\circ}&=m_4.\end{aligned}\right\} \quad (28)$$

where

$$a_{11}=\frac{\sin\theta_1}{\cos\theta_\circ},\quad a_{12}=1,\quad a_{13}=-\frac{\sin\theta_3}{\cos\theta_\circ},\quad a_{14}=\frac{\cos\theta_4}{\cos\theta_\circ},$$

$$a_{21}=\overline{\tau}_1\frac{\left[C_{44}\sin 2\theta_1-\kappa_1e_{15}\sin\theta_1\right]}{\left[C_{44}\cos 2\theta_\circ-\kappa_\circ e_{15}\sin\theta_\circ\right]},\quad a_{22}=1,$$

$$a_{23}=\overline{\tau}_2\frac{\left[C'_{44}\sin 2\theta_3+\kappa_3e'_{15}\sin\theta_3\right]}{\left[C_{44}\cos 2\theta_\circ-\kappa_\circ e_{15}\sin\theta_\circ\right]},$$

$$a_{24}=-\overline{\tau}_3\frac{\left[C'_{44}\cos 2\theta_4-\kappa_4e'_{15}\sin\theta_4\right]}{\left[C_{44}\cos 2\theta_\circ-\kappa_\circ e_{15}\sin\theta_\circ\right]},$$

$$a_{31}=\overline{\tau}_1\frac{\left[C_{13}\sin^2\theta_1+C_{33}\cos^2\theta_1-\kappa_1e_{33}\cos\theta_1\right]}{\left[(C_{13}-C_{33})\sin\theta_\circ\cos\theta_\circ-\kappa_\circ e_{33}\cos\theta_\circ\right]},\quad a_{32}=1,$$

$$a_{33}=-\overline{\tau}_2\frac{\left[C'_{13}\sin^2\theta_3+C'_{33}\cos^2\theta_3+\kappa_3e'_{33}\cos\theta_3\right]}{\left[(C_{13}-C_{33})\sin\theta_\circ\cos\theta_\circ-\kappa_\circ e_{33}\cos\theta_\circ\right]},$$

$$a_{34} = \bar{\tau}_3 \frac{\left[\left(C'_{13} - C'_{33}\right)\sin\theta_4 \cos\theta_4 - \kappa_4 e'_{33} \cos\theta_4\right]}{\left[\left(C_{13} - C_{33}\right)\sin\theta_\circ \cos\theta_\circ - \kappa_\circ e_{33} \cos\theta_\circ\right]},$$

$$a_{41} = \frac{\kappa_1}{\kappa_\circ}, \quad a_{42} = 1, \quad a_{43} = -\frac{\kappa_3}{\kappa_\circ}, \quad a_{44} = -\frac{\kappa_4}{\kappa_\circ},$$

$$m_1 = 1, \quad m_2 = -1, \quad m_3 = 1, \quad m_4 = -1.$$

Solving Eqs. (28), we can determine the reflection and refraction coefficients $A_1/A_\circ$, $A_2/A_\circ$, $A_3/A_\circ$, $A_4/A_\circ$ as:

$$\frac{A_1}{A_\circ} = \frac{T_1}{D}, \quad \frac{A_2}{A_\circ} = \frac{T_2}{D}, \quad \frac{A_3}{A_\circ} = \frac{T_3}{D}, \quad \frac{A_4}{A_\circ} = \frac{T_4}{D} \tag{29}$$

where

$$T_1 = \begin{vmatrix} m_1 & a_{12} & a_{13} & a_{14} \\ m_2 & a_{22} & a_{23} & a_{24} \\ m_3 & a_{32} & a_{33} & a_{34} \\ m_4 & a_{42} & a_{43} & a_{44} \end{vmatrix} \quad T_2 = \begin{vmatrix} a_{11} & m_1 & a_{13} & a_{14} \\ a_{21} & m_2 & a_{23} & a_{24} \\ a_{31} & m_3 & a_{33} & a_{34} \\ a_{41} & m_4 & a_{43} & a_{44} \end{vmatrix}$$

$$T_3 = \begin{vmatrix} a_{11} & a_{12} & m_1 & a_{14} \\ a_{21} & a_{22} & m_2 & a_{24} \\ a_{31} & a_{32} & m_3 & a_{34} \\ a_{41} & a_{42} & m_4 & a_{44} \end{vmatrix} \quad T_4 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & m_1 \\ a_{21} & a_{22} & a_{23} & m_2 \\ a_{31} & a_{32} & a_{33} & m_3 \\ a_{41} & a_{42} & a_{43} & m_4 \end{vmatrix},$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}.$$

$$\frac{B_1}{B_\circ} = \frac{\kappa_1}{\kappa_\circ}\frac{A_1}{A_\circ}, \quad \frac{B_2}{B_\circ} = \frac{A_2}{A_\circ}, \quad \frac{B_3}{B_\circ} = \frac{\kappa_3}{\kappa_\circ}\frac{A_3}{A_\circ}, \quad \frac{B_4}{B_\circ} = \frac{\kappa_4}{\kappa_\circ}\frac{A_4}{A_\circ}. \tag{30}$$

Special case

In this case, the initial stresses are neglected, i.e., $\sigma_{zz}^{(\circ)} = 0$ in Eqs. (29) and (30). So, one may get the reflection and refraction coefficients without the effect of initial stresses which are computed and plotted in Figures (10)-(13).

V. Numerical results and discussion

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results. The materials chosen for this purpose are (PZT-5H) ceramics for the upper medium and Aluminum Nitride (AlN) for the lower medium which fall in the category of transversely isotropic materials. The physical data for those two materials is given in the following Table 1 [3].

TABLE I Physical data for two materials

Aluminum Nitride (AlN).	**PZT-5H Ceramics**	**Units**
$\rho = 3230$	$\rho' = 7500$	kg/m^3
$C_{11} = 41\times10^{10}$	$C'_{11} = 12.1\times10^{10}$	N/m^2
$C_{12} = 14\times10^{10}$	$C'_{12} = 7.95\times10^{10}$	N/m^2
$C_{13} = 10\times10^{10}$	$C'_{13} = 8.41\times10^{10}$	N/m^2
$C_{33} = 39\times10^{10}$	$C'_{33} = 11.7\times10^{10}$	N/m^2
$C_{44} = 12\times10^{10}$	$C'_{44} = 2.30\times10^{10}$	N/m^2
$e_{15} = -0.48$	$e'_{15} = 17$	N/m^2
$e_{31} = -0.58$	$e'_{31} = -6.5$	N/m^2
$e_{33} = 1.55$	$e'_{33} = 23.3$	N/m^2

Since the above mentioned materials are anisotropic elastic especially of the kind of transversely isotropic, so one may calculate the velocities of the fallen and reflective *qSV*-waves and the reflective *qP*-waves of the lower medium from the following equations (31).

$$c_{T\circ} = c_{T2} = \left(\sqrt{C_{44} + C_{11}\sin^2\alpha + C_{33}\cos^2\alpha - v_1}\right)/\sqrt{2\rho},$$
$$c_{L1} = \left(\sqrt{C_{44} + C_{11}\sin^2\alpha + C_{33}\cos^2\alpha + v_1}\right)/\sqrt{2\rho}. \tag{31}$$

with

$$v_1 = \sqrt{[(C_{11} - C_{44})\sin^2\alpha + (C_{44} - C_{33})\cos^2\alpha]^2 + (C_{13} + C_{44})^2 \sin^2 2\alpha}.$$

One also may compute the velocities of the refractive *qP*-waves and *qSV*- waves of the upper medium through the following definition:

$$c_{L3} = \left(\sqrt{C'_{44} + C'_{11}\sin^2\beta + C'_{33}\cos^2\beta + v_2}\right)/\sqrt{2\rho'},$$
$$c_{T4} = \left(\sqrt{C'_{44} + C'_{11}\sin^2\beta + C'_{33}\cos^2\beta - v_2}\right)/\sqrt{2\rho'}. \tag{32}$$

where

$$v_2 = \sqrt{\left[\left(C'_{11} - C'_{44}\right)\sin^2\beta + \left(C'_{44} - C'_{33}\right)\cos^2\beta\right]^2 + \left(C'_{13} + C'_{44}\right)^2 \sin^2 2\beta}.$$

Using equations (31) and (32) and the physical constants which are presented in Table (1), the reflection coefficients $A_i/A_\circ$, $B_i/B_\circ$ (i=1,2) and the refraction coefficients $A_j/A_\circ$, $B_j/B_\circ$ (j=3,4) are calculated, for various values of initial stresses $\sigma_{zz}^{\circ} = (2,3,4,5)\times10^{11}$, as a function of the angle of incident of $\theta_\circ$. The reflection and refraction coefficients have been presented on curves in figures (2)–(13) which have the following observations:

1) In numerically calculating the reflection coefficients of the *qSV*–waves, it is found that when the angle of incidence $\theta_\circ$ takes the values (1^0 to 35^0), the values of the reflection coefficients are real values at this period, while they become complex quantities when the angle of incidence becomes bigger than that.

2) Figures (2) and (4) represent the relations of the reflection and transmission coefficients $A_1/A_\circ$ and $A_3/A_\circ$ with an angle of incidence $\theta_\circ$. As it is shown in the figures, the curves behavior the same manner where the values of

each one decrease with the increase of the value of the angle of incidence $\theta_{\circ}$. Additionally, we notice the effect of the initial stress on them where their values decrease with increasing of the value of the initial stress component.

3) Figures (3) and (7) show the relations of the reflection coefficients $A_2 / A_{\circ}$ and $B_2 / B_{\circ}$ with the angle of incidence $\theta_{\circ}$. While Figures (5) and (8) show the relations of the transmission coefficients $A_4 / A_{\circ}$ and $B_3 / B_{\circ}$ with the angle of incidence $\theta_{\circ}$. It is easy to see that their values increase with increasing the value of angle of incidence, after that their values decrease at the end of the above mentioned period. It is also noticed that the values of the reflection coefficients increase as the value of the initial stress component increases .

4) Figure (6) represents the relation of the reflection coefficient $B_1 / B_{\circ}$ with the angle of incidence $\theta_{\circ}$. The value of the reflection coefficient decreases with increasing the value of the angle of incidence then it rapidly increases at the end of the period till it reaches its maximum value when $\theta_{\circ} = 35^0$, where the value of reflection coefficient decreases with the increase of the value of the initial stress component.

5) Figure (9) represents the relation of the transmission coefficient $B_4 / B_{\circ}$ with the angle of incidence $\theta_{\circ}$. The value of the refraction coefficient increases with the increase of the angle of incidence and the initial stresses components.

6) Figures (10)-(13) represent the special case of this study, i.e. when the initial stresses are neglected. As it is shown, the values of the coefficients $A_1 / A_{\circ}$, $A_3 / A_{\circ}$, $A_4 / A_{\circ}$, $B_1 / B_{\circ}$, $B_3 / B_{\circ}$, $B_4 / B_{\circ}$ increase with the angle of incidence increasing. This is contrary to what happen to the reflection coefficients $A_2 / A_{\circ}$ and $B_2 / B_{\circ}$.

7) The component of the initial stress σ°_{xx} has been neglected in the numerical calculation because the effect of this component on the reflection and transmission coefficients is very small .

8) We didn't make a comparison with any other cases because there are no preceding studies or numerical computations for this case or any other special cases of it. therefore, it is found sufficient to display the results of this study without making comparisons .

9) from the preceding notes, we notice that reflection and relative transmission coefficients between two anisotropic piezoelectric media are considerably affected by the angle of incidence the physical constants of the two media (whether they are piezoelectric or elasticity constants) and by the initial stresses .

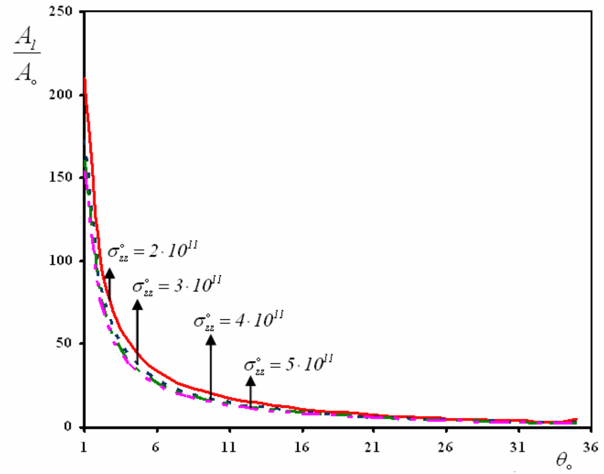

Fig. 2 $A_1 / A_{\circ}$ versus $\theta_{\circ}$ for different values of initial stresses.

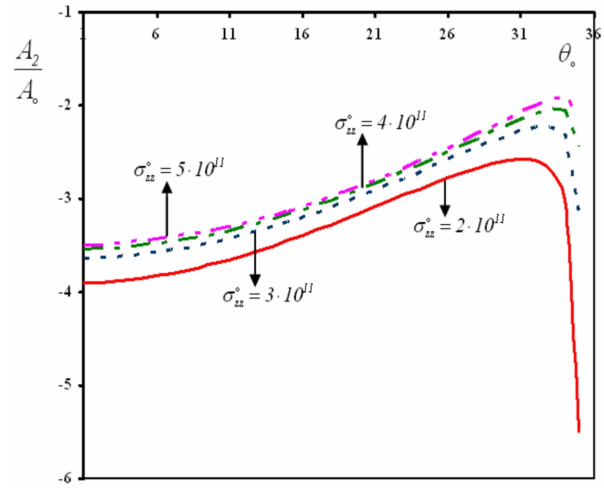

Fig. 3 $A_2 / A_{\circ}$ versus $\theta_{\circ}$ for different values of initial stresses.

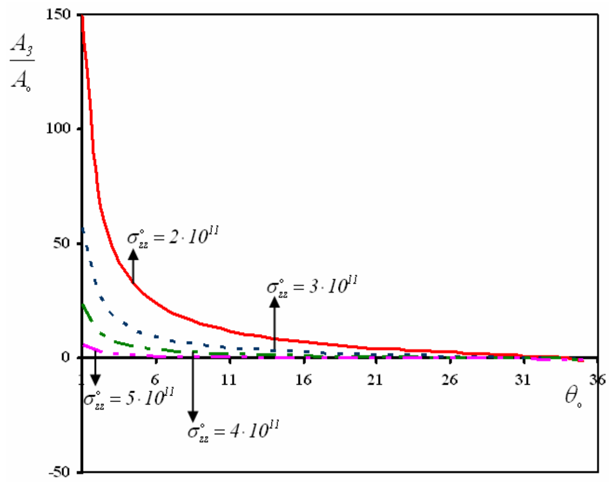

Fig. 4 $A_3 / A_{\circ}$ versus $\theta_{\circ}$ for different values of initial stresses.

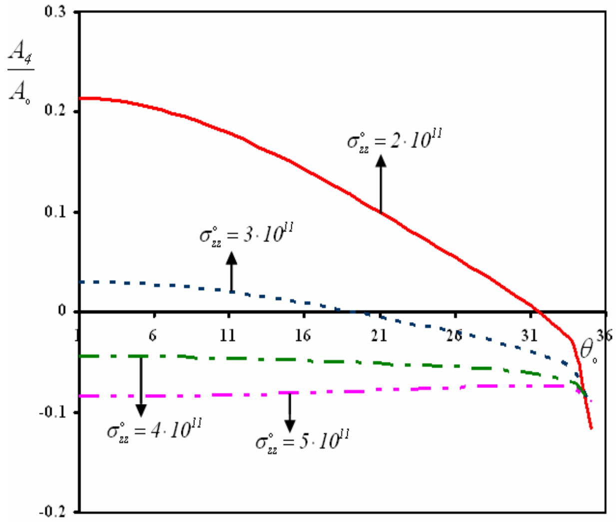

Fig. 5 $A_4 / A_\circ$ versus $\theta_\circ$ for different values of initial stresses.

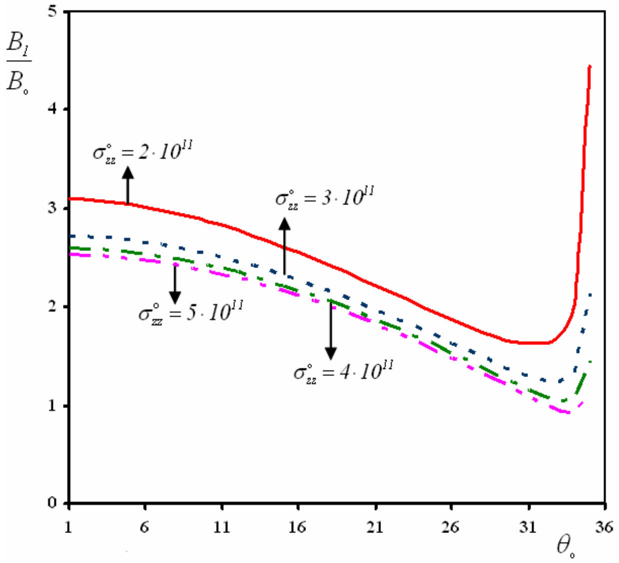

Fig. 6 $B_1 / B_\circ$ versus $\theta_\circ$ for different values of initial stresses.

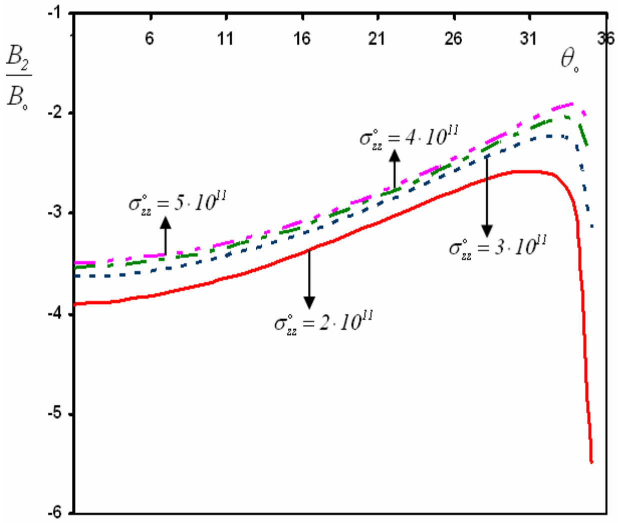

Fig. 7 $B_2 / B_\circ$ versus $\theta_\circ$ for different values of initial stresses.

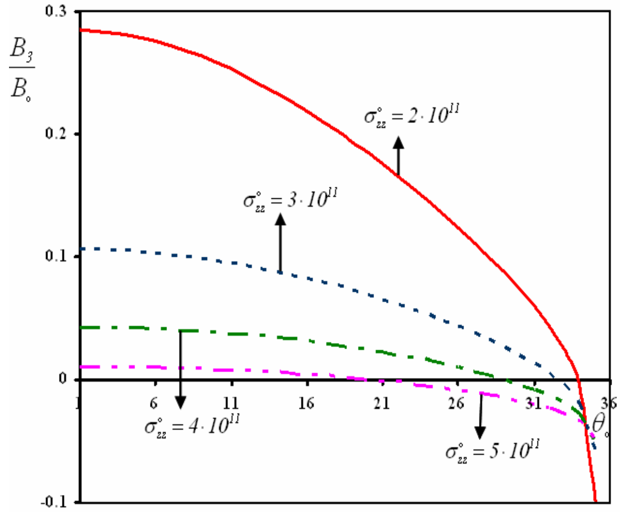

Fig. 8 $B_3 / B_\circ$ versus $\theta_\circ$ for different values of initial stresses.

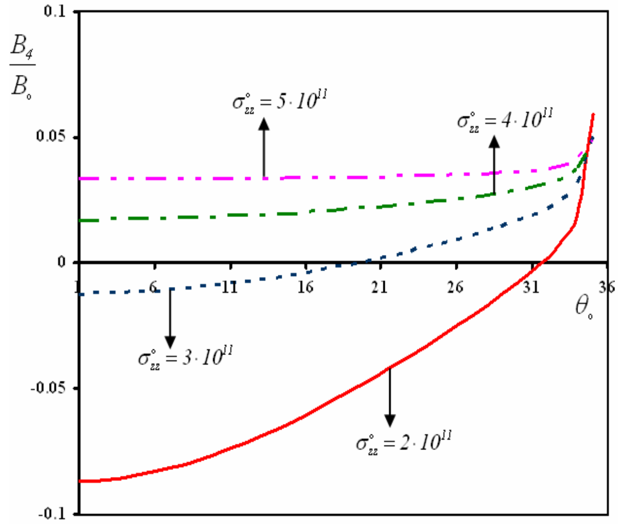

Fig. 9 $B_4 / B_\circ$ versus $\theta_\circ$ for different values of initial stresses.

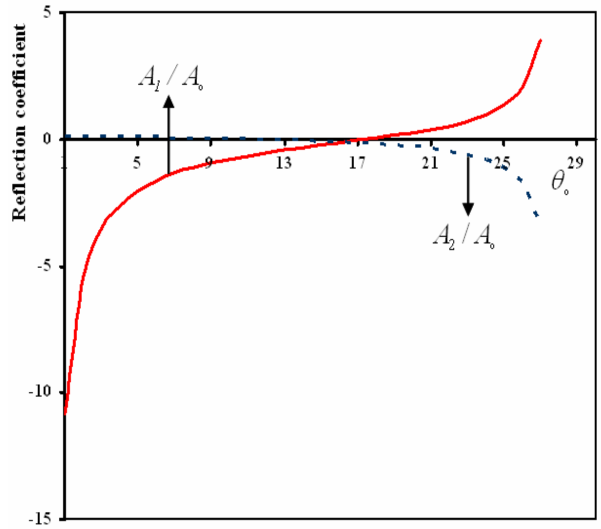

Fig. 10 $A_1 / A_\circ$ and $A_2 / A_\circ$ versus $\theta_\circ$ without initial stresses influence (Special case).

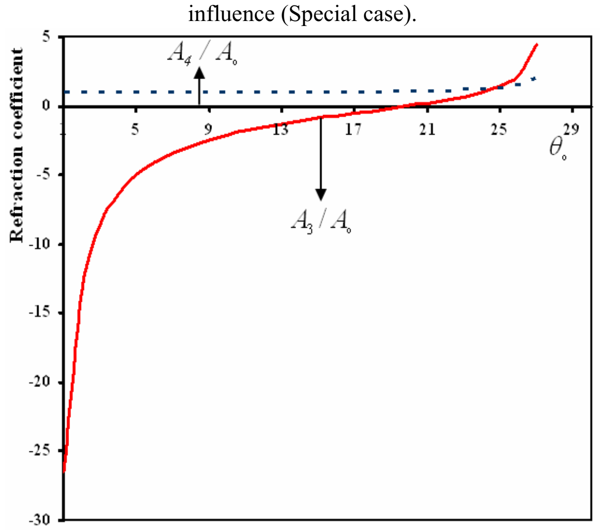

Fig. 11 $A_3/A_\circ$ and $A_4/A_\circ$ versus $\theta_\circ$ without initial stresses influence (Special case).

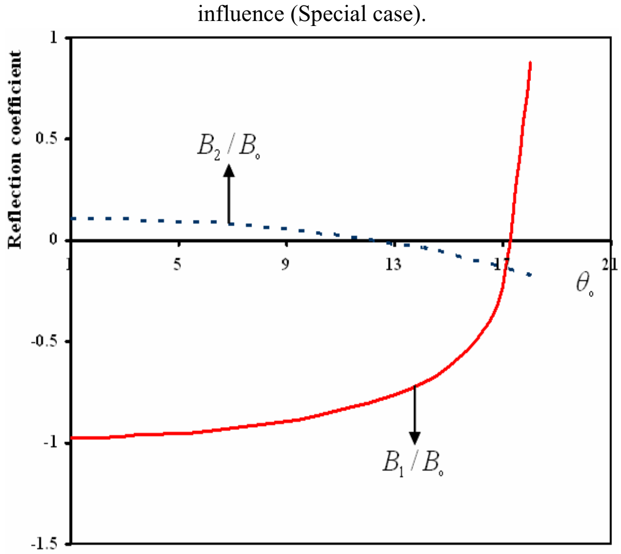

Fig. 12 $B_1/B_\circ$ and $B_2/B_\circ$ versus $\theta_\circ$ without initial stresses influence (Special case).

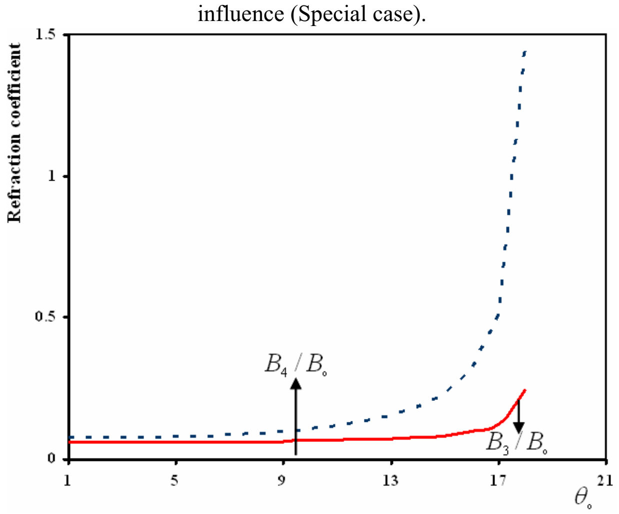

Fig. 13 $B_3/B_\circ$ and $B_4/B_\circ$ versus $\theta_\circ$ without initial stresses influence (Special case).

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