World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:2, No:12, 2008

# Adaptive Equalization using Controlled Equal Gain Combining for Uplink/Downlink MC-CDMA Systems

Miloud Frikel , Boubekeur Targui, François Hamon and Mohammed M'SAAD



***Abstract*—In this paper we propose an enhanced equalization technique for multi-carrier code division multiple access (MC-CDMA). This method is based on the control of Equal Gain Combining (EGC) technique. Indeed, we introduce a new level changer to the EGC equalizer in order to adapt the equalization parameters to the channel coefficients. The optimal equalization level is, first, determined by channel training. The new approach reduces drastically the mutli-user interferences caused by interferes, without increasing the noise power. To compare the performances of the proposed equalizer, the theoretical analysis and numerical performances are given.

***Keywords*—MC-CDMA, Equalization, EGC, Single User Detection.

## I. Introduction

RECENTLY a new CDMA system based on the combination of CDMA and OFDM has been proposed [1], [2] which is potentially robust to channel frequency selectivity. Furthermore, it has a good spectral efficiency, multiple access capability and it's easy to be implemented with FFT. For the scheme based on a combination of CDMA and multi-carrier technique spreads the original data stream over different sub-carriers using a spreading code in the frequency domain [3]. Indeed, in the past decade, there is a growing need for technological innovations to satisfy the increase demand for personal wireless radio communications. This technology must be able to allow users to efficiency share common resources, whether it involves the frequency spectrum and computational load. That why the MC-CDMA [3], the one of representative of the multi-carrier techniques, has been considered as a promising system for the next generation of wireless communication. One large advantage of this technology is its robustness in case of multi-path propagation, and it's capable to combat frequency selective fading, flexible to generate different data rates and provides bandwidth efficiency.

The principles of MC-CDMA [3] is that a single data symbol is transmitted on multiple narrow band sub-carriers. Indeed, in MC-CDMA systems, spreading codes are applied in the frequency domain and transmitted over independent sub-carriers. However, multicarrier systems are very sensitive to synchronization errors such as carrier frequency offset and phase noise. Synchronization errors cause loss of orthogonality among subcarriers and considerably degrade the performance especially when large number of subcarriers presents. There have been a lot of approaches on synchronization algorithms in literature [4], [5], [6]. This paper describes an adaptive method of equal gain combining equalizer for MC-CDMA. Performance analysis of different detection techniques will be presented.

This work was supported by R-Interface and Normandie Incubation (France): www.r-interface.com

Miloud Frikel, Boubekeur TARGUI and M. M'Saad are with the GREYC Lab UMR 6072 CNRS, Equipe Automatique, Avenue 6 juin, 14053 Caen, email: mfrikel@greyc.ensicaen.fr

François Hamon is with R-Interface, France

## II. MC-CDMA Model

In order to satisfy a large number of users, the frequency band should be, optimally, used. The objective is to transmit, in simultaneous over the same channel the maximum of informations. So the use of multiplexing. Indeed, in CDMA [2], the users have access, in the same time, to the totality of the frequency band, in the receiver, to distinguish between them, we use a different codes affected for each user.

That was possible thanks to the technique of spectral spreading, in condition that the emitted signals by different users have some proprieties allowing them to separate.

In opposition of the others techniques of multi-access such FDMA (Frequency division Multiple Access) and TDMA (Time division Multiple Access), where the capacity of the number of users is limited by the frequency and time resources, respectively, the number of users in CDMA is fixed by the proprieties of used spreading codes. That why the CDMA is an alternative to the others multiplexing techniques to increase the reuse frequency factor and eventually the spectral efficiency of communication systems. A different approach to further increase the system capacity without allocating additional frequency spectrum is the use of code multiplexing.

The MC-CDMA modulator spreads the data of each user in frequency domain. In addition, precisely, the complex symbol $g_j$ of each user $j$ is, first, multiply by each chips $c_{j,k}$ of spreading code $SC_j$, and then apply to the modulator of multi-carriers. Each sub-carrier transmits an element of information multiply by a code chip of that sub-carrier.

We consider, for example, the case where the length $L_c$ of spreading code is equal to the number $N$ of sub-carriers. The optimum space between two adjacent sub-carriers is equal to inverse of duration $T_c$ of chip of spreading code in order to guaranty the orthogonality between sub-carriers. The occupied spectral band is, then equal: $B = \frac{(Np+1)}{T_c}$.

The MC-CDMA signal is:

$$s(t) = \frac{g_j}{\sqrt{N}} \sum_{k=0}^{N-1} c_{j,k} e^{2if_k t} \quad (1)$$

We consider the channel is invariant in time and he is characterized by $P$ paths of magnitudes $\beta_p$ and phase $\theta_p$. The impulse response is given by:

$$h(\tau) = \sum_{p=0}^{P-1} \beta_p e^{i\theta_p} \delta(\tau - \tau_p)$$

The relationship between the emitted signal $s(t)$ and the received signal $r(t)$ is given by: $r(t) = h(t) * s(t) + n(t)$.

$$\begin{aligned} r(t) &= \int_{-\infty}^{+\infty} \sum_{p=0}^{P-1} \beta_p e^{i\theta_p} \delta(\tau - \tau_p) s(t-\tau) d\tau + n(t) \\ &= \sum_{p=0}^{P-1} \beta_p e^{i\theta_p} s(t-\tau) + n(t) \end{aligned} \quad (2)$$

where $n(t)$ is the additive white gaussian noise and $P$ is the number of paths.

In a system of $M$ users, the emitted signal through a channel is given by:

$$s(t) = \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} g_m c_{m,k} e^{2if_k t} \quad (3)$$

The received signal after passing through the channel is:

$$r(t) = \frac{1}{\sqrt{N}} \sum_{p=0}^{P-1} \sum_{k=0}^{N-1} \beta_p e^{i\theta} g_u c_{u,k} e^{2i\pi(f_0 + k/T_s)(t-\tau_p)} + n(t) \quad (4)$$

At the reception, we demodulate the signal according the $N$ sub-carriers, and then we multiply the received sequence by the code of the user. Some techniques of equalization and, then, applied to estimate the frame $g_j$.

When there are M active users, the received signal is

$$r(t) = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} h_{m,i} C_m[i] a_m[k] cos(2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,i}) + n(t)$$

where the effects of the channel have been included in $h_{m,i}$ and $\theta_{m,i}$ and $n(t)$ is additive white Gaussian noise (AWGN) with a one-sided power spectral density of $N_0$.

Where it has been assumed that $m = 0$ corresponds to the desired signal. With this model, there are $N$ matched filters with one matched filter for each subcarrier. The output of each filter contributes one component to the decision variable, $\vartheta_0$. Each matched filter consists of an oscillator with a frequency corresponding to the frequency of the particular BPSK modulated subcarrier that is of interest and an integrator. In addition, a phase offset equal to the phase distortion introduced by the channel, $\theta_{m,i}$, is included in the oscillator to synchronize the receiver to the desired signal in time. To extract the desired signal's component, the orthogonality of the codes is used. For the $i^{th}$ subcarrier of the desired signal, the corresponding chip, $C_0[i]$, from the desired user's code is multiplied with it to undo the code. If the signal is undistorted by the channel, the interference terms will cancel out in the decision variable due to the orthogonality of the codes. As the channel will distort the subcarrier components, an equalization gain, $g_{0,i}$, may be included for each matched filter branch of the receiver.

Applying the receiver model to the received signal given in equation (4) yields the following decision variable for the $k^{th}$ data symbol assuming the users are synchronized in time:

$$\vartheta_0 = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} h_{m,i} C_m[i] g_{0,i} a_m[k] \frac{2}{T_b} \int_{kT_b}^{(k+1)T_b} cos(2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,i}) cos(2\pi f_c t + 2\pi i \frac{F}{T_b} t + \widehat{\theta}_{0,i}) dt + \eta \quad (5)$$

where $\widehat{\theta}_{0,i}$ denotes the receiver's estimation of the phase at the $i^{th}$ subcarrier of the desired signal and the corresponding AWGN term, $\eta$ is given as:

$$\eta = \sum_{i=0}^{N-1} \int_{kT_b}^{(k+1)T_b} n(t) \frac{2}{T_b} g_{0,i} cos(2\pi f_c t + 2\pi i \frac{F}{T_b} t + \widehat{\theta}_{0,i}) dt \quad (6)$$

Assuming perfect phase correction, $\widehat{\theta}_{0,i} = \theta_{0,i}$, the decision variable reduces to

$$\vartheta_0 = a_0[k] \sum_{i=0}^{N-1} h_{0,i} g_{0,i} + \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} a_m[k] C_m[i] C_0[i] h_{m,i} g_{0,i} cos(\widetilde{\theta}_{m,i}) + \eta \quad (7)$$

where $\widetilde{\theta}_{m,i} = \theta_{0,i} - \theta_{m,i}$. Not that if $\theta_{0,i}$ and $\theta_{m,i}$ are iid uniform r.v.'s on the interval $[0, 2\pi]$, then $\widetilde{\theta}_{m,i}$ is also uniformly distributed on the interval $[0, 2\pi]$. Note that the decision variable consists of three term. The first term corresponds to the desired signal's component, the second corresponds to the interference and the last corresponds to a noise term.

$$\vartheta_0 = \xi_{inf} + \beta_{int} + \eta \quad (8)$$

where, $\xi_{inf}$ and $\beta_{int}$ are the terms of informations and interferences, respectively, defined by,

$$\begin{aligned} \xi_{inf} &= a_0[k] \sum_{i=0}^{N-1} h_{0,i} g_{0,i} \\ \beta_{int} &= \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} a_m[k] C_m[i] C_0[i] h_{m,i} g_{0,i} cos(\widehat{\theta}_{m,i}) \\ \eta &= \sum_{i=0}^{N-1} \int_{kT_b}^{(k+1)T_b} n(t) \frac{2}{T_b} g_{0,i} cos(2\pi f_c t + 2\pi i \frac{F}{T_b} t + \widehat{\theta}_{0,i}) dt \end{aligned}$$

The noise can be approximated by a zero-mean Gaussian random variable with the following variance (see the Appendix I):

$$\sigma_\eta^2 = N \frac{N_0}{T_b} E[g_{0,i}^2] \quad (9)$$

Below the studied term for the equalization is $g_{0,i}$.

## III. Classical EGC Equalization

The goal of the equalization techniques should be to reduce the effect of the fading and the interference while not enhancing the effect of the noise on the decision of what data symbol was transmitted. Whenever there is a diversity scheme involved whether it may involve receiving multiple copies of a signal from time, frequency or antenna diversity, the field of classical diversity theory can be applied. These equalization techniques may be desirable for their simplicity as they involve simple multiplications with each copy of the signal. However, they may not be optimal in a channel with interference in the sense of minimizing the error under some criterion.

It should be noted that while there are some decision making techniques, such as Viterbi decoding and Wiener filtering, that are optimal in the sense that they minimize the mean-squared error, the actual implementation of these methods may be prohibitive complex for a channel equivalent to the one that is being analyzed in this paper. By assuming that the fading at the N subcarriers are independent, it is assumed that there are N degrees of freedom in one form or another. It could mean that there are N taps in the impulse response of the channel and a very large number of states in a Viterbi decoder for large N. It could also mean there are N taps in a LMS implementation of a Wiener filter.

In the analysis, the EGC equalization technique will be evaluated. This technique may be associated with classical diversity theory as it involved multiplying each copy of the signal by some gain factor. As it can be seen from equation (7), the EGC equalization technique will affect the distribution of the noise component differently.

With EGC, the gain factor of the $i^{th}$ subcarrier is chosen to be:

$$g_{0,i} = \frac{h_{0,i}^*}{\mid h_{0,i} \mid}$$

This technique does not attempt to equalize the effect of the channel distortion in any way. This technique may be desirable for its simplicity as the receiver does not require the estimation of the channel's transfer function. Using this scheme, the decision variable of equation (8) is given as

$$\vartheta_0 = \xi_{inf}^{egc} + \beta_{int}^{egc} + \eta^{egc} \tag{10}$$

with $\xi_{inf}^{egc} = a_0[k]\sum_{i=0}^{N-1} h_{0,i}$ and $\beta_{int}^{egc} = \sum_{m=1}^{M-1} a_m[k] \sum_{i=0}^{N-1} C_m[i]C_0[i]h_{m,i}cos(\widehat{\theta}_{m,i})$.

where the noise can be approximated by a zero-mean Gaussian random with a variance of: $\sigma_\eta^2 = N\frac{N_0}{T_b}$.

## IV. Controlled EGC : C-EGC

For the controlled EGC (C-EGC) equalizer, the gain factor is given by,

$$g_{0,i} = \alpha_{0,i}\frac{h_{0,i}^*}{\mid h_{0,i} \mid} \tag{11}$$

where, $\alpha_{0,i} = q^{i+1}, \quad i = 0...N-1$, with $q$ is a deterministic controlled gain of the equalization chosen in function of the desired performance (BER).

In this case, the decision variable is given by,

$$\vartheta_0 = \xi_{inf}^{c-egc} + \beta_{int}^{c-egc} + \eta^{c-egc} \tag{12}$$

With $\xi_{inf}^{c-egc} = a_0[k]\sum_{i=0}^{N-1} h_{0,i}\alpha_{0,i}$, and $\beta_{int}^{c-egc} = \sum_{m=1}^{M-1} a_m[k] \sum_{i=0}^{N-1} C_m[i]C_0[i]h_{m,i}h_{0,i}\alpha_{0,i}cos(\widehat{\theta}_{m,i})$.

and $\sigma_\eta^2 = \frac{N_0}{T_b}\sum_{i=0}^{N-1}\alpha_{0,i}^2$, and $\sigma_{\beta int}^2 = \frac{(M-1)}{N}\overline{P}_m\sum_{i=0}^{N-1}\alpha_{0,i}^2$.

where $\overline{P}_m$ is the power of each user.

(see the Appendices II and III for the proofs of those formulas).

## V. Performance Analysis: uplink case

As mentioned above, there are several combining schemes for a MC-CDMA system, we analyze, here, the BER of the system with the following two combining schemes.

### A. classical EGC

We have calculated the theoretical Bit Error Rate (BER) for the classical equalizer EGC, and for the new developed equalizers.

Below, some the theoretical performance results obtained.

As $\sigma_{\beta int}^2 = (M-1)\overline{P}_m$ (Variance of interferences) and $\sigma_\eta^2 = N \times \frac{N_0}{T_b}$ (variance of noise).

We have the general form of BER [3] :

$$BER = \frac{1}{2}.erfc\left(\sqrt{\frac{0.5(\sum_{i=0}^{N-1} h_{0,i}g_{0,i})^2}{\sigma_{\beta int}^2 + \sigma_\eta^2}}\right) \tag{13}$$

In the case of EGC we have :

$$BER_{EGC} = \frac{1}{2}erfc\left(\sqrt{\frac{\frac{1}{2}(\sum_{i=0}^{N-1} h_{0,i})^2}{(M-1)\overline{P}_m + N \times \frac{N_0}{T_b}}}\right) \tag{14}$$

The objective is to find an approximation of $h_0 = \sum_{i=0}^{N-1} h_{0,i}$.

*1) Law of Large Numbers (LLN) approximation:* In the limiting case of a large number of subcarriers, $(\sum_{i=0}^{N-1} h_{0,i})$ can be approximated by the LLN to be the constant $NE[h_{0,i}]$. The advantage of using the LLN is that it requires low computational complexity. Using the LLN simplifies the expression for the probability of error to [3]

N is large $\Longrightarrow \gamma_0 = \sum_{i=0}^{N-1} h_{0,i} \simeq N \times E[h_{0,i}]$.

$$BER_{LLN}^{EGC} = \frac{1}{2} erfc \left( \sqrt{\frac{\frac{1}{2} N^2 . E^2[h_{0,i}] . T_b}{(M-1)\overline{P}_m T_b + N \times N . N_0}} \right)$$

$$BER_{LLN}^{EGC} = \frac{1}{2} erfc \left( \sqrt{\frac{\frac{\pi}{4} N . SNR}{(M-1) . SNR + N}} \right) \quad (15)$$

Where N is the number of sub-carriers and M is the number of users.

*2) CLT: Central Limit Theorem:* A third possible approximation can be obtained by applying the CLT for the limiting case of large N. Using the CLT results in a BER of [3],

$$BER_{CLT}^{EGC} = \frac{1}{2} erfc \left( \sqrt{\frac{\frac{\pi}{4} . N . SNR}{(2 - \frac{\pi}{2}) SNR + (M-1) SNR + N}} \right)$$

We have the BER function of N, M and the SNR.

Thus, these equalization techniques have two conflicting goals: to combat noise and to combat interference. In the process of combating one form of degradation, the receiver becomes more susceptible to the other. EGC may be desirable for its simplicity. It should be noted that future communication systems tend to be headed in the direction of interference-limited channels, as they attempt to multiplex as many users as possible using the same resources.

While these techniques are not optimal in the sense that they do not address the minimization of some performance parameter [4], they are noteworthy for their relative simplicity and their intuitive feel for the underlying effects of the coding of the subcarriers. It should be noted that although CE is not optimal, it produces low BERs that are close to the theoretical minimum that are obtainable in a noise-only Rician fading channel with K = 10. A more formal (and more complicated) approach may be taken with multi-signal/multi-user detection to minimize the interference plus noise power in the decision process. With these schemes, each receiver considers the signals designated for all users simultaneously to decide what was transmitted from the desired user. In the new equalizers, we add the following elements: $q, q^2, ...., q^{(N-2)}, q^{(N-1)}, q^N$, the optimal value of $\alpha$ is given by:

$$\alpha = \frac{(1-q^N).(1+q)}{(1+q^N).(1-q)} \quad (16)$$

- The BER of C-EGC with LLN approximation is given by:

$$BER_{\alpha}^{C-EGC} = \frac{1}{2} . erfc \left( \sqrt{\frac{0.5 (\sum_{i=0}^{N-1} h_{0,i} g_{0,i})^2}{\sigma_{\beta int}^2 + \sigma_{\eta}^2}} \right) \quad (17)$$

where:

$$\sigma_{\beta int}^2 = \frac{(M-1)}{N} \overline{P}_m \sum_{i=0}^{N-1} \alpha_{0,i}^2 \quad (18)$$

$$\sigma_{\eta}^2 = \frac{N_0}{T_b} \sum_{i=0}^{N-1} \alpha_{0,i}^2$$

$$\sum_{i=0}^{N-1} \alpha_{0,i} = \sum_{i=0}^{N-1} q^{i+1} = \frac{1-q^N}{1-q}$$

$$\sum_{i=0}^{N-1} q^{2i+2} = \frac{1-q^{2N}}{1-q^2}$$

$$\frac{\left(\sum_{i=0}^{N-1} \alpha_i\right)^2}{\sum_{i=0}^{N-1} \alpha_i^2} = \frac{(1-q^N)(1+q)}{(1-q)(1-q^N)} = \alpha$$

We have then,

$$BER_{\alpha}^{C-EGC} = \frac{1}{2} . erfc \left( \sqrt{\frac{\alpha . \pi / 4 . N . SNR}{(M-1) . SNR + N}} \right)$$

$$BER_{\alpha}^{C-EGC} = \frac{1}{2} . erfc \left( \sqrt{\frac{\alpha . \pi / 4 . N . SNR}{(M-1) . SNR + N}} \right) \quad (19)$$

## VI. The Equalization Methods: Downlink case

For the transmissions in the downlink, i.e., the transmission from the base station to the terminals through the same channel.

In this section, we'll use the notation of (7), and we assume perfect phase correction for interference. The generalized decision variable given in (7), simplifies to :

$$\vartheta_0 = a_0[k] \sum_{i=0}^{N-1} h_{0,i} g_{0,i} + \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} a_m[k] C_m[i] C_0[i] h_{0,i} g_{0,i} + \eta \quad (20)$$

The used codes are orthogonal, the product, $c_{k,i} c_{k,j}$ is, then, equal to 1 with the probability $1/2$ and $-1$ with the same probability for $i \neq j$.

$$\vartheta_0 = a_0[k] \sum_{i=0}^{N-1} h_{0,i} g_{0,i} + \sum_{m=0}^{M-1} a_m[k] \left( \sum_{i=0}^{N/2-1} h_{0,i} g_{0,i} - \sum_{j=0}^{N/2-1} h_{0,j} g_{0,j} \right) + \eta \quad (21)$$

### A. Controlled Equal Gain combining (C-EGC)

With this technique of combining each branch of diversity is balanced equally by :

$$g_k = \alpha_{0,i} \frac{h_{0,i}^*}{|h_{0,i}|}$$

where $\alpha_{0,i} = q_{i+1}$, $i = 0, 1, ..., N-1$.
and the decision variable $\vartheta_0$ becomes:

$$\vartheta_0 = a_0[k] \sum_{i=0}^{N-1} q^{i+1}|h_{0,i}| +$$

$$\sum_{m=0}^{M-1} a_m[k] \left( \sum_{i=0}^{N/2-1} q^{i+1}|h_{0,i}| - \sum_{j=0}^{N/2-1} q^{j+1}|h_{0,j}| \right) + \eta \quad (22)$$

The decision variable can be written as:

$$\vartheta_0 = Info + Inter + Noise$$

where "$Info$" is the utile information, "$Inter$" is the interferences term, and "$Noise$" represents the gaussian noise, with:

$$Info = a_0[k] \sum_{i=0}^{N-1} q^{i+1}|h_{0,i}|$$

$$Inter = \sum_{m=0}^{M-1} a_m[k] \left( \sum_{i=0}^{N/2-1} q^{i+1}|h_{0,i}| - \sum_{j=0}^{N/2-1} q^{j+1}|h_{0,j}| \right)$$

$$Noise = \eta$$

*B. The performances of the C-EGC Equalizer*

The estimation of the error probability is based on the effect that the interferences of multiple access are from several independents sources.

These interferences are approximated by a gaussian variable.

The error probability by binary element is given by:

$$BER = 0.5erfc(\sqrt{\frac{0.5(\sum_{i=0}^{N-1} h_{0,i}g_{0,i})^2}{\sigma_{inter}^2 + \sigma_{noise}^2}})$$

with: $\sigma_{inter}^2 = 4(M-1)q^2(\frac{1-q^N}{1-q^2} - \frac{\pi}{4}\frac{(1-q^{N/2})^2}{(1-q^2)^2})P_0$.

and, $\sigma_{noise}^2 = 2\frac{N_0}{T_b}\frac{1-q^N}{1-q^2}$. (See the Appendix V for more details).

with $SNR = \frac{P_0 T_b}{N_0}$ and the approximation using LLN, we have:

$$BER = 0.5erfc(\sqrt{\frac{\pi Q_1^2}{4(M-1)(Q_2 - \frac{\pi}{4}Q_1^2) + 2\frac{1}{SNR}Q_2}})$$

with: $Q_1 = \frac{1-q^{N/2}}{1-q}$, and $Q_2 = \frac{1-q^N}{1-q^2}$. for more details (see the Appendix IV).

For $q = 1$ we have the classical EGC equalization, from [4] and with a LLN approximation, the BER is given by:

$$BER = 0.5erfc(\sqrt{\frac{\pi/4}{2\frac{(M-1)}{N}(1-\frac{\pi}{4}) + \frac{1}{SNR}}})$$

The following figure shows the performances comparison between the two techniques for a variation of the parameter $q$ in the interval $0 < q < 1$, so there is a $q_0$ that gives good results for the second method.

## VII. Numerical and Simulation Results

In this section, the approximations for the BER using the LLN and the CLT will be evaluated numerically. Using the expressions for the BER obtained for uplink transmissions in a Rayleigh fading channel, the average BER versus the number of co-channel interferers with a spreading factor $N = 128$ of is shown below. To calculate the BER, it is assumed that the local-mean power of each interferer is equal to the local-mean power of the desired signal. The SNR, which is assumed to be 10dB, is defined to be: $SNR = \frac{\overline{p}_0 T_b}{N_0}$. Where $\overline{p}_0$ is the power of each user supposed equal for all users.

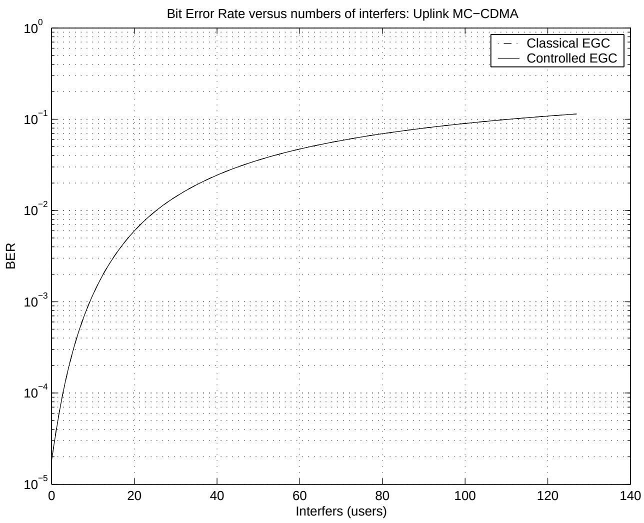


Fig. 1. BER of EGC and Controlled EGC for MC-CDMA : Uplink Case for $q = 0$, $\alpha = 1$ and $\sigma^2 = 0$.

The figure (1) shows the BER performance of EGC and Controlled EGC for MC-CDMA in the case of uplink transmission for different values of controlled equalizers : $q = 0$, $\alpha = 1$ and $\sigma^2 = 0$. We remark in this case, the EGC and C-EGC are equivalent, because we don't have any control of EGC when $q = 0$ and $\alpha = 1$.

The figure (2) presents the BER performance of EGC and Controlled EGC for MC-CDMA in the case of uplink transmission for different values of controlled equalizers : $q = 0.2$, $\alpha = 1.5$ and $\sigma^2 = 0.0417$. We remark that, in this case, C-EGC outperforms the EGC technique.

The figure (3) describes the BER in function of interferes (users) for the equalizers EGC and for different values of controlled equalizers $q = 0.5$, $\alpha = 3$ and $\sigma^2 = 0.3333$. The new equalizer introduces a control parameter to eliminate, simultaneously, the interferences and the noise. We observe that the new equalizers improve the performances in comparison to the classical EGC technique.

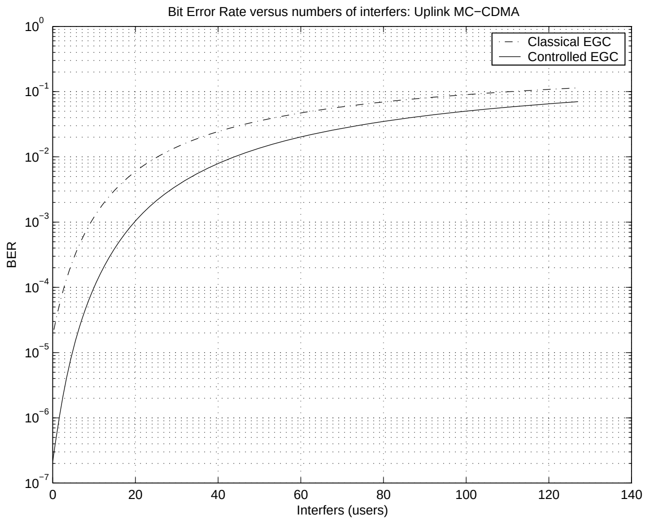


Fig. 2. BER of EGC and Controlled EGC for MC-CDMA : Uplink Case for $q = 0.2$, $\alpha = 1.5$ and $\sigma^2 = 0.0417$.

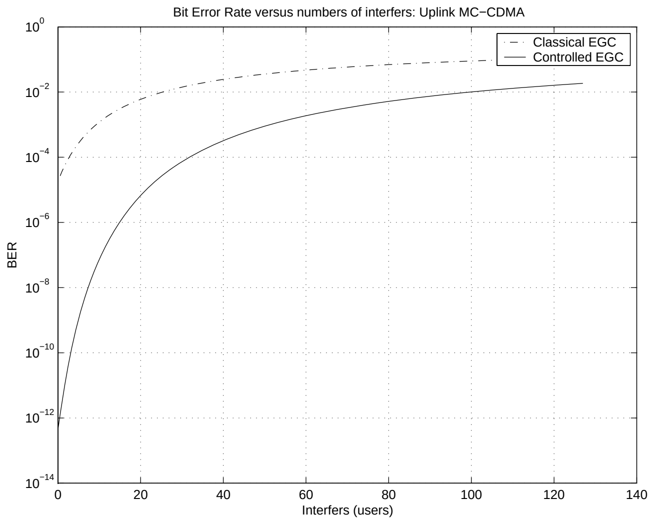


Fig. 3. BER of EGC and Controlled EGC for MC-CDMA : Uplink Case for $q = 0.5$, $\alpha = 3$ and $\sigma^2 = 0.3333$.

The figure (1) shows the BER performance of EGC and Controlled EGC for MC-CDMA in the case of downlink transmission for different values of controlled equalizers : $q = 0$, $\alpha = 1$ and $\sigma^2 = 0$. We remark in this case, the EGC and C-EGC are equivalent, because we don't have any control of EGC when $q = 0$ and $\alpha = 1$.

The figures (4 and 5) describe the BER function of interferes (users) of the two equalizers EGC and C-EGC and for a downlink transmission in the case of different control parameters of EGC equalizer : $q = 0.2$, $\alpha = 1.5$ and $\sigma^2 = 0.0417$. As the number of users is increased, C-EGC ourperforms the classical EGC technique. Although, the interference increases, the increase is not great enough to balance the adverse effects of noise amplification. We remark that C-EGC has better performance than EGC for all number of interferes for the downlink.

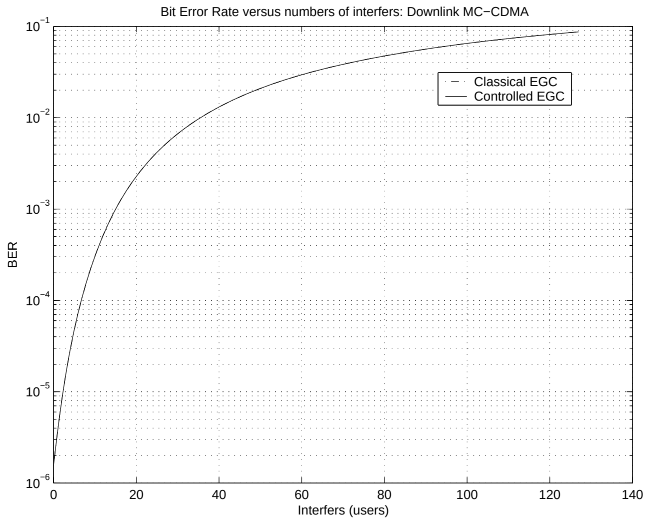


Fig. 4. BER of EGC and Controlled EGC for MC-CDMA : Downlink Case for $q = 0$, $\alpha = 1$ and $\sigma^2 = 0$.

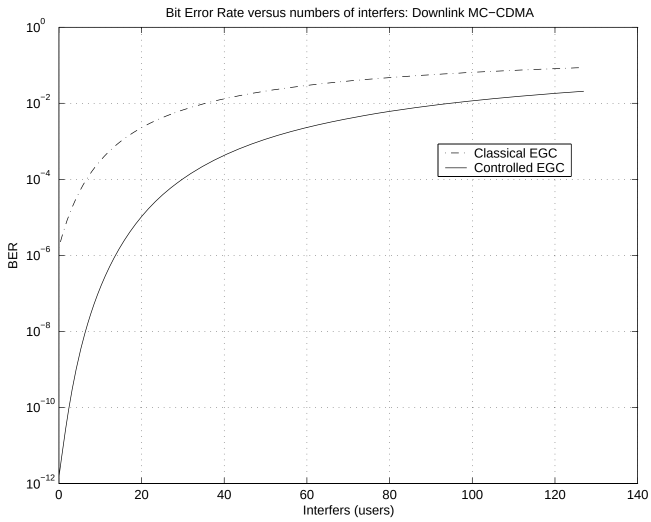


Fig. 5. BER of EGC and Controlled EGC for MC-CDMA : Downlink Case for $q = 0.2$, $\alpha = 1.5$ and $\sigma^2 = 0.0417$.

The figure (6) presents the BER in function of the signal to Noise Ration (SNR) for the two equalizers EGC and C-EGC in the case of uplink transmission. We remark that the BER of C-EGC is very weak compare to the BER of EGC.

## VIII. Conclusion

A digital modulation technique called Multi-Carrier Code Division Multiple Access was analyzed in Rayleigh fading channel. The performance of this technique, gauged by the average bit error rate, was analytically and numerically evaluated for some equalization techniques that fall under classical diversity techniques. These techniques, Equal Gain Combining, and Controlled EGC, perform equalization in the frequency domain, taking the component of each subcarrier (which represents the fading of the channel at a corresponding frequency) and performing a multiplicative operation on this component.

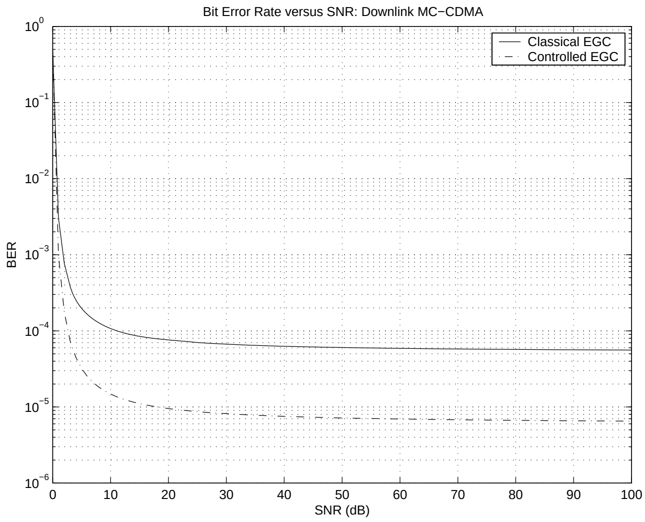


Fig. 6. BER of EGC and Controlled EGC for MC-CDMA versus SNR.

## APPENDIX A
## PROOF OF THE NOISE VARIANCE

$$\sigma_\eta^2 = N\frac{N_0}{T_b}E[g_{0,i}^2] \tag{23}$$

$$\sigma_\eta^2 = E[\eta^2] = \sum_{i=0}^{N-1} E\{[\sum_{i=0}^{N-1}\int_{kT_b}^{(k+1)T_b} n(t_1)\frac{2}{T_b}g_{0,i}$$

$$cos(2\pi f_c t_1 + 2\pi i\frac{F}{T_b}t_1 + \widehat{\theta}_{0,i})dt_1].\sum_{i=0}^{N-1}\int_{kT_b}^{(k+1)T_b} n(t_2)\frac{2}{T_b}g_{0,i}$$

$$cos(2\pi f_c t_2 + 2\pi i\frac{F}{T_b}t_2 + \widehat{\theta}_{0,i})dt_2\} = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\int_{kT_b}^{(k+1)T_b}\int_{kT_b}^{(k+1)T_b}$$

$$\frac{4}{T_b^2}E\{n(t_1)n(t_2)\}E\{g_{0,i}g_{0,j}\}.E\{cos(2\pi f_c t_1 + 2\pi i\frac{F}{T_b}t_1 + \widehat{\theta}_{0,i})$$

$$cos(2\pi f_c t_2 + 2\pi i\frac{F}{T_b}t_2 + \widehat{\theta}_{0,i})\}dt_1 dt_2$$

$$E\{n(t_1)n(t_2)\} = \frac{N_0}{2}\delta(t_1 - t_2)$$

$$\sigma_\eta^2 = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\int_{kT_b}^{(k+1)T_b}\int_{kT_b}^{(k+1)T_b}\frac{4}{T_b^2}\frac{N_0}{2}\delta(t_1 - t_2)$$

$$E\{g_{0,i}g_{0,j}\}.E\{cos(2\pi f_c t_1 + 2\pi i\frac{F}{T_b}t_1 + \widehat{\theta}_{0,i}) \tag{24}$$

$$cos(2\pi f_c t_2 + 2\pi i\frac{F}{T_b}t_2 + \widehat{\theta}_{0,i})\}dt_1 dt_2$$

$$\sigma_\eta^2 = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\int_{kT_b}^{(k+1)T_b}\frac{2}{T_b^2}N_0 E\{g_{0,i}g_{0,j}\}$$

$$.E\{cos(2\pi f_c t_1 + 2\pi i\frac{F}{T_b}t_1 + \widehat{\theta}_{0,i})cos(2\pi f_c t_2 + 2\pi i\frac{F}{T_b}t_2 + \widehat{\theta}_{0,i})\}dt_1$$

$$\sigma_\eta^2 = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\frac{2}{T_b^2}N_0 E\{g_{0,i}g_{0,j}\} \tag{25}$$

$$E\{\int_{kT_b}^{(k+1)T_b}[\frac{1}{2}cos(2\pi F\frac{i-j}{T_b}t_1 + 2\pi + \theta_{0,i} - \theta_{0,j})$$

$$+\frac{1}{2}cos(4\pi F\frac{i-j}{T_b}t_1 + 2\pi + \theta_{0,i} - \theta_{0,j})\}dt_1 \tag{26}$$

$$\begin{aligned}\sigma_\eta^2 &= \frac{2}{T_b^2}N_0 E\{g_{0,i}^2\}\frac{1}{2}T_b \\ &= N.\frac{N_0}{T_b}E\{g_{0,i}^2\}\end{aligned} \tag{27}$$

## APPENDIX B
## PROOF OF THE NOISE VARIANCE $\xi_{inf}^{c-egc}$

$$\xi_{inf}^{c-egc} = a_0[k]\sum_{i=0}^{N-1}h_{0,i}\alpha_{0,i}$$

$$\beta_{int}^{c-egc} = \sum_{m=1}^{M-1}a_m[k]\sum_{i=0}^{N-1}C_m[i]C_0[i]h_{m,i}h_{0,i}\alpha_{0,i}cos(\widehat{\theta}_{m,i})$$

## APPENDIX C
## THE VARIANCE OF INTERFERENCES $\sigma_{inter}^2$

$$Inter = \sum_{k=0}^{M-1}a_m[k]\left(\sum_{i=0}^{N/2-1}h_{0,i}g_{0,i} - \sum_{j=0}^{N/2-1}h_{0,j}g_{0,j}\right)$$

$$\sigma_{inter}^2 = (M-1)E\left[\sum_{i=0}^{N/2-1}h_{0,i}g_{0,i} - \sum_{j=0}^{N/2-1}h_{0,j}g_{0,j}\right]^2$$

$$\sigma_{inter}^2 = 2(M-1)\left[E(\sum_{i=0}^{N/2-1}h_{0,i}g_{0,i})^2 - E^2(\sum_{j=0}^{N/2-1}h_{0,j}g_{0,j})\right]$$

In the case of the EGC equalizer, we have:

$$g_{0,i} = q^{i+1}\frac{h_{0,i}^*}{|h_{0,i}|}, i = 0, 1, \ldots, \frac{N}{2} - 1$$

$$g_{0,j} = q^{j+1}\frac{h_{0,j}^*}{|h_{0,j}|}, j = 0, 1, \ldots, \frac{N}{2} - 1$$

that gives: $\sigma^2_{inter} = 2(M-1)[E(\sum_{i=0}^{N/2-1} q^{i+1}|h_{0,i}|)^2 - E^2(\sum_{j=0}^{N/2-1} q^{j+1}|h_{0,j}|)]$.

$$\sigma^2_{inter} = 2(M-1)[\sum_{i=0}^{N/2-1} q^{2(i+1)}E|h_{0,i}|^2 - (\sum_{j=0}^{N/2-1} q^{j+1}E|h_{0,j}|)^2].$$

In the case of Rayleigh channel, we have:

$E|h_{0,i}| = \frac{\sqrt{\pi}}{\sqrt{2}}\sqrt{\bar{P}_{0,i}}$.

and $E|h_{0,i}|^2 = 2\bar{P}_{0,i}$.

where $\bar{P}_{0,i}$ is the power of each user.

that gives:

$$\sigma^2_{inter} = 2(M-1)[2\sum_{i=0}^{N/2-1} q^{2(i+1)} - \frac{\pi}{2}(\sum_{i=0}^{N/2-1} q^{i+1})^2]\bar{P}_{0,i}.$$

let: $S_1 = \sum_{i=0}^{N/2-1} q^{2(i+1)} = q^2\frac{1-q^N}{1-q^2}$.

and $S_2 = \sum_{i=0}^{N/2-1} q^{i+1} = q\frac{1-q^{N/2}}{1-q}$.

then we have:

$$\sigma^2_{inter} = 4(M-1)\left[q^2\tfrac{1-q^N}{1-q^2} - \tfrac{\pi}{4}q^2\tfrac{(1-q^{N/2})^2}{(1-q)^2}\right]\bar{P}_{0,i}.$$

with the hypothesis that the all users have the same powers, i.e., $\bar{P}_{0,i} = P_0$, we have:

$$\sigma^2_{inter} = 4(M-1)q^2\left[\frac{1-q^N}{1-q^2} - \frac{\pi}{4}\frac{(1-q^{N/2})^2}{(1-q)^2}\right]\bar{P}_0$$

## APPENDIX D
## THE NOISE VARIANCE

we have, $\sigma^2_{noise} = \frac{N_0}{T_b}\sum_{i=0}^{N-1} E(g^2_{0,i})$.

$$\sigma^2_{noise} = \frac{N_0}{T_b}(\sum_{i=0}^{N/2-1} E(g^2_{0,i}) + \sum_{i=0}^{N/2-1} E(g^2_{0,i})).$$

In the case of EGC equalizer, we have: $g_{0,i} = q^{i+1}\frac{h^*_{0,i}}{|h_{0,i}|}, i = 0, 1, \ldots, \frac{N}{2}-1$.

that implies:

$$\sigma^2_{noise} = \frac{N_0}{T_b}\left(\sum_{i=0}^{N/2-1} q^{2(i+1)} + \sum_{i=0}^{N/2-1} q^{2(i+1)}\right).$$

that gives: $\sigma^2_{noise} = 2\frac{N_0}{T_b}q^2\frac{1-q^N}{1-q^2}$.

## REFERENCES

[1] Project Broadband Radio Access Networks (BRAN); HYPERLAN Type 2, Technical specification; Physical Layer.
[2] MC-CDMA for next generation systems. The IST projects Matrice and 4MORE.
[3] N.Yee, Jean-Paul M. G. Linnartz and G. Fettweis, Multi-Carrier-CDMA in indoor wireless networks, Conference Proceedings PIMRC '93, Yokohama, September, 1993. p 109-113.
[4] Jean-Paul M. G. Linnartz, Performance Analysis of Synchronous MC-CDMA in Mobile Rayleigh Channel With Both Delay and Doppler Spreads. IEEE Transactions on Vehecular Technology, Vol. 50, No. 6, November 2001.
[5] M. Frikel, B. Targui, M. M'Saad, and F. Hamon, Bit Error Rate Analysis of the Controlled Equalization for MC-CDMA. IEEE ICSPC'07.
[6] S. Kaiser, OFDM-CDMA versus DS-CDMA: performance evaluation for fading channels. IEEE International Conference on Communications, pages 1722-1726, June 1995.
[7] S. Kaiser, Multi-carrier CDMA mobile radio system-analysis and optimization of detection, decoding, and channel estimation, Ph.D. dissertation, University of Munich, Munich, Germany, 1998.
[8] K. Fazel et L. Papke, On the performance of convolutionnally-coded CDMA/OFDM for mobile communication system. IEEE Personal, Indoor and Mobile Radio Communications Symposium, pages 468-472, Septembre 1993.

**Miloud Frikel** Dr. received his Ph.D. degree from the center of mathematics and scientific computation, France, in array processing. Currently, Miloud FRIKEL is with the GREYC laboratory at the university of Caen and with the R/T (Telecom and Networks) Department of the IUT of Caen. From 1998 to 2003, Dr. Frikel was with the Signal Processing Lab, Institute for Systems and Robotics, Institute Superior Tecnico, Lisbon, as a researcher in the field of wireless location and statistical array processing, after been a research engineer in a software company in Munich, Germany. And he worked in the Institute for Circuit and Signal Processing of the Technical University of Munich, Germany.Dr. Frikels research interests span several areas, including statistical signal and array processing, cellular geolocation (wireless location), space-time coding, direction finding and source localization, blind channel identification for wireless communication systems, and MC-CDMA systems.