

# Correlation of Viscosity in Nanofluids using Genetic Algorithm-neural Network (GA-NN)

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**Abstract**—An accurate and proficient artificial neural network (ANN) based genetic algorithm (GA) is developed for predicting of nanofluids viscosity. A genetic algorithm (GA) is used to optimize the neural network parameters for minimizing the error between the predictive viscosity and the experimental one. The experimental viscosity in two nanofluids Al<sub>2</sub>O<sub>3</sub>-H<sub>2</sub>O and CuO-H<sub>2</sub>O from 278.15 to 343.15 K and volume fraction up to 15% were used from literature. The result of this study reveals that GA-NN model is outperform to the conventional neural nets in predicting the viscosity of nanofluids with mean absolute relative error of 1.22% and 1.77% for Al<sub>2</sub>O<sub>3</sub>-H<sub>2</sub>O and CuO-H<sub>2</sub>O, respectively. Furthermore, the results of this work have also been compared with others models. The findings of this work demonstrate that the GA-NN model is an effective method for prediction viscosity of nanofluids and have better accuracy and simplicity compared with the others models.

**Keywords**—genetic algorithm, nanofluids, neural network, viscosity

## I. INTRODUCTION

NANOFUIDS are mixtures of solid nanoparticles with average particle size smaller than 100 nm dispersed in base fluids [1] such as water, ethylene glycol, propylene glycol or engine oil. Research on nanofluids has received great attention in the last decade due to the prospect of enhanced transport properties. Among of transport properties viscosity is a fundamental characteristic property of a fluid that influences flow and heat transfer phenomena. Determining the viscosity of nanofluids is essential for optimizing flow transport devices in energy supply.

Many experimental and theoretical works have been dedicated to the thermal conductivity of nanofluids [2-6]. However, experimental data for the effective viscosity of nanofluids are limited to certain nanofluids. [7-13]. The ranges of the investigated variables such as the particle volume concentration, particle size and temperature are also limited. More correlations reported for determining of nanofluids viscosity are based on simple Einstein model [14]. A few theoretical models have been developed for the determination of a nanoparticles suspension viscosity [15, 16]. Still, the experimental data show the trend that the effective viscosities of nanofluids are higher than the existing theoretical predictions [12]. In an attempt to modify this situation, researchers proposed models applied to specific applications, e.g., Al<sub>2</sub>O<sub>3</sub> in water [12, 17], Al<sub>2</sub>O<sub>3</sub> in ethylene

glycol [17], and CuO in water with temperature change [18]. However, the problem with these models is that they do not reduce to the Einstein model [14] at very low particle volume concentrations and, hence, lack a sound physical basis. Moreover, many of the deterministic or conceptual viscosity models need a sufficient amount of data for calibration and validation purposes that makes them computationally incompetent. As a result this has caused the attention of the researcher to focus on a separate category of models called systems theoretic models. Systems theoretic models such as artificial neural networks (ANN), also known as the black box models, attempt to develop relationships among input and output variables involved in a physical process without considering the underlying physical process. The ANN technique has applied successfully in various fields of modeling and prediction in many engineering systems, mathematics, medicine, economics, metrology and many others. It has become increasingly popular in during last decade. The advantage of ANN compared to conceptual models are its high speed, simplicity and large capacity which reduce engineering attempt. Some recent applications are made in thermophysical properties [19-21]. Back propagation neural network (BPNN) is widely used because it can effectively solve non-linear problem. However, there are some deficiencies for BP neural network, such as getting into local extreme and convergence is slow. This is disadvantageous under limited experiment data of nanofluid viscosity. To overcome these problems and improve reliability of network, this study attempts to combine genetic algorithm (GA), avoiding local minima and achieving global convergence quickly and correctly by searching in several regions simultaneously, with BPNN to minimize the total mean squared error (MSE) between output of the network and the desired output through optimizing the weights and thresholds of neural network. The presented model determines the effective viscosity of nanofluids as a function of the temperature, nanoparticle volume fraction, nanoparticle size and the base fluid physical properties.

## II. SOFT COMPUTING TECHNIQUES

The soft computing techniques employed in this study include the artificial neural networks and the genetic algorithms. Implementation algorithm of the network training is programmed within the Matlab environment using GA and Neural Networks Tool Boxes. A brief overview of these techniques is presented here.

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### III. ARTIFICIAL NEURAL NETWORKS

Artificial neural networks have been introduced as a flexible mathematical, which can imitate complex and non-linear relationships through the application of many non-linear processing units called neurons. The relationship can be learned by a neural network through adequate training from the experimental data [22]. Artificial neural network provides a parameterized, non-linear mapping between inputs and outputs. Neural networks are clearly extremely useful in recognizing patterns in complex data. The resulting quantitative models are transparent; they can be interrogating to reveal the patterns and the model parameters can be studied to illuminate the significance of particular variables [23]. Several authors have discussed the history, architecture, and operation of ANN.

Hormik et al. [24] has proved a three layered feed-forward neural network with back propagation algorithm can map any non-linear relationship with a desired degree of accuracy. In this study, a three layer back propagation network (figure 1) is developed to predict viscosity, where the transfer functions in hidden and output layer are sigmoid and linear, respectively. The four variables (temperature, particle volume concentration, particle size and fluid base viscosity) are chosen as input to the BPNN, the nanofluid viscosity is the output of the network.

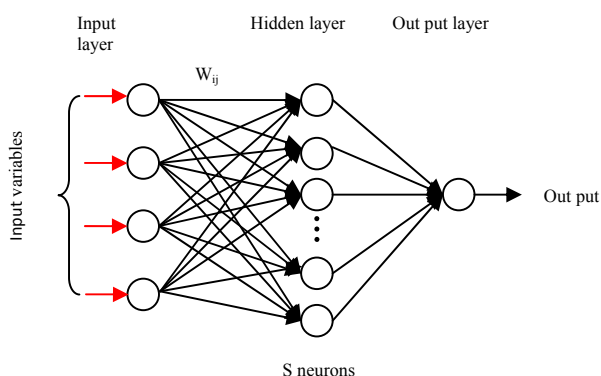


Fig. 1 architecture of three layer back propagation neural network for nanofluids viscosity prediction

### IV. THE GA TECHNIQUE

GA has been proved to be capable of finding global optima in complex problems by exploring virtually all regions of the state space and exploiting promising areas through mutation, crossover and selection operations applied to individuals in the populations [25]. It applies selection, crossover and mutation operators to construct fitter solutions. A genetic algorithm processes the populations of chromosomes by replacing unsuitable candidates according to the fitness function. It can solve how to optimize weight and threshold of BP network. It is employed to optimize the weight and threshold of the network in this study. The operation is divided into five steps as follows.

#### A Initialization of population

After coding the weights  $W_{ij}$  and thresholds  $b_i$  of the network, chromosome is generated at random and makes up an initial population. We start iterative search using initial population as a start point. Finally, population size, selection probability, crossover probability and mutation probability are determined by experiments.

#### B Fitness function

Fitness function is an important principle on evaluating individual. In this study, the fitness is function of the average deviation between experimental and predicted values of viscosity ( $\mu$ ). For do this, firstly the objective function defined in Eq.( 1) According to mean square error of neural network. The fitness value of a chromosome is calculated using the objective function BPNN architecture. F is given by

$$\delta (W_{ij}, b_i) = \frac{1}{N} \sum_{i=1}^N (\mu_i^{\text{exp}} - \mu_i^{\text{net}})^2 \quad (1)$$

$$F_{\text{fitness}} = \frac{1}{1 + \delta} \quad (2)$$

Where N stands for the number of the training data,  $\mu_i^{\text{cal}}$  is the expected output, and  $\mu_i^{\text{exp}}$  is the experimental output.

#### C Selection operation

The selection operation is to choose the individual who has the large fitness from the population. It has the chance to propagate offspring. A roulette wheel selection method is used to choose new individual in this study..

#### D Crossover operation

The crossover operation for GA creates variation in the population by producing new offspring that consists of the parts taking from each parent. It is that some genes of two chromosomes are changed to produce new individual. In this study, two parental chromosomes and bunch's crossing position are determined by random.

#### E Mutation operation

The mutation operation introduces random changes in structure in the population. It is to change some values of chromosomes of weight and threshold with little probability. It cannot only avoid some information be lost perpetually resulting from selection and crossover operation, but also ensure validity of genetic arithmetic. By using the above genetic algorithm operation, appropriate network weight and threshold are obtained

### V. BPNN LEARNING BASED ON GA OPTIMIZATION

According to GA method the optimization is to maximize the fitness values ( $F_{\text{fitness}}$ ) which would lead to the minimization of the total mean squared error from Eq. (1). This makes optimize the weights and thresholds of the BPNN and successively BPNN be achieved in training.

The BPNN learning algorithm composed of two stages: firstly employing GA to search for optimal or approximate optimal connection weights and thresholds for the network, then uses the back-propagation learning rule and training algorithm to adjust the final weights and thresholds. The operations are as follows:

The BPNN weights  $W_{ij}$  and thresholds  $b_i$  are initialized as genes of chromosome, and then the global optimum is searched through selection, crossover and mutation operators of genetic algorithm. This procedure is completed by applying a BP algorithm on the GA established initial connection weights and thresholds. If the BP network's total mean squared error is larger than the expected error, the weights and thresholds will be updated; otherwise, they are saved as initial value of BP network training. After that, they are further adjusted under BP learning rule to the best results, by which the viscosity can be accurately predicted. The overall GA-NN optimization, training, learning and prediction procedure for the experimental dataset is represented in figure 2.

## VI. TRAINING AND TESTING DATABASE

Experimental data of Nguyen C.T., et al. [26] for viscosity in two nanofluids  $Al_2O_3-H_2O$  and  $water-CuO-H_2O$  were used in this study. The summary of variables and those ranges of varieties are given in Table 1.

## VII. RESULTS AND ANALYSIS

The GA-NN is trained and tested based on the database. Randomly 75% of the data points were used for training and the rest 25% was used for testing. As the dimension and magnitude of original experimental data are different, the training and testing data should be normalized before fed to GA-NN.

It should be noted that a potential difficulty with the use of powerful non-linear regression methods is the possibility of over-training data. The proper selection of the number of neurons in the hidden layer can avoid the overtraining of neural network effectively. In the above developed technique in order to optimize the net, achieve generalization of the model and avoid over-training, the number of hidden nodes was determined by trial and error, and the sensitivity of the network to this number was studied. For the given training subset, we started with 2 neurons in the hidden layer and gradually increased the number neurons. the performance of network in training phase (simulating the input-output response embedded in the training subset) was observed to increase with increasing number of neuron, while the performance of network in testing (simulating the input-

output response embedded in the testing subset) was observed to be optimum at an optimal number of hidden neurons. As network was used for generalizing an input-output response, network testing was given higher preference, and the use of more than the optimal number of hidden nodes was discouraged. For this study, the mean square error (MSE) was chosen as a measure of the performance of the nets. The network model with a hidden layer and seven neurons in hidden layer resulted in the best prediction.

We run GA-NN algorithm using initial population as a start point for experimental data set of two nanofluids. Finally, population size, selection probability, crossover probability and mutation probability are determined by trial. They are 20, 0.8, 0.87 and 0.001, respectively.

As using the optimized neural network model to predict the effective viscosity, the comparisons between predictive values and experimental values for testing data are shown in figures 3 and 4.

The prediction results of the GA-BPNN model are shown in Table 2. The results demonstrated good agreement between the predicted and the experimental values of diffusivity by relatively low mean absolute errors (MARE=0.0122 and 0.0177), and high correlation coefficients  $R=0.999$  for  $Al_2O_3-H_2O$  and  $CuO-H_2O$ , respectively.

The computational results were compared with the results obtained using a conventional back-propagation neural network (BPNN). For comparison of GA-NN and conventional neural network based on average error criteria we defined advantage of GA-NN as follow.

$$Ad = \frac{MARE_{BPPN} - MARE_{GA-NN}}{MARE_{BPPN}} \times 100 \quad (3)$$

The analysis of results showed that the mean absolute relative error (MARE) corresponding to conventional BPN for two nanofluids,  $Al_2O_3-H_2O$  and  $CuO-H_2O$ , are 0.0296 and 0.0270 respectively. According to Table 2 and using formula (3), for  $Al_2O_3-H_2O$ ,  $Ad= 58\%$ , it means GA-NN model can reduce the error by 58%. Similarly for  $CuO-H_2O$ ,  $Ad= 34\%$ . This is further proof that the proposed procedure can build a useful and robust model.

So the conclusion can be drawn that the proposed GA-NN model has more accurate prediction ability over two nanofluids,  $Al_2O_3-H_2O$  and  $CuO-H_2O$ , but slightly less accuracy can be seen in the case of  $CuO-H_2O$ .

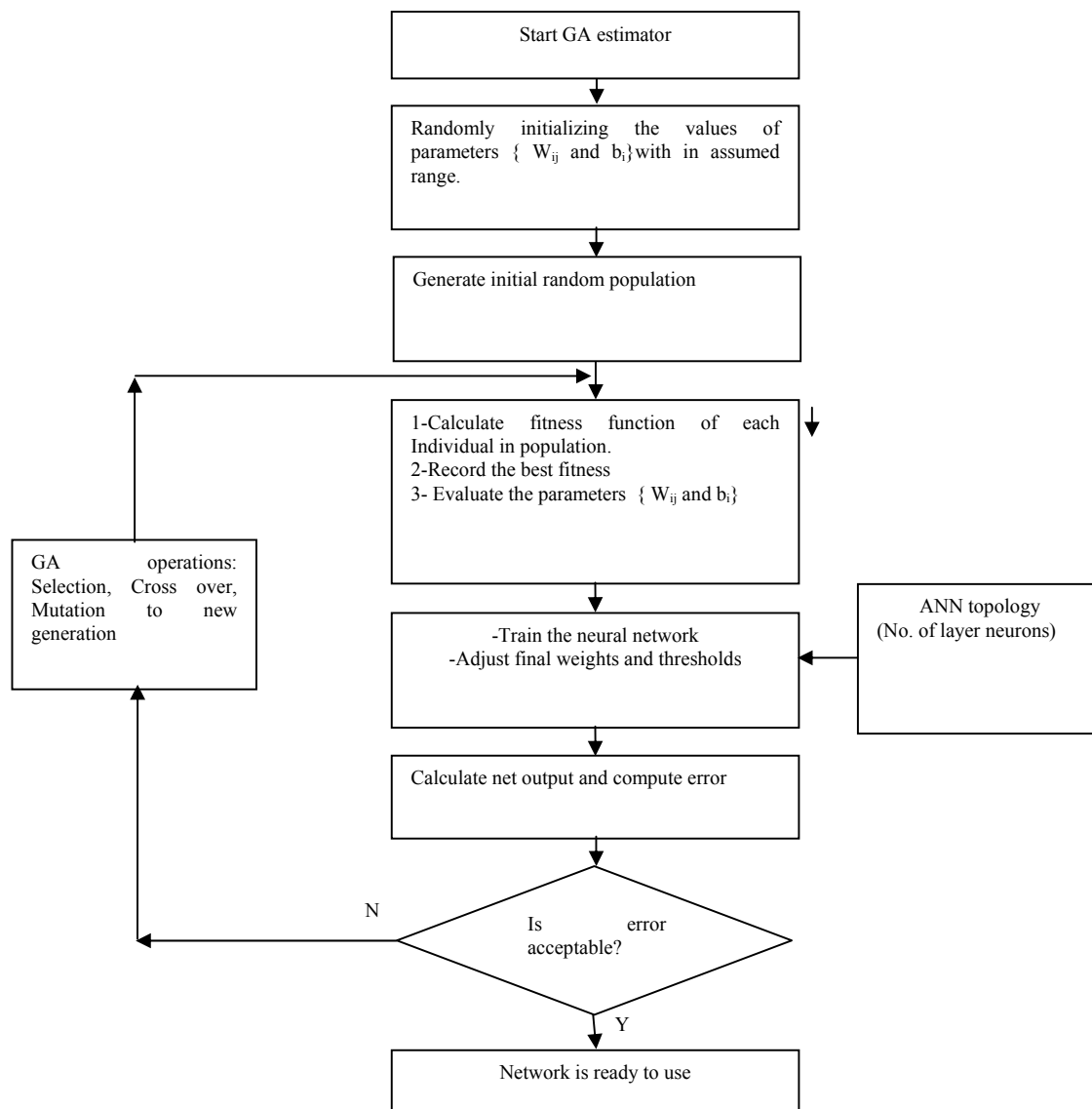


Fig. 2 The block diagram of hybrid genetic algorithm and neural network (GA-NN) procedure to prediction of nanofluids

TABLE I SUMMARY OF THE NANOFUIDS VISCOSITY DATASET CHARACTERIZATION

nanofluids	Temperature [K]	Volume fraction	Particle size(nm)	Base fluid viscosity (pa.s)	Nanofluids viscosity(pa.s)
Al2O3-H2O	293.15-343.15	0.005-0.094	13-47	0.00039-0.0010	0.00044-0.0022
CuO-H2O	278-339.25	0.050-0.150	29	0.00042-0.00151	0.00046-0.288

TABLE II RESULTS OBTAINED FOR TEST DATA MODELED WITH GA-NN AND BPNN MODEL FOR  $Al_2O_3$ - $H_2O$  AND  $CuO$ - $H_2O$  NANOFUIDS, WITH RESPECTIVE ERRORS

$Al_2O_3$ - $H_2O$			$CuO$ - $H_2O$		
Effective viscosity[Pa.s]	GA-NN	BPNN	Effective viscosity[Pa.s]	GA-NN	BPNN
Experimental	ARE %	ARE%	Experimental	ARE%	ARE%
0.00068	1.44	4.58	0.00312	0.37	1.44
0.00205	0.83	0.57	0.00188	1.73	0.71
0.00088	1.27	0.10	0.00046	0.01	1.22
0.00045	1.49	1.32	0.00468	0.41	0.27
0.00092	0.60	4.63	0.00176	1.39	2.39
0.00145	1.35	6.49	0.00092	0.42	3.52
0.00094	0.29	1.09	0.00257	0.41	1.19
0.00117	0.53	5.84	0.00207	2.55	8.04
0.00064	0.74	4.92	0.00710	2.24	2.41
0.00065	2.76	4.16	0.00185	3.84	3.02
0.00109	1.64	0.73	0.00101	2.45	0.61
0.00240	0.87	2.76	0.00241	0.74	0.02
0.00092	1.82	6.46	0.00049	2.97	2.75
0.00047	1.00	0.90	0.00800	2.68	3.39
0.00109	2.49	2.74	0.00142	0.50	2.98
0.00048	0.29	0.94	0.00055	5.53	9.18
0.00127	0.15	5.47	MARE	1.77	2.70
0.00063	1.82	3.78			
0.00369	0.34	0.26			
0.00209	1.57	1.16			
0.00107	0.61	1.59			
0.00136	0.38	1.84			
0.00118	0.44	1.79			
0.00150	0.60	2.14			
0.00105	1.55	6.23			
0.00177	0.54	1.82			
0.00083	0.20	0.54			
0.00088	1.33	7.95			
0.00345	0.84	1.00			
0.00105	0.91	5.58			
0.00055	6.10	3.35			
0.00074	2.52	1.33			
0.00142	1.94	3.55			
0.00092	0.86	4.37			
0.00299	1.16	3.06			
0.00220	0.61	1.46			
MARE	1.22	2.96			

In the following, we compared the results obtained by GA-NN with other analytical and empirical correlations.

Figure 5 compares the temperature variation of the predicted results of  $Al_2O_3$ - $H_2O$  and  $CuO$ - $H_2O$  nanofluids with the experimental results and with other models [14, 15, and 26]. As is shown the GA-NN model could well predict the effect of the temperature variations on the nanofluid viscosity. There are a great deal of deviation between other models prediction and experimental data especially in figures 5a, 5c and 5d. In Enistien and Bachelor model the effect of

nanoparticle is not considered in effective viscosity correlation. Therefore as showed in figures 5a, 5c and 5d, the accuracy of those models reduce seriously in large volume fraction of nanoparticle in suspension.

In Masoumi et al.[15] the model parameters are adjusted for limited nanoparticle size, consequently this model not usable for wide range of nanoparticle size.

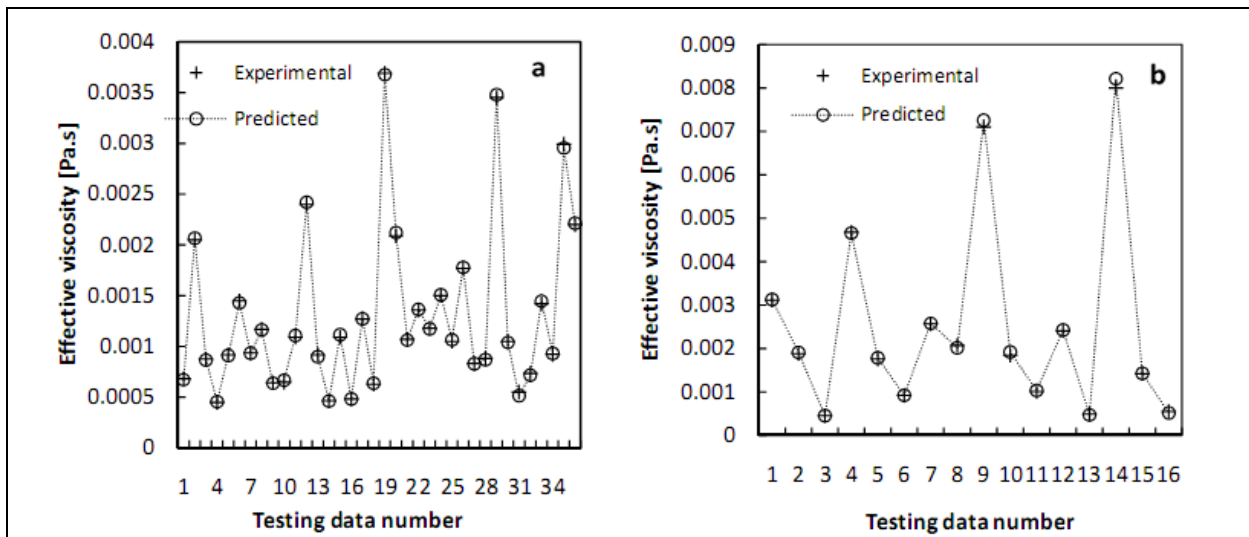


Fig. 3 Comparisons between predictive values and experimental values for testing data a) the predicted results of  $\text{Al}_2\text{O}_3\text{-H}_2\text{O}$  nanofluid; b) the predicted results of  $\text{CuO-H}_2\text{O}$  nanofluid

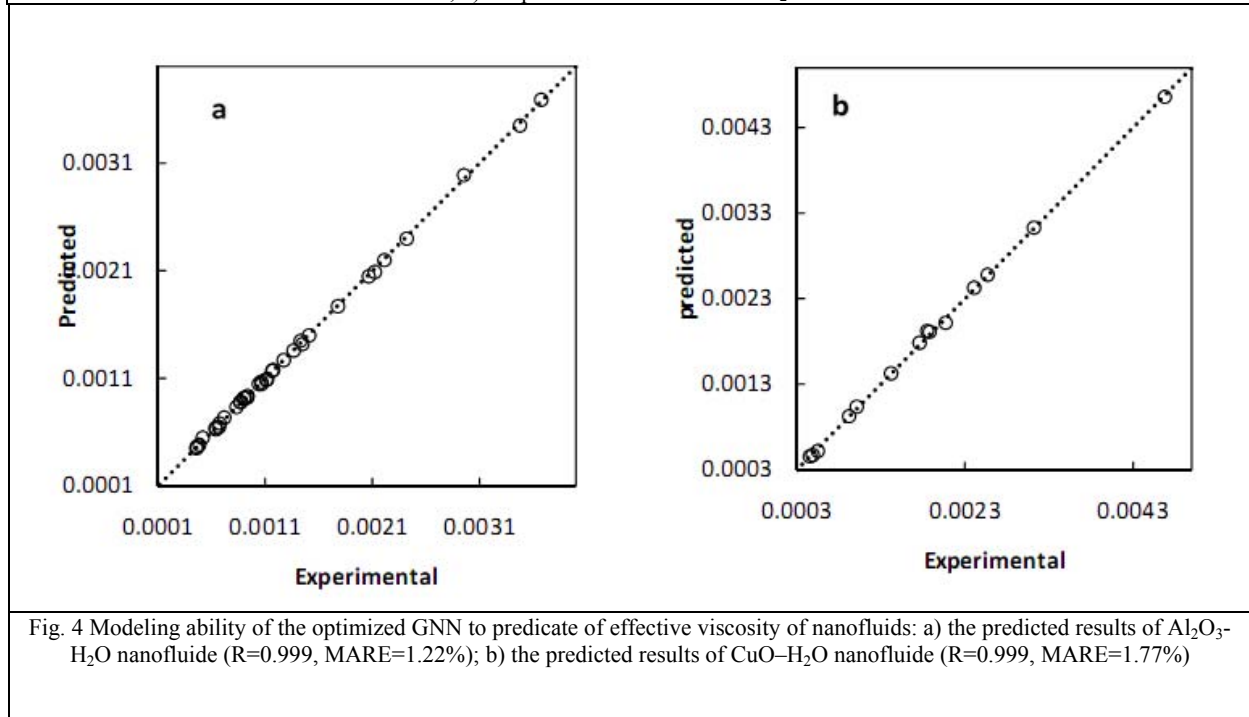


Fig. 4 Modeling ability of the optimized GNN to predicate of effective viscosity of nanofluids: a) the predicted results of  $\text{Al}_2\text{O}_3\text{-H}_2\text{O}$  nanofluid ( $R=0.999$ ,  $\text{MARE}=1.22\%$ ); b) the predicted results of  $\text{CuO-H}_2\text{O}$  nanofluid ( $R=0.999$ ,  $\text{MARE}=1.77\%$ )

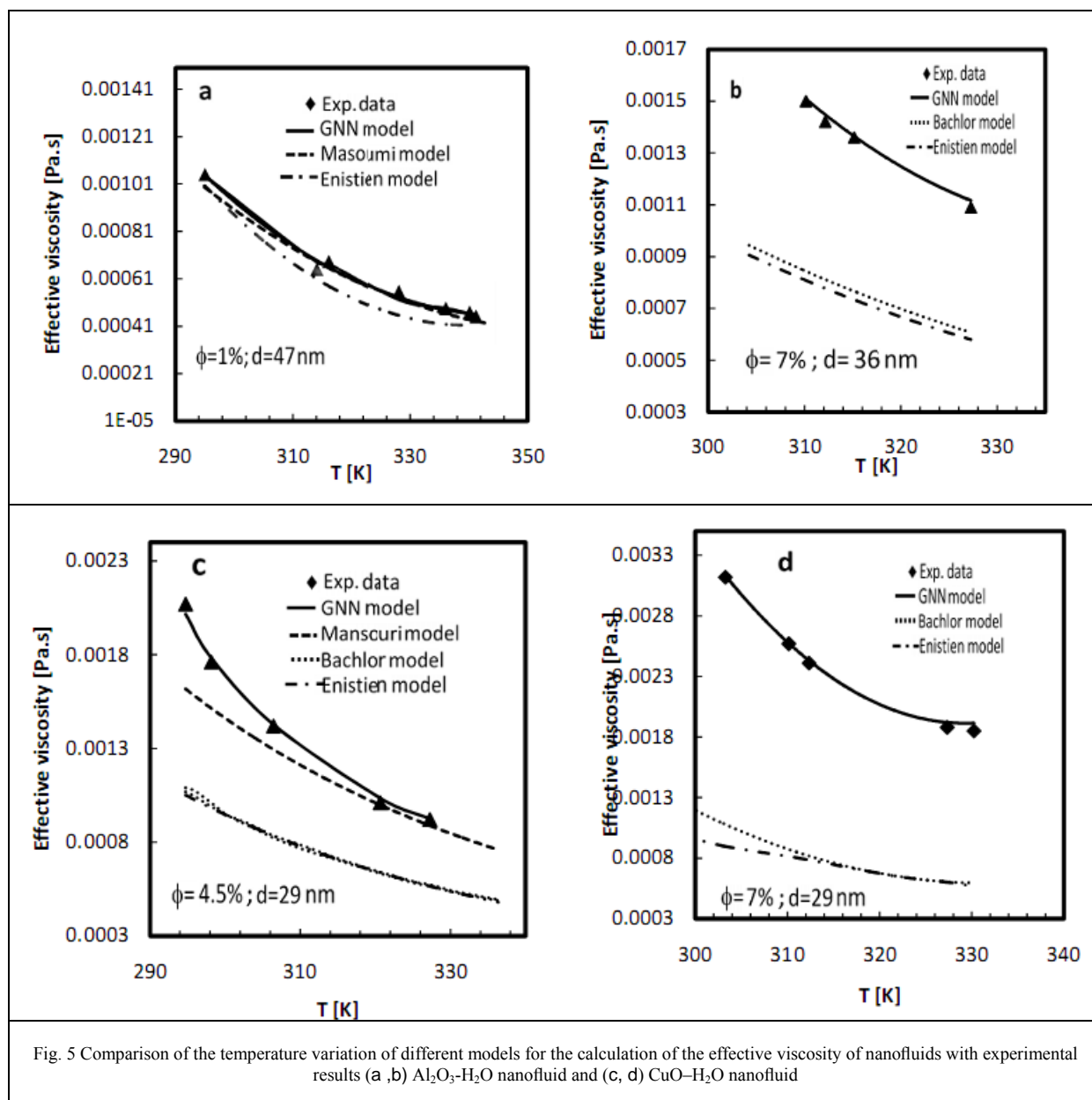


Fig. 5 Comparison of the temperature variation of different models for the calculation of the effective viscosity of nanofluids with experimental results (a, b)  $\text{Al}_2\text{O}_3\text{-H}_2\text{O}$  nanofluid and (c, d)  $\text{CuO-H}_2\text{O}$  nanofluid

### VIII. CONCLUSION

In this study we successfully employed the hybrid neural network and genetic algorithm for the prediction of the effective viscosity of nanofluids. The model could calculate the effective viscosity as a function of the temperature, the mean particle size, the nanoparticle volume fraction and the base fluid viscosity. The GA-NN due to advantages including relatively low MARE in prediction (1.22% for  $\text{Al}_2\text{O}_3\text{-H}_2\text{O}$  and 1.77% for  $\text{CuO-H}_2\text{O}$  nanofluids), the high correlation coefficient between predicted and experimental value is a powerful tool for accurate determination of viscosity of nanofluids. GA-NN model results were compared with conventional back propagation neural network (BPNN), for this purpose an advantage factor (Eq. 3) base on GA-NN

model was defined. For  $\text{Al}_2\text{O}_3\text{-H}_2\text{O}$  nanofluids,  $A_d = 58\%$ , it means GA-NN model can reduce the error by 58%, for  $\text{CuO-H}_2\text{O}$ ,  $A_d = 34\%$ . This is another proof that the proposed MDNN can build a useful and robust model. Compared with the other theoretical and imperial models that are available in the literature, the presented model in general has higher accuracy and precision

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