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Rarefactive and Compressive Solitary Waves in Warm Plasma with Positrons and Nonthermal Electrons

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***Abstract*—**Ion-acoustic solitary waves in a plasma with nonthermal electrons, thermal positrons and warm ions are investigated using Sagdeev's pseudopotential technique. We study the effects of non-thermal electrons and ion temperature on solitons and show both negative and positive potential waves are possible.

***Keywords*—**Ion acoustic waves, Solitons, Nonlinear phenomena, Sagdeev potential

I. INTRODUCTION

SOLITARY waves are nonlinear and localized structures that propagate when the nonlinearity and dispersion are balanced. They are a subject of continuing interest because of their practical importance. Ion-acoustic solitary waves and their attributes have been the subject of many researches in plasma physics and complex plasma as well. The existence of a considerable number of ions in the Earth's ionosphere [1] and cometary comae [2] is well known. Electron-positron plasmas are found in early universe, active galactive nuclei, magnetosphere of pulsars [3-5]. These waves have been studied both theoretically and experimentally [6,7]. Theory of nonlinear wave-wave and wave-particle interactions in this plasma has been studied by Machabeli et al. [8]. Their results can be applied to real astrophysical plasmas, in particular, pulsar magnetospheres. They also considered the instability resulting from superluminal Langmuir waves interacting with two transverse waves and discussed a new model for pulsar g-ray emission [9]. However, ions may be present in most of plasmas, and the presence of ions leads to the existence of several low frequency waves which otherwise do not propagate on electron-positron plasmas. Some authors have studied different types of linear and nonlinear wave structures such as solitons, double layers and vortices in e-p plasmas [10-13]. The effect of nonthermal electron distribution on ion acoustic solitary waves in e-p plasma has been investigated in [14]. Recently, a great deal of attention has been devoted to the study of different types of collective processes in electron-positron-ion plasmas [15-21]. S. Popel et.al [17] have studied e-p-i plasmas with Boltzmann distribution for electron. Space plasma observations indicate the presence of ion and electron populations which are not in thermodynamic [22,23]. The motivation for this came from the observations of solitary structures with density depletions made by the Freja [22] and Viking satellites [23]. Recently, motivated by the latter class of events, Cairns et al. [24] have considered a nonthermal plasma model and shown that the presence of a nonthermal distribution of electrons may change the nature of ion acoustic solitary structures and allow the existence of structures very like those observed in [22,23]. Mamun investigated the nonthermal electrons and warm ion effects on ion acoustic waves in [25]. We study ion acoustic solitary structures in plasma containing nonthermal electrons, warm positive ions and thermal positrons with respective subscripts e, i and p. The presence of positrons and warm ions with nonthermal electron distribution in a e-p-i plasma introduces a new aspect of the nonlinear ion-acoustic waves.The manuscript is organized as follows: In the next section, we present the basic equations of our theoretical model and derive the pseudo-potential associated to localized ion-acoustic solitary waves. Our results are presented and discussed in Sec. 3. A summary of our results and conclusions is given in Sec. 4.

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II. BASIC EQUATION

Let us consider a collisionless unmagnetized plasma consisting of positrons, electrons obeying a nonthermal distribution and stationary warm ions. The nonlinear dynamics of the ion acoustic solitary waves is governed by the following set of normalized basic equations

$$\frac{\partial n}{\partial t}+\frac{\partial (nu)}{\partial x}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{\sigma}{n}\frac{\partial P}{\partial x}+\frac{\partial \phi}{\partial x}=0 \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n-n_p \tag{3}$$

where n is the number density of ions and it is normalized by $n_\circ$, which $n_\circ$ is the unperturbed ion number density. $\sigma=\frac{T_i}{T_{eff}}$, in which T_i is the temperature of ion and T_{eff} is the effective temperature $T_{eff}=\frac{n_\circ}{\frac{n_{\circ e}}{T_e}+\frac{n_{\circ p}}{T_P}}$. u is velocity of ion fluid and ϕ is electrostatic potential, respectively. u is normalized by the ion acoustic speed $c_i=\sqrt{\frac{kT_{eff}}{m}}$ and ϕ is normalized by $(\frac{kT_{eff}}{e})$, where *k* is Boltzmann's constant and *m* is mass of ion and e is the electron charge. The time *t* and the distance *x* are normalized by the ion plasma frequency $\omega_{pi}^{-1} = \sqrt{\frac{m}{4\pi n_{\circ} e^2}}$ and by the electron Debye length $\lambda_D = \sqrt{\frac{kT_{eff}}{4\pi n_{\circ} e^2}}$, respectively which T_e is temperature of electron. P is the pressure and electrical potential and is normalized by $n_{\circ} kT_i$. We also take the equation of state as $P = n^3$ for adiabatic process. The electron number density is given

$$n_e = \frac{1}{1-p}\left[1 - \beta\phi + \beta\phi^2\right]e^{\phi} \tag{4}$$

where $\beta = \frac{4\alpha}{1+3\alpha}$. α is a parameter that determines the population of energetic nonthermal electrons and characterizes the degree of nonthermality. And for positrons with Boltzmann distribution

$$n_p = \frac{1}{1-p}e^{-\delta\phi} \tag{5}$$

where $\delta = \frac{T_e}{T_p}$, where T_p is the positron temperature, $p = \frac{n_{\circ p}}{n_{\circ e}}$, which $n_{\circ e}$ and $n_{\circ p}$ are the unperturbed number densities of electrons and positrons, respectively.

In order to find the Sagdeev's pseudopotential from Eqs. (1)–(5), we assume that all dependent variables depend on a single independent variable $\xi = x - Mt$, where *M* being the soliton velocity normalized by c_i. The variable ξ is the special coordinate in the coordinate system moving with the solitary wave velocity, i.e., the wave frame. Equations (1) and (2) in the stationary frame can be integrated to give [27]

$$n = \frac{\sqrt{2}M}{\sqrt{M^2 - 2\phi + 3\sigma + \sqrt{(M^2 - 2\phi + 3\sigma)^2 - 12\sigma M^2}}} \tag{6}$$

where we have used boundary conditions for localized disturbance, viz, $u \to 0, n \to 1, \phi \to 0$, $\mathrm{P} \to 1$, when $\xi \to \infty$. Substituting *n* from Eq. (6) in Eq. (3) and following Sagdeev's pseudopotential method along with appropriate boundary conditions, we obtain [27]

$$\frac{1}{2}\left[\frac{d\phi}{d\chi}\right]^2 + V(\phi) = 0 \tag{7}$$

where

$$V(\phi) = (\frac{1}{1-p})\left\{\begin{matrix}\left[(1-p)(M^2+\sigma)+1+3\beta+\frac{p}{\delta}\right]- \\ \left[1+3\beta+\beta\phi^2-3\beta\phi\right]e^{\phi}-\frac{p}{\sigma}e^{-\delta\phi}\end{matrix}\right\} - \frac{\sqrt{2}}{2}M \times$$
$$\left\{\begin{matrix}\left[M^2-2\phi+3\sigma+\sqrt{(M^2-2\phi+3\sigma)^2-12M^2\sigma}\right]^{\frac{1}{2}}+ \\ 4M^2\sigma\left[M^2-2\phi+3\sigma+\sqrt{(M^2-2\phi+3\sigma)^2-12\sigma M^2}\right]^{\frac{-3}{2}}\end{matrix}\right\} \tag{8}$$

In the absence of positrons (p=0) and for cold plasma ($\sigma = 0$), (8) is reduced to [14]. Also, in the absence of nonthermal electrons in cold plasma; i.e. for $\alpha = 0, \sigma = 0$ (8) is reduced to [17].

III. Results and Discussion

Equation (8) can be regarded as an 'energy integral' of an oscillating particle of unit mass with a velocity $d\phi / d\xi$ and position ϕ in a potential $V(\phi)$. Further it is clear that $V(\phi) = \circ$ and $dV(\phi)/d\phi = \circ$ at $\phi = 0$ [28]. Solitary wave solution for exists if $d^2V/d\phi^2 < 0$ at $\phi = 0$, so that the zero as a fixed point is unstable. All the specified conditions are satisfied. Besides that $V(\phi)$ should be negative between $\phi = 0$ and ϕ_m where ϕ_m is some maximum or minimum potential for compressive or refractive solitons respectively. To find the range of compressive and rarefactive solitons, one has to study the nature of the function $V(\phi)$. Clearly the functional dependence of $V(\phi)$ is very sensitive to the variation in parameters σ, α, p, δ and M. Since temperature of the ion and nonthermal electron distribution are the main parameters considered here, it is useful to investigate the dynamics of solitons as function of variation of these parameters. Figures 1 and 2 show $V(\phi)$ as a function of ϕ for different values of α. These figures show the comparison of the Sagdeev pseudo-potential profiles for different values of α by choosing the values of σ=0.03 and 0.01, respectively. It is observed there is a shift in the value of $\phi_{\circ}$ as α is decreased. So it is found that the nonthermal parameter (α) has a significant role on the formation compressive and rarefactive solitons.

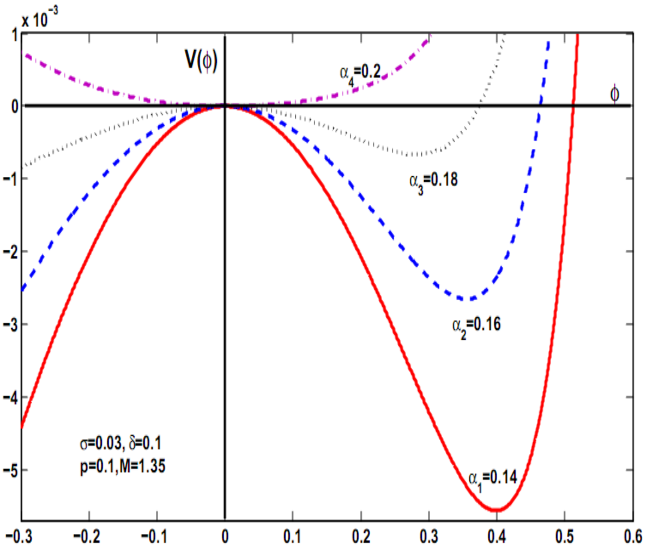

Fig. 1 The Sagdeev potential $V(\phi)$ with respect to ϕ for fixed value of σ =0.03, p=0.1, δ =0.1, M=1.45 and different values of α

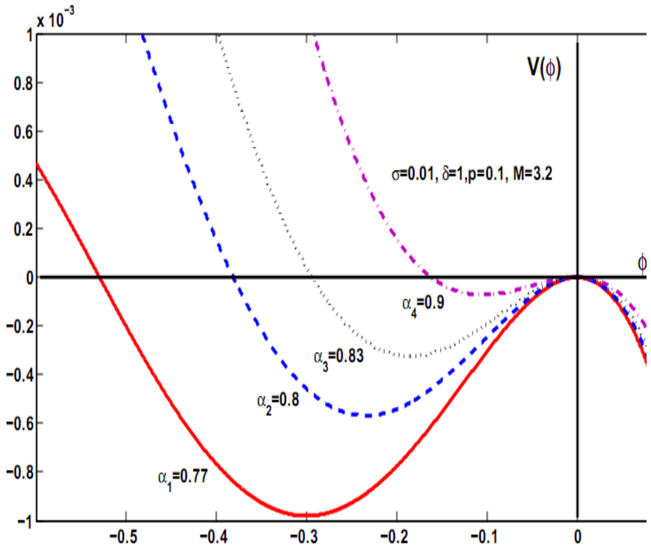

Fig. 2 The Sagdeev potential $V(\phi)$ with respect to ϕ for fixed value of M, σ =0.01, p=0.1, δ =1, M=3.2 and different values of α

To see the effect of the ion temperature σ , in Figures 3 and 4, $V(\phi)$ are plotted against ϕ by choosing the values of α =0.25 and 0.78 but varying the value of ion temperature. It is seen that both rarefactive and compressive solitons exist. It is obvious from these figures that the increase in the ion temperature has significantly effect on both the compressive and rarefactive solitons. It is also observed when the value of σ is increased, both compressive and rarefactive solitons are disappeared. Thus, there are critical values of σ (σ_c) for both region of rarefactive and compressive solitons, so that for $\sigma > \sigma_c$ the soliton is not formed. It is obvious the critical temperatures in the Figs. (3) and (4) are σ_c =0.085 and σ_c =0.8, respectively. It can be concluded from the investigation that there are critical values of σ and α for each of solitons (rarefactive and compressive solitons).

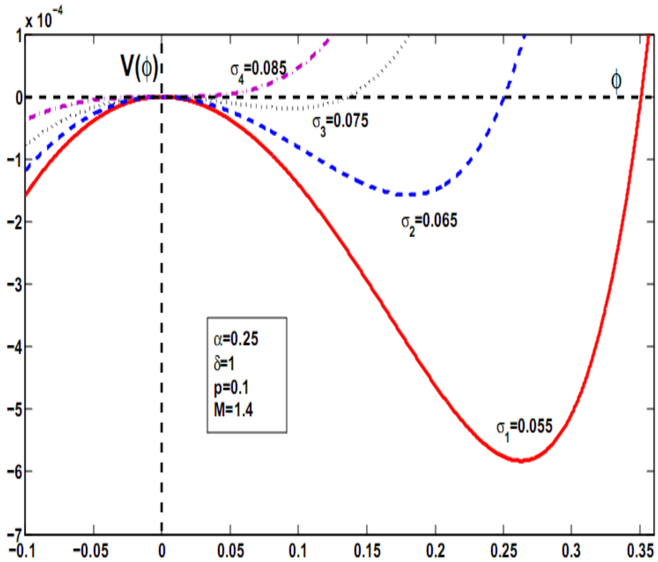

Fig. 3 The Sagdeev potential $V(\phi)$ with respect to ϕ for fixed value of α =0.25, p=0.1, δ =1, M=1.4 and different values of σ

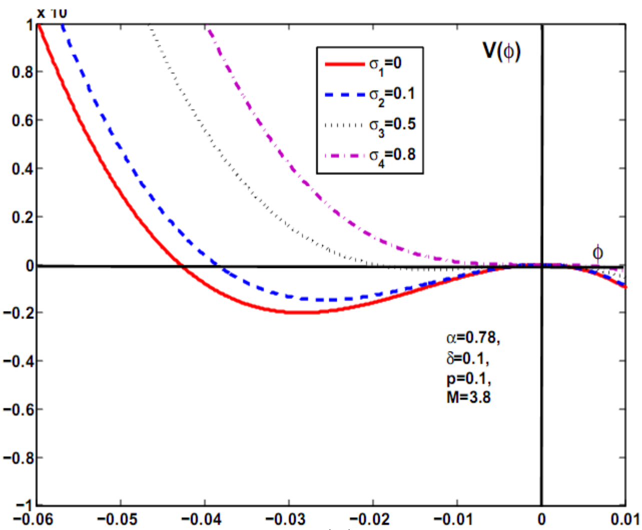

Fig. 4 The Sagdeev potential $V(\phi)$ with respect to ϕ for fixed value of α =0.78, p=0.1, δ =0.1, M=3.8 and different values of σ

Now let us examine the existence of rarefactive and compressive solitons, numerically. As apparent from Table 1 and 2, the range of existence of compressive and rarefactive solitons shift. It is seen from Table 1, for low values of M, neither compressive nor rarefactive solitons exist. Nevertheless, there is a range of α and M where co-existence of rarefactive as well as compressive solitons is possible. Table II shows the range of rarefactive and compressive solitons for fixed values of α =0.2, p=0.1 and δ =0.1 and different values of σ and M. It is obvious that the both rarefactive and compressive solitons exist in the specific range of Mach number (1.33<M<1.5).

TABLE I
EFFECT OF α , THE NONTHERMAL PARAMETER, ON THE RANGE OF RAREFACTIVE AND COMPRESSIVE SOLITONS. THE OTHER PARAMETERS HAVE BEEN FIXED AS σ =0.0001, P=0.1 AND δ =0.1

$M \setminus \alpha$	0.15	0.2	0.25	0.3
1.23	-	-	-	-
1.25	C	-	-	-
1.32	R,C	-	-	-
1.33	R,C	C	-	-
1.36	R,C	R,C	-	-
1.42	R,C	R,C	C	-
1.54	R,C	R,C	R,C	C
1.6	R,C	R,C	R,C	R,C

TABLE II
EFFECT OF σ , THE ION TEMPERATURE, ON THE RANGE OF RAREFACTIVE AND COMPRESSIVE SOLITONS. THE OTHER PARAMETERS HAVE BEEN FIXED AS α =0.2, P=0.1 AND δ =0.1

$\sigma \backslash M$	1.32	1.33	1.34	1.37	1.38	1.4	1.45	1.5	1.6
0	-	C	R,C	R,C	C	C	C	C	-
0.0001	-	C	R,C	R,C	C	C	C	C	-
0.001	-	C	R,C	R,C	C	C	C	-	-
0.01	-	-	C	R,C	R,C	C	C	-	-
0.1	-	-	-	-	-	-	-	C	-
0.3	-	-	-	-	-	-	-	-	-
0.5	-	-	-	-	-	-	-	-	-
0.8	-	-	-	-	-	-	-	-	-

There is only an important note in the Table II, when σ and M are 0.1 and 1.5, respectively. For low temperature of ions and M=1.5, only rarefactive solitons exist. Then, when the values of σ is increased, the rarefactive solitons disappeared.But, the rarefactive soliton appears for σ =0.1, again.The limitation of the present analysis is that the rarefactive and compressive solitary waves have been studied as separately. In the same way, one can easily show (after a more numerical analysis with different values of σ , α and M) there is a region in parameter space where both rarefactive and compressive solitons can coexist. It is clear that it is possible to investigate more nonlinear wave structures over a wider range of parameters space. It can be also shown that there is double layer solitons in our model for specific values of α and σ . This case might be studied in a new work.

IV. CONCLUSION

In this paper, we have studied the effect of non-thermal electrons and ion temperature on solitary waves in a plasma consisting warm ions, positrons and nonthermal electron distribution. The pseudo-potential approach has been used. Our results show that in such a plasma spatially localized ion-acoustic structures, the height and nature of which depend sensitively on the plasma parameters, can exist. The spatial patterns of the ion acoustic solitary waves are significantly modified by the effects of electron nonthermal and ion temperature. It was shown that the rarefactive and compressive solitons can be appeared in our model. The particular regions of space parameter where the compressive and/or rarefactive solitary waves exist were cleared. We found that the amplitude of both rarefactive and compressive solitons decreases with an increasing in the nonthermal parameter α and ion temperature σ . On the other hand, we also found that there are the critical values of α and σ (α_c , σ_c), in which Sagdeev potential don't behavior well when $\alpha \rightarrow \alpha_c$ (or $\sigma \rightarrow \sigma_c$), that is for $\alpha \geq \alpha_c$ ($\sigma \geq \sigma_c$) there is no soliton. The critical value depends on the plasma parameters (σ, δ, p , M and different values of α). When we choose other values of the parameters, which a changing nature occurs, the critical value for α corresponds to an other different value. Considering the wide relevance of nonlinear oscillations, we stress that the results of the present investigation should be useful in understanding the nonlinear features of localized ion-acoustic structures in different regions of the astrophysical and space invironments as well as other physical phenomena like condensation of double layers [29,30].

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