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Mean square stability of impulsive stochastic delay differential equations with markovian switching and poisson jumps

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Abstract—In the paper, based on stochastic analysis theory and Lyapunov functional method, we discuss the mean square stability of impulsive stochastic delay differential equations with markovian switching and poisson jumps, and the sufficient conditions of mean square stability have been obtained. One example illustrates the main results. Furthermore, some well-known results are improved and generalized in the remarks.

Keywords—impulsive, stochastic, delay, Markovian switching, Poisson jumps, mean square stability.

I. INTRODUCTION

MANY evolution processes which are changed at certain moments are always affected by impulsive, such as, medicine, economics, biology, mechanics and so on. In recent years, the stability and other properties of impulsive differential equations have been investigated and many criteria of stability for these systems have been obtained [see[1]-[4]]. Stochastic effects are often taken into account, which is very necessary for good results, and some results of stability for impulsive stochastic delay differential equations (SDDE) have been gotten [see[10]-[13]]. However, the results of impulsive SDDE with jumps are very few, so the investigation is very necessary and valuable.

To the best of author's knowledge, the stability of impulsive SDDE have been studied. But the investigation of these equations which are embedded markov chains and poisson jumps are blank. In this paper, we will have a try to study them to fill the gap.

The markov chain and poisson jumps become very popular in recent years, because they are extensively used to model on many phenomena emerging in a lot of areas. So the first attempt that we investigate the mean square stability of impulsive SDDE with markovian switching and poisson jumps is very necessary.

This paper is organized as follows: In section II, we present some basic preliminaries; In section III, the main result of mean square stability and the proof have been given; In section IV, some well-known results are generalized in the remarks and an example is given to illustrate our conclusion.

II. PRELIMINARIES

Let $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P}\}$ be a probability space with a filtration satisfying the usual conditions, i.e., the filtration is continuous on the right and $\mathcal{F}_0$-contains all $\mathbf{P}$-zero sets. Let $B(t) = (B_1(t), B_2(t), ..., B_m(t))^T$ be an m-dimensional Brownian motion defined on the probability space. $||\bullet||$ is the Euelidean norm in R^n and $\|x_(t)\|_\tau = \sup\limits_{-\tau\leq\theta\leq 0} \|x(t+\theta)\|$.

Let $PC(I, R^n) = \{\phi : I \to R^n | \phi(t^+) = \phi(t) \quad for \quad t \in I; \phi(t^-) \quad exists \quad for \quad t \in (t_0, \infty), \phi(t^-) = \phi(t) \quad for \quad all \quad but \quad points \quad t_k \in (t_0, \infty)\}$, where $I \subset R$ is an interval, $\phi(t^-)$ and $\phi(t^+)$ denote the left-hand and right-hand limits of function. Let $PC(\delta) = \{\phi : \phi \in PC([-\tau, 0], R^n) \quad and \quad \|\phi\|_\tau \leq \delta\}$ and $PC_{\mathcal{F}_0}([-\tau, 0], R^n)$ denote the family of all $\mathcal{F}_0$-measurable $PC([-\tau, 0], R^n)$-valued stochastic process $\varphi = \{\varphi(s) : -\tau \leq s \leq 0\}$ such that $\sup\limits_{-\tau\leq s\leq 0} E\|\varphi(s)\|^2 < \infty$, and $PC^b_{\mathcal{F}_0}(\delta) = \{\varphi : \varphi \in PC^b_{\mathcal{F}_0}([-\tau, 0], R^n), \quad and \quad E\|\varphi(s)\|^2 < \delta\}$.

Let $\{r(t), t \in R_{t_0} = [t_0, +\infty)\}$ be a right-continuous Markov chain on the probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathbf{P}\}$ taking values in a finite state space $S = \{1, 2, ..., N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$P(r(t+\Delta) = j | r(t) = i) = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & if \quad i \neq j \\ 1 + \gamma_{ii}\Delta + o(\Delta), & if \quad i = j \end{cases}$$

where $\Delta > 0$.Here $\gamma_{ij} \geq 0$ is the transition rate from i to j,if $i \neq j$.while

$$\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}.$$

We assume that Markov chain $r(\cdot)$ is independent of the Brownian motion $B(\cdot)$.It is known that almost every sample path of $r(t)$ is right continuous step function with a finite number of simple jumps in any finite sub-interval of R_{t_0}.

Let $\{v(dt, du), t \in R_{t_0}, u \in R\}$ be a centered Poisson random measure with parameter $\pi(du)dt$.

Consider the following impulsive stochastic delay differential equations with markovian switching and poisson jumps:

$$\begin{aligned} dx(t) =& f(t, x(t), x_t, r(t))dt + g(t, x(t), x_t, r(t))dB(t) \\ &+ \int_{-\infty}^{+\infty} h(t, x(t), u)v(dt, du) \quad t \geq t_0, t \neq t_k \quad (1) \\ x(t_k) =& H_k(x(t_k^-)) \quad k = 1, 2, 3... \end{aligned}$$

with the initial condition $x_0 = x(t_0 + s) = \varphi(s) \in PC^b_{\mathcal{F}_0}(\delta)$, where $s \in [-\tau, 0]$ and $H_k(x(t_k^-)) = (H_{1k}(x(t_k^-)), H_{2k}(x(t_k^-)), ..., H_{nk}(x(t_k^-)))^T$ represents the

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impulsive perturbation and satisfies the global Lipschitz condition as follows:

$$\|H_k(x(t_k^-))\| \le M_k\|x(t_k^-)\| \quad M_k \ge 0, k = 1, 2, ..., \tag{2}$$

the fixed moments of time t_k satisfies $0 \le t_1 \le t_2 \le ... \le t_k \le ..., \lim_{k\to\infty} t_k = \infty$.

In the paper, we always assume that under some conditions the system (1) has a unique solution $x(t) = (x_1(t), ..., x_n(t))^T$ and $x_t = (x_{1t}, ..., x_{nt})^T, x_{it} = x_i(t - \tau_i), i = \{1, 2, ..., n\}$, and $\tau = \max_{0\le i\le n}\{\tau_i\}$.

Assume that:

$$f : R \times R^n \times R^n \times S \to R^n;$$

$$g : R \times R^n \times R^n \times S \to R^{n\times m};$$

$$h : R \times R^n \times R \to R^n.$$

Further, assume that $f(t, 0, 0, i) \equiv 0$ and $g(t, 0, 0, i) \equiv 0$ for all $i \in S$, and $h(t, 0, \cdot) \equiv 0$, then system (1) has a trivial solution $x(t) \equiv 0$.

Denote by $C^{2,1}(R^n \times [t_0,\infty) \times S; R_+)$ the family of all non-negative function $V(x, t, i)$ on $R^n \times [t_0,\infty) \times S$ which are continuously twice differential with respect to x and once differential with respect to t.

For any $(x, t, i) \in R^n \times [t_0,\infty) \times S$, define an operator L by

$$\begin{aligned} &LV(x, y, t, i)\\ &= V_t(x, t, i) + V_x(x, t, i)f(t, x, y, i)\\ &+ \frac{1}{2}trace[g^T(t, x, y, i)V_{xx}(x, t, i)g(t, x, y, i)]\\ &+ \sum_{j=1}^{N}\gamma_{ij}V(x, t, j) + \int_{-\infty}^{+\infty}[V(x + h(t, x, u), t, i)\\ &- V(x, t, i) - V_x(x, t, i)h(t, x, u)]\pi(du), \end{aligned} \tag{3}$$

where

$$V_t(x, t, i) = \frac{\partial V(x, t, i)}{\partial t};$$
$$V_x(x, t, i) = (\frac{\partial V(x, t, i)}{\partial x_1}, ..., \frac{\partial V(x, t, i)}{\partial x_n});$$
$$V_{xx}(x, t, i) = (\frac{\partial^2 V(x, t, i)}{\partial x_i\partial x_j})_{n\times n}.$$

The generalized $Itô$ formula reads as follows:

$$\begin{aligned} &EV(x(t + h), t + h, r(t + h))\\ &= EV(x(t), t, r(t)) + E\int_t^{t+h} LV(x(s), x_s, s, r(s))ds. \end{aligned} \tag{4}$$

Definition 2.1 The solution of system (1) is mean square stability if for any $\varepsilon > 0$, there exists a scalar $\delta > 0$ and the initial function $\varphi \in PC^b_{\mathcal{F}_0}(\delta)$, such that

$$E\|x(t)\|^2 < \varepsilon, \quad t \ge t_0.$$

III. Main results

Theorem 3.1 Assume that there exit $\lambda_1 > 0, \lambda_2 > 0, \lambda_4 > 0, \lambda_3 \in R$ and a Lyapunov function $V(x, t, i) \in C^{2,1}(R^n \times [t_0,\infty) \times S; R_+)$, such that

$(i)\lambda_1\|x(t)\|^2 \le v(x(t), t, i) \le \lambda_2\|x_t\|^2_\tau$;
$(ii)LV(x(t), x_t, t, i) \le \lambda_3V(x(t), t, i) + \lambda_4V(x_t, t, i)$
$t \in [t_{k-1}, t_k), \quad k = 1, 2, ...;$
$(iii)0 < \lambda < 1, \quad where \quad \lambda = sup\{\lambda_k|\lambda_k = \frac{\lambda_2}{\lambda_1}M_k^2,$
$k = 1, 2, ...\};$
$(iv)(\lambda_3 + \frac{\lambda_4}{\lambda})(t_k - t_{k-1}) < -ln\lambda, \quad k = 1, 2, ...$

where $M_k, k = 1, 2, 3...$ have been defined in (2).Then the trivial solution of system (1) is mean square stability.

Proof For any $\varepsilon > 0$, there exists a scalar $\delta = \delta(\varepsilon) > 0$, such that $\delta < \frac{\lambda_1\lambda}{\lambda_2}\varepsilon$. For any $t_0 \ge 0$ and $x_0 = \varphi \in PC^b_{\mathcal{F}_0}(\delta)$, let $x(t) = x(t, t_0, \varphi)$ be the solution of system (1).
Due to (4), we obtain that

$$\begin{aligned} &EV(x(t), t, r(t))\\ &= EV(x(t_k), t_k, r(t_k)) + E\int_{t_k}^{t} LV(x(s), x_s, s, r(s))ds,\\ &t \in [t_k, t_{k+1}) \end{aligned} \tag{5}$$

For sufficiently small $\Delta t > 0$, such that $t + \Delta t \in t \in [t_k, t_{k+1})$. We get

$$\begin{aligned} &EV(x(t + \Delta t), t + \Delta t, r(t + \Delta t))\\ &= EV(x(t_k), t_k, r(t_k))\\ &+ E\int_{t_k}^{t+\Delta t} LV(x(s), x_s, s, r(s))ds, \quad t \in [t_k, t_{k+1}] \end{aligned} \tag{6}$$

Using (5), (6) and condition (ii), we observe that

$$\begin{aligned} &EV(x(t + \Delta t), t + \Delta t, r(t + \Delta t)) - EV(x(t), t, r(t))\\ &= E\int_t^{t+\Delta t} LV(x(s), x_s, s, r(s))ds,\\ &\le \int_t^{t+\Delta t}(\lambda_3EV(x(s), s, r(s)) + \lambda_4EV(x_s, s, r(s)))ds,\\ &t \in [t_k, t_{k+1}) \end{aligned}$$

therefore,

$$\begin{aligned} D^+EV(x(t), t, r(t)) &\le \lambda_3EV(x(t), t, r(t))\\ &+ \lambda_4EV(x_t, t, r(t)), \quad t \in [t_k, t_{k+1}). \end{aligned}$$

Now we claim that

$$EV(x(t), t, r(t)) \le \frac{\lambda_2}{\lambda}\delta, \quad t_0 \le t \le t_1. \tag{7}$$

Due to $x_0 \in PC^b_{\mathcal{F}_0}(\delta)$ and condition (i), it's obvious that

$$\begin{aligned} &EV(x(t), t, r(t))\\ &= EV(x(t_0 + \theta), t_0 + \theta, r(t_0 + \theta))\\ &\le \lambda_2E\|x_0\|^2_\tau \le \lambda_2\delta \le \frac{\lambda_2}{\lambda}\delta, \quad t_0 - \tau \le t \le t_0. \end{aligned}$$

If (7) does not hold, then there exists some $s \in (t_0, t_1)$, such that

$$EV(x(s), s, r(s)) > \frac{\lambda_2}{\lambda}\delta > \lambda_2\delta \geq EV(x(t_0), t_0, r(t_0)).$$

Let

$$s_1 = inf\{s \in [t_0, t_1) | EV(x(s), s, r(s)) > \frac{\lambda_2}{\lambda}\delta\}.$$

For any $t \in [t_0 - \tau, t_0], EV(x(t), t, r(t)) < \frac{\lambda_2}{\lambda}\delta$, note that $EV(x(t), t, r(t))$ is continuous for variable on $[t_0, t_1)$, then

$$\begin{array}{l} EV(x(s_1), s_1, r(s_1)) = \frac{\lambda_2}{\lambda}\delta; \\ EV(x(t), t, r(t)) \leq \frac{\lambda_2}{\lambda}\delta, \quad t_0 - \tau \leq t \leq s_1; \\ D^+EV(x(s_1), s_1, r(s_1)) \geq 0. \end{array} \tag{8}$$

From the inequalities $\frac{\lambda_2}{\lambda}\delta > \lambda_2\delta$, then there exists $s_2 \in [t_0, s_1)$, such that

$$\begin{array}{l} EV(x(s_2), s_2, r(s_2)) = \lambda_2\delta; \\ EV(x(t), t, r(t)) \geq \lambda_2\delta, \quad s_2 \leq t \leq s_1; \\ D^+EV(x(s_2), s_2, r(s_2)) \geq 0. \end{array} \tag{9}$$

Combing (8) and (9), we get

$$EV(X_t, t, r(t)) \leq \frac{\lambda_2}{\lambda}\delta \leq \frac{1}{\lambda}EV(x(t), t, r(t)), \quad t \in [s_2, s_1],$$

and

$$\begin{aligned} & D^+EV(x(t), t, r(t)) \\ & \leq \lambda_3 EV(x(t), t, r(t)) + \lambda_4 EV(x_t, t, r(t)) \\ & \leq (\lambda_3 + \frac{\lambda_4}{\lambda})EV(x(t), t, r(t)) \end{aligned} \tag{10}$$

Therefore, for any $t \in [s_2, s_1]$

$$\int_{s_2}^{s_1} \frac{D^+EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds \leq \int_{s_2}^{s_1} (\lambda_3 + \frac{\lambda_4}{\lambda})ds.$$

Applying condition (iii) and (iv), we have

$$\begin{aligned} \int_{s_2}^{s_1} (\lambda_3 + \frac{\lambda_4}{\lambda})ds & \leq \int_{t_0}^{t_1} (\lambda_3 + \frac{\lambda_4}{\lambda})ds \\ & = (\lambda_3 + \frac{\lambda_4}{\lambda})(t_1 - t_0) < -ln\lambda. \end{aligned}$$

So

$$\int_{s_2}^{s_1} \frac{D^+EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds < -ln\lambda.$$

At the same time,

$$\begin{aligned} \int_{s_2}^{s_1} \frac{D^+EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds & = \int_{EV(x(s_2), s_2, r(s_2))}^{EV(x(s_1), s_1, r(s_1))} \frac{du}{u} \\ & = \int_{\lambda_2\delta}^{\frac{\lambda_2}{\lambda}\delta} \frac{du}{u} \\ & = ln(\frac{\lambda_2}{\lambda}\delta) - ln(\lambda_2\delta) \\ & = -ln\lambda, \end{aligned}$$

which is a contradiction, so (7) holds.
Combing (2),(7) and condition (i), we get

$$\begin{aligned} EV(x(t_1), t_1, r(t_1)) & = EV(H_1(x(t_1^-)), t_1, r(t_1)) \\ & \leq \lambda_2 E\|H_1(x(t_1^-))\|_\tau^2 \\ & \leq \lambda_2 M_1^2 E\|x(t_1^-)\|_\tau^2 \\ & \leq \frac{\lambda_2 M_1^2}{\lambda_1} \sup_{-\tau \leq \theta \leq 0} EV(x(t_1^- + \theta), \\ & t_1^- + \theta, r(t_1^- + \theta)) \\ & \leq \lambda \frac{\lambda_2}{\lambda}\delta \leq \lambda_2\delta \leq \frac{\lambda_2}{\lambda}\delta \end{aligned}$$

Now we assume that for $m = 1, 2, ..., k$, the following inequalities hold,

$$\begin{array}{l} EV(x(t), t, r(t)) \leq \frac{\lambda_2}{\lambda}\delta, \quad t_{m-1} \leq t \leq t_m; \\ EV(x(t_k), t_k, r(t_k)) \leq \frac{\lambda_2}{\lambda}\delta, \quad k = 1, 2, ..., \end{array} \tag{11}$$

for $m = k + 1$, we claim that

$$EV(x(t), t, r(t)) \leq \frac{\lambda_2}{\lambda}\delta, \quad t_k \leq t \leq t_{k+1}. \tag{12}$$

If (12) does not hold, then there exists some $p \in (t_k, t_{k+1})$, such that

$$EV(x(p), p, r(p)) > \frac{\lambda_2}{\lambda}\delta > \lambda_2\delta \geq EV(x(t_k), t_k, r(t_k)).$$

Let

$$p_1 = inf\{p \in [t_0, t_1) | EV(x(p), p, r(p)) > \frac{\lambda_2}{\lambda}\delta\}.$$

For any $t \in [t_{k-1}, t_k], EV(x(t), t, r(t)) < \frac{\lambda_2}{\lambda}\delta$, note that $EV(x(t), t, r(t))$ is continuous for variable on $[t_k, t_{k+1})$, then

$$\begin{array}{l} EV(x(p_1), p_1, r(p_1)) = \frac{\lambda_2}{\lambda}\delta; \\ EV(x(t), t, r(t)) \leq \frac{\lambda_2}{\lambda}\delta, \quad t_0 - \tau \leq t \leq p_1; \\ D^+EV(x(p_1), p_1, r(p_1)) \geq 0. \end{array} \tag{13}$$

From the inequalities $\frac{\lambda_2}{\lambda}\delta > \lambda_2\delta$, then there exists $p_2 \in [t_k, p_1)$, such that

$$\begin{array}{l} EV(x(p_2), p_2, r(p_2)) = \lambda_2\delta; \\ EV(x(t), t, r(t)) \geq \lambda_2\delta, \quad p_2 \leq t \leq p_1; \\ D^+EV(x(p_2), p_2, r(p_2)) \geq 0. \end{array} \tag{14}$$

Combing (13) and (14), we get

$$EV(X_t, t, r(t)) \leq \frac{\lambda_2}{\lambda}\delta \leq \frac{1}{\lambda}EV(x(t), t, r(t)), \quad t \in [p_2, p_1],$$

and

$$\begin{aligned} & D^+EV(x(t), t, r(t)) \\ & \leq \lambda_3 EV(x(t), t, r(t)) + \lambda_4 EV(x_t, t, r(t)) \\ & \leq (\lambda_3 + \frac{\lambda_4}{\lambda})EV(x(t), t, r(t)) \end{aligned} \tag{15}$$

Therefore, for any $t \in [p_2, p_1]$

$$\int_{p_2}^{p_1} \frac{D^+EV(x(s), s, r(s))}{EV(x(s), s, r(s))} ds \leq \int_{p_2}^{p_1} (\lambda_3 + \frac{\lambda_4}{\lambda})ds.$$

Applying condition (iii) and (iv), we have

$$\begin{aligned} \int_{p_2}^{p_1} (\lambda_3 + \frac{\lambda_4}{\lambda})ds & \leq \int_{t_0}^{t_1} (\lambda_3 + \frac{\lambda_4}{\lambda})ds \\ & = (\lambda_3 + \frac{\lambda_4}{\lambda})(t_1 - t_0) < -ln\lambda. \end{aligned}$$

So

$$\int_{p_2}^{p_1} \frac{D^+EV(x(s),s,r(s))}{EV(x(s),s,r(s))}ds < -ln\lambda.$$

At the same time,

$$\begin{aligned}\int_{p_2}^{p_1} \frac{D^+EV(x(s),s,r(s))}{EV(x(s),s,r(s))}ds &= \int_{EV(x(p_2),p_2,r(p_2))}^{EV(x(p_1),p_1,r(p_1))} \frac{du}{u}\\ &= \int_{\lambda_2\delta}^{\frac{\lambda_2}{\lambda}\delta} \frac{du}{u}\\ &= ln(\frac{\lambda_2}{\lambda}\delta) - ln(\lambda_2\delta)\\ &= -ln\lambda,\end{aligned}$$

which is a contradiction, so (12) holds.
Combing (2),(12) and condition (i), we get

$$\begin{aligned}&EV(x(t_{k+1}),t_{k+1},r(t_{k+1}))\\ &= EV(H_{k+1}(x(t_{k+1}^-)),t_{k+1},r(t_{k+1}))\\ &\le \lambda_2 E\|H_{k+1}(x(t_{k+1}^-))\|_\tau^2\\ &\le \lambda_2 M_{k+1}^2 E\|x(t_{k+1}^-)\|_\tau^2\\ &\le \frac{\lambda_2 M_{k+1}^2}{\lambda_1} \sup_{-\tau\le\theta\le 0} EV(x(t_{k+1}^-+\theta),t_{k+1}^-+\theta,r(t_{k+1}^-+\theta))\\ &\le \lambda\frac{\lambda_2}{\lambda}\delta \le \lambda_2\delta \le \frac{\lambda_2}{\lambda}\delta\end{aligned}$$

By the mathematical induction, we can conclude that

$$\begin{aligned}&EV(x(t),t,r(t)) \le \tfrac{\lambda_2}{\lambda}\delta, \quad t_{k-1} \le t \le t_k;\\ &EV(x(t_k),t_k,r(t_k)) \le \tfrac{\lambda_2}{\lambda}\delta, \quad k = 1,2,....\end{aligned}$$

Therefore

$$EV(x(t),t,r(t)) \le \frac{\lambda_2}{\lambda}\delta, \quad t \ge t_0,$$

which yields

$$E\|x(t)\|^2 \le \frac{\lambda_2}{\lambda_1\lambda} < \varepsilon, \quad t \ge t_0.$$

Now, we can obtain that the solution of system (1) is mean square stability by definition 2.1.

IV. Remarks and an Example

Remark 4.1 When $r(t) \equiv 0$ and $h(t,x(t),\cdot) \equiv 0$, the system (1) reduces to

$$\begin{aligned}dx(t) =& f(t,x(t),x_t)dt + g(t,x(t),x_t)dB(t)\\ & t \ge t_0, t \ne t_k \qquad (16)\\ x(t_k) =& H_k(x(t_k^-)) \quad k = 1,2,3...\end{aligned}$$

with the initial condition $x_0 = x(t_0+s) = \varphi(s) \in PC_{\mathcal{F}_0}^b(\delta)$, where $s \in [-\tau, 0]$, which is recently studied in the similar literatures.That is to say, we generalize the results of the similar literatures.

Example 4.1 Consider the following impulsive stochastic delay differential equations:

$$\begin{aligned}\begin{pmatrix} dx_1(t)\\ dx_2(t)\end{pmatrix} =& [\begin{pmatrix} -10.5 & 0\\ 0 & -12.2\end{pmatrix}\begin{pmatrix} x_1(t)\\ x_2(t)\end{pmatrix}\\ &+ \begin{pmatrix} 1.2 & -0.2\\ 0.6 & 2.4\end{pmatrix}\begin{pmatrix} sinx_1(t)\\ arctanx_2(t)\end{pmatrix}\\ &+ \begin{pmatrix} 1.6 & 0.3\\ -0.5 & 1.8\end{pmatrix}\begin{pmatrix} sinx_1(t-\frac{1}{2})\\ arctanx_2(t-\frac{1}{3})\end{pmatrix}]dt\\ &+ \begin{pmatrix} 2x_1(t) & x_2(t-\frac{1}{3})\\ x_1(t-\frac{1}{2}) & -x_2(t)\end{pmatrix}\begin{pmatrix} dB_1(t)\\ dB_2(t)\end{pmatrix},\\ & t \ge t_0, t \ne t_k\\ \begin{pmatrix} x_1(t_k)\\ x_2(t_k)\end{pmatrix} =& e^{-0.1k}\begin{pmatrix} 0.5 & -0.15\\ 0.12 & 0.6\end{pmatrix}\begin{pmatrix} x_1(t_k^-)\\ x_2(t_k^-)\end{pmatrix}\\ & k = 1,2,3...\end{aligned} \qquad (17)$$

where $t_0 = 0$ and $t_k = t_{k-1} + 0.15$ (k=1,2,...).
Let $\lambda_1 = 0.5600, \lambda_2 = 0.6800, \lambda_3 = -1.8670, \lambda_4 = 2.1071$ and $M_k = 0.62e^{-0.1k}$, $0 < \lambda = 0.313 < 1$, then $(\lambda_3 + \frac{\lambda_4}{\lambda})(t_k - t_{k-1}) = 0.7293 < 1.1608 = -ln\lambda$. So the solution of system (17) is mean square stability by our theory.

Remark 4.2 With the process of example 4.1, we can obtain that the conditions of mean square stability have become much easier to be satisfied than the similar literatures.

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References

[1] Zhang Q Q. On a linear delay difference equation with impulses. Ann.Differential Equations,2002,18:197-204.
[2] Braverman E. On a discrete model of population dynamics with impulsive harvesting or recruitment. Nonlinear Anal.,2005,63(5):751-759.
[3] Wei G P. The persistence of nonoscillatory solutions of difference equations under impulsive perturbations. Comput.andMath.Appl.,2005,50(10):1579-1586.
[4] Peng M S. Oscillation criteria for second-order impulsive delay difference equations. Appl.Math.Comput., 2003,146(1):227-235.
[5] Mao X. Stability of stochastic differential equations with respect to semimartingales. NewYork:Longman Scientific and Technical,1991.
[6] Luo J W. Comparison principle and stability of Ito stochastic differential delay equations with poisson jumps and Markovian swithching. Nonlinear Analysis,2006,64:253-262.
[7] Mao X. Exponential stability of stochastic differential equations. NewYork:Marcel Dekker, 1994.
[8] Mao X. Stochastic differential equations and applications. NewYork:Horwood,1997.
[9] Li R H. Convergence of numerical solutions to stochastic delay differential equations with jumps. App.Math.Comp.,2006,172(2):584-602.
[10] Yang Z, Xu D. Mean square exponential stability of impulsive stochastic difference equations. Appl.math.Letter,2007,20:938-945.
[11] Yang J, Zhong S. Mean square stability of impulsive stochastic differential equations with delays. J.comp.appl.math.,in press.
[12] Yang Z, Xu D. Exponential p-stability of impulsive stochastic differential equations with delays. Physics Letters A,2006,359(3):129-137.
[13] Wu S. The Euler scheme for random impulsive differential equations. Appl.math.comp.,in press.