Abstract—The study of the generated defects on manufactured parts shows the difficulty to maintain parts in their positions during the machining process and to estimate them during the pre-process plan. This work presents a contribution to the development of 3D models for the optimization of the manufacturing tolerances. An experimental study allows the measurement of the defects of part positioning for the determination of $\varepsilon$ and the choice of an optimal setup of the part. An approach of 3D tolerance based on the small displacements method permits the determination of the manufacturing errors upstream. A developed tool, allows an automatic generation of the tolerance intervals along the three axes.

Keywords—Manufacturing tolerances, 3D modeling, optimization, errors.

I. INTRODUCTION

In a chain of dimensional ratings, more angular and radial defects of the same coin can be influential on a requirement, making it impossible to employ a statistical method based on simply the sum of the variances associated with each tolerance. This difficulty has necessitated the development of a particular method to separate the possible influence of defects in each part. The method of lines analysis establishes, for each requirement, an inequality that links the nominal dimensions and tolerances of parts specifications, the limit value of the requirement. Y.S. Hong [1] proposed a very broad review of the tolerancing method in 2002. Bourdet [2,3] has developed a computational model of tolerance unidirectional based on machining dispersions. Anselmetti [4] extended this approach for faces lying in any direction. Such approaches are not able to handle small angular deviations from one machining phase to the next. The purpose of three-dimensional approaches is to manage the small angular deviations occurring between the various machining phases. P. Bourdet, E. Ballot and F. Thiebaut [5,6] have developed the $\Delta$Tol tolerancing method, which relies upon the concept of a small displacement torsor. This model considers that hape defects affecting the raw and machined surfaces are in act negligible, which offers the possibility of replacing real surfaces containing defects by perfect substitution surfaces, n association with certain criteria to be defined (e.g., least quares, mini max, cylinder inscribed).

II. EXPERIMENTAL STUDY

The purpose of this step is determining the optimal position isostatic. To achieve this objective, a series of 100 tests for measuring defects in positioning load by varying the distance between the normals on a gauge block of dimensions 100 x 35 x 9 mm was done (figure 1).

Fig. 1 Support variation

Fig. 2 shows an example of a statement of metrological errors into normal position due to the support plan. Figure 3 represents the evolution of deviations from the support 1 according to the number of trials. Note that the chosen positions vary between 0.002 and 0.014. The optimal position is given by the smallest deviation from the support 1 at position 9.
Figure 2 Example of measurement

Figure 3 Standard deviations of the support 1 according to the number of tests

Figure 4 shows the evolution of deviations from the support 2 by the number of trials. Note that the chosen positions vary between 0.002 and 0.012. The optimal position is given by the smallest deviation from the support 2 is located at position 10.

Figure 4 Standard deviations of the support 2 by the number of tests

Figure 5 shows the evolution of deviations from the support 3 depending on the number of trials. Note that the chosen positions vary between 0.002 and 0.016. The optimal position of the support 2 is in position 6.

Figure 5 Standard deviations of the support 3 according to the number of tests

Figure 6 shows the evolution of deviations from the support 4 against the number of trials. Note that the chosen positions vary between 0.002 and 0.035. The optimal position is given by the smallest deviation from the support 4 is located at position 10.

Figure 6 Standard deviations of the support 4 against the number of tests

Figure 7 represents the evolution of deviations from the support 5 in terms of number of trials. Note that the chosen positions vary between 0.002 and 0.025. The optimal position is given by the smallest deviation from the support 5 located in position 10.

Figure 7 Standard deviations of the support 5 against the number of tests
Figure 8 represents the evolution of standard deviations of the support 6 in terms of number of trials. Note that the chosen positions vary between 0 and 0.025. The optimal position is given by the smallest deviation from the support 6 is found in positions 3, 8 and 10.

According to the graphs shown in Figures 3, 4, 5, 6, 7 and 8, we deduce the distribution of normal best way to find the slightest error. Figure 9 represents the optimal distribution of normal (X=\(\frac{1}{6}L\), Y=\(\frac{1}{6}H\) and Z=\(\frac{1}{2}L\)).

III. MODELING POSITIONING

Several researchers have focused their work on the contribution to the development of mathematical models on the positioning. Nnaji and Cabaday propose a model of decision-piece using a kinematic approach. Nnaji [11,12] likens the piece to a prism. It provides a rationale for modeling taking part in analyzing an equal number of faces of the prism modeling part. The inventory of the various forces exerted on the workpiece can check the balance of the play on his feet and conclude on the uniqueness of the position of the part in the repository of the machine tool. The passage of a piece of complex geometry modeling in the form of a prism is not addressed and does not seem easy. Cabaday [13] proposes a model to evaluate the forces exerted by the mounting part machining.

Liu [14] propose a strategy for placing the media. It models the actions exerted by the cutting tool on the workpiece in order to calculate the deformations of walls machined. The number and placement of materials are determined to minimize distortion. Donoghue and Cutkosky model the contact between the part and assembly machining. Cutkosky [15] studied the friction between the part and assembly machining. It calculates the friction surface limits that ensure the balance of the play when subjected to cutting forces. Donoghue [16] models the pressure and deformations in contact-part assembly machine. In our study, we focus on modeling of deviations from the room without clamping, using the torques of small displacements and the tightening effect on the positioning, using the torsor reaction support.

IV. SMALL DISPLACEMENT TORSOR

The concept of small displacement torsor (TPD) has been developed in the 70s by Pierre Bourdet and Andrew Clement. It allows to define any point M of a rigid body moving a small [17,18,19]. The displacement of a solid can be characterized by a point O by a translation vector and rotation matrix, equation (1).

\[ D_a = \vec{r}_o + M\vec{O} \times \vec{\omega} \]  
(1)

With \(\vec{r}_o (u,v,w)\) translation vector at point O around the axes x, y, z, and \(\vec{\omega} (\alpha, \beta, \gamma)\) vector rotation around the axes x, y, z.

The translation vector and rotation are given according to \(\varepsilon\), equations 7, 8, 9, 10 and 11. The pair of vectors \(\varepsilon\) is called a small displacement torsor. In this work, this small displacement torsor in characteristic deviations of the workpiece on the support.
\[
\begin{align*}
\varepsilon_2 &= \begin{bmatrix} u & v + \beta A & x_2 \\ w & z_2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\varepsilon_3 &= \begin{bmatrix} u & v + \beta A & x_3 \\ w & z_3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\varepsilon_4 &= \begin{bmatrix} u & v + \beta A & x_4 \\ w & z_4 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\varepsilon_5 &= \begin{bmatrix} u & v + \beta A & x_5 \\ w & z_5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\varepsilon_6 &= \begin{bmatrix} u & v + \beta A & x_6 \\ w & z_6 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\end{align*}
\]

\[
\begin{align*}
\varepsilon_2 &= v - (\alpha z_2 - \delta x_2) \\
\varepsilon_3 &= v - (\alpha z_3 - \delta x_3) \\
\varepsilon_4 &= w + (\alpha y_4 - \beta x_4) \\
\varepsilon_5 &= w + (\alpha y_5 - \beta x_5) \\
\varepsilon_6 &= w + (\alpha y_6 - \beta x_6)
\end{align*}
\]

\[
U = \varepsilon_6 - \begin{bmatrix} \varepsilon_1 - \varepsilon_2 \\ \varepsilon_1 - \varepsilon_3 \\ \varepsilon_1 - \varepsilon_4 \\ \varepsilon_1 - \varepsilon_5 \\ \varepsilon_1 - \varepsilon_6 \\ Y_2 - Y_3 \\ X_4 - X_3 \end{bmatrix}
\]

\[
V = \varepsilon_3 - \begin{bmatrix} \varepsilon_3 - \varepsilon_2 \\ \varepsilon_3 - \varepsilon_4 \\ \varepsilon_3 - \varepsilon_5 \\ \varepsilon_3 - \varepsilon_6 \\ X_3 \end{bmatrix}
\]

\[
\begin{align*}
\alpha &= \frac{(\varepsilon_1 - \varepsilon_2) + [(\varepsilon_2 - \varepsilon_3), (1 - Z_1), (1 - Z_2), (1 - Z_3)]}{(1 - Z_1)} \\
\beta &= \frac{(\varepsilon_1 - \varepsilon_2) + [(\varepsilon_2 - \varepsilon_3), (1 - Z_1), (1 - Z_2), (1 - Z_3)]}{(1 - Z_1)} \\
\gamma &= \frac{(\varepsilon_2 - \varepsilon_3) + [(\varepsilon_2 - \varepsilon_3), (1 - Z_1), (1 - Z_2), (1 - Z_3)]}{(1 - Z_1)} \\
\delta &= \frac{(\varepsilon_3 - \varepsilon_4) + [(\varepsilon_3 - \varepsilon_4), (1 - Z_1), (1 - Z_2), (1 - Z_3)]}{(1 - Z_1)} \\
\end{align*}
\]

V. DEVELOPING A MODULE FOR CALCULATING ERRORS IN 3D

Module has been developed in VB6, figure 13, based on formulas 7, 8, 9, 10 and 11 to calculate the errors of the fixture workpiece.

A concrete example, figure 11 and 12, was treated to calculate the errors in three directions X, Y, and Z.

\[\text{Fig. 11 3D Drawing}\]
The tolerancing of mechanical systems is a key stage of the creation of a product. The effects induced by the influence tolerancing quality and cost of the finished product. Mastering dimensional and geometrical defects in manufacture of mechanical parts can ensuring final product quality during the realization of a piece, it is necessary implement several procedures and processes manufacturing. Dispersions are manufacturing due process and the manufacturing process; that is to say, the choice of machine, type of machining, door parts and the best positioning, and tools that will help realize the room. A condition of no waste remains always control for positioning defects or random dispersions.

This work has allowed us to identify errors (geometric imperfections) in three dimensions. To calculate these errors, it is sufficient to introduce the dimension of the workpiece in the developed module.

REFERENCES


Fig. 12 Drawing definition

Fig. 13 Module developed