A Wind Farm Reduced Order Model Using Integral Manifold Theory

M. Sedighizadeh, and A. Rezazadeh

Abstract—Due to the increasing penetration of wind energy, it is necessary to possess design tools that are able to simulate the impact of these installations in utility grids. In order to provide a net contribution to this issue a detailed wind park model has been developed and is briefly presented. However, the computational costs associated with the performance of such a detailed model in describing the behavior of a wind park composed by a considerable number of units may render its practical application very difficult. To overcome this problem integral manifolds theory has been applied to reduce the order of the detailed wind park model, and therefore create the conditions for the development of a dynamic equivalent which is able to retain the relevant dynamics with respect to the existing a.c. system. In this paper integral manifold method has been introduced for order reduction. Simulation results of the proposed method represents that integral manifold method results fit the detailed model results with a higher precision than singular perturbation method.

Keywords—Wind, Reduced Order, Integral Manifold.

I. INTRODUCTION

WIND energy conversion system (WECS) have been identified as promising renewable energy sources mainly due to either its negligible impact on the environment and pre-competitive production costs. The advances in technology in this field in recent years have confirmed the expectations placed in this form of energy conversion.

The production of a significant amount of power from the wind has required both the development of larger more efficient and reliable wind turbines and the use of more than one machine at each site constituting the popular wind farms. Currently, the most used configuration of wind parks consists of a set of wind turbines each one driving a double output induction generator (DOIG) or synchronous machine [1], which, in turn, are assembled in group(s) and directly coupled to the existing a.c. system.

Actually, wind energy being a reliable source of energy on a year to year basis, is an intermittent sources of energy on a day to day basis, meaning that it is a non-dispatch able form of energy. Moreover, the power spectral density (PSD) of the WECS power output fluctuations vary in amplitude over a wide frequency range which may have a non-negligible

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impact in the quality of the utility power, especially in cases where wind is a significant component of the generation mix.

To address these technical issues it is necessary to possess design tools that are able to correctly simulate the impact of the wind energy penetration in the utility distribution grids. Currently these tools do exist whenever the case of a single WECS is to be assessed.

In order to give a net contribution to solve the problems that are still pending , namely in what concerns the impact of the integration of wind parks in the utility distribution system, some computational tools have been developed with the aim of assisting both wind park and distribution system planners and designers. These computational tools are based on models that are able to accurately simulate the behavior of wind parks under transient situations.

The paper is concerned with the development and application of wind parks reduced order Models that are able to simulate relevant dynamics of the park with respect to the utility system, henceforth denoted by Wind Park reduced order models. This is an important issue, since to study the interaction between a wind park and the gird the inclusion of the fast dynamics does not add relevant information when compared with the use of reduced order models. Therefore, the need for wind park dynamic equivalents which are able to retain the relevant dynamics of the park with respect to the utility grid , taking into account the effects of wind speed fluctuations.

Firstly, a wind park detailed model will be presented. Then, following previous studies on this domain, integral manifolds technique will be applied to the model in order to achieve a reduced order model which describes with a reasonable degree of accuracy the transient behavior of the wind park in what concerns the interaction with the grid.

II. WIND PARK FULL ORDER MODEL

Detailed models previously developed for each component of the system have been conveniently adapted and linked together in order to form an integrated wind park detailed model[2].The different elements modeled, namely , wind turbine , induction generator , reactive power compensation system , transformers , interconnection feeder and possible local loads connected to the feeder , are shown in Fig. 1.

A wind model able to generate correlated wind time series was developed. Currently, this model addresses the wind turbine side-by- side and along-wind configurations and is being improved in order to take account for wake effects.

World Academy of Science, Engineering and Technology
International Journal of Energy and Power Engineering
Vol:2, No:1, 2008

A wind turbine model was built after the blade aerodynamic characteristics and taking into account the effects of variable rotational speed, thus enabling torque/speed characteristics to be obtained. An improvement of this model is currently being performed and was not used in this paper. The new wind rotor model has a flexible approach enabling these blades to have both flap and lead-lag freedom concentrated on the hub.

Particular care was taken when considering the model of the double cage DOIG shown in Fig. 2. A double cage type of DOIG was considered with saturation effects, as this is the type of induction machine generally used in WECS. Details on the park model can be found in following:
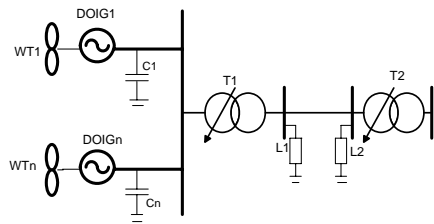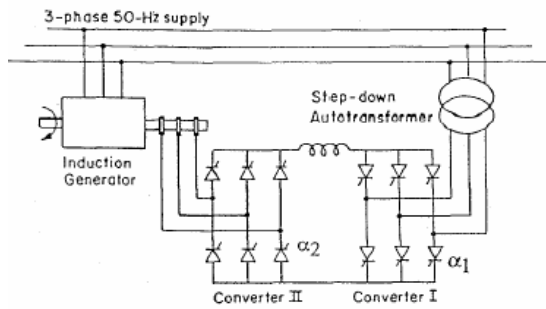

Fig. 1 System studied

Fig. 2 Schematic representation of DOIG

A. Wind and Wind Turbine Models

A wind turbine model that considers the rotor as a rigid body was used, the details of which can be found in [2]. Each wind rotor performance was described by a characteristic mechanical torque equation $T_m = f(\omega_r, \upsilon)$ that accounts both for the shaft rotational speed, ω_r, and the wind speed, υ, as follows:

$$T_{mi} = f(\omega_{ri}, \upsilon_i) = A_i(\upsilon_i)\omega_{ri} + B_i(\upsilon_i) \qquad (1)$$

where

$$A_i(\upsilon_i) = k_{a1i}\upsilon^{ea1i} + k_{a2i}\upsilon^{ea2i} + k_{a3i}\upsilon^{ea3i}$$

$$B_i(\upsilon_i) = k_{b1i}\upsilon^{eb1i} + k_{b2i}\upsilon^{eb2i} + k_{b3i}\upsilon^{eb3i}$$

In order to provide the simultaneous wind velocity time series at the various turbine locations, including not just the average wind velocity but also the superimposed turbulence, a wind model was developed and presented in [2].

B. Double Cage DOIG Model

The DOIG model is a generalization of the model presented in [3] to the case of a number n of double cage DOIGs. Standard assumptions of the generalized theory of electrical machines are adopted and the resultant constant-parameter model of a double cage DOIG is given with the following set of complex differential and algebraic equations:

$$\begin{aligned}
\overline{U}_s &= -R_s\overline{i}_s + d\overline{\phi}_s/dt + j\omega_a\overline{\phi}_s \\
\overline{U}_r &= -R_{r1}\overline{i}_{r1} + d\overline{\phi}_{r1}/dt + j(\omega_a - \omega_r)\overline{\phi}_{r1} + R_c(\overline{i}_{r1} + \overline{i}_{r2}) \\
0 &= -R_{r2}\overline{i}_{r2} + d\overline{\phi}_{r2}/dt + j(\omega_a - \omega_r)\overline{\phi}_{r2} + R_c(\overline{i}_{r1} + \overline{i}_{r2}) \\
T_e &= p\,\mathrm{Im}(\overline{\phi}_s^*\,\overline{i}_s) \\
\overline{\phi}_s &= -L_s\overline{i}_s - L_m(\overline{i}_{r1} + \overline{i}_{r2}) \\
\overline{\phi}_{r1} &= -L_{r1}\overline{i}_{r1} - L_{12}\overline{i}_{r2} - L_m\overline{i}_s \\
\overline{\phi}_{r2} &= L_{r2}\overline{i}_{r2} - L_{12}\overline{i}_{r1} - L_m\overline{i}_s
\end{aligned} \qquad (2)$$

Due to existence of the common end-ring in the double-cage induction machine used in this study, equations (2) contain terms which describe voltage drop on common resistance R_c.

The model is developed based on the assumptions that it is possible to decompose the total flux linking each winding into leakage and magnetization components, and that it is possible to characterize the machine through a unique magnetization characteristic.

C. Swing Equation

For each unit i, the rotor speed ω_r may be evaluated using the swing equation:

$$\omega(T_m - T_e) = 2H(d\omega_r/dt) \qquad (3)$$

III. REDUCTION ORDER PRELIMINARY ANALYSIS

In a first step, some of the methodologies currently available to reduce the order of power systems models have been applied to the specific case of wind parks. With the aim of gaining some insight regarding the behavior of each technique, a steady –state preliminary analysis has been carried out, using linear models in order to take advantage of its inherent simplicity [4].

Modal truncation is one first reduction schemes that has been applied to electric power systems [5]. This technique is based on the pole location of the system. The state variables are transformed in modal variables and the fast decay poles and/or those associated with high frequencies are neglected, thus enabling a reduction in the order of the system.

Balanced reduction techniques take a slightly different approach, because they are based in the input/output behavior of the system [6]. Actually, the original state-space system is transformed into a new representation that has the property that each state-space variable is both controllable and observable. In order to achieve a reduced order model, states that are strongly influenced by the inputs and strongly connected to the outputs are retained, whereas states that are weakly controllable and observable are truncated.

World Academy of Science, Engineering and Technology
International Journal of Energy and Power Engineering
Vol:2, No:1, 2008

Another method used in power systems order reduction is the so called optimal Henkel –norm approximation [7]. This criterion tries achieving a compromise between a small worst case error and a small energy error.

Another technique used in power system as singular perturbations decomposes the system according to its fast and slow dynamics and then lowers the model order by first neglecting the fast dynamics phenomena [8]. The effect of fast dynamics are then reintroduced as boundary layer correction calculated in separated time scales, which leads to correct static gains.

The technique known in the literature is the concept of integral manifolds; a nonlinear generalization of the notion of invariant subspace in linear systems [9].This paper employs the manifold concept as a tool for reduced order modeling and decomposition of Wind Park.

Both Henkel-norm and balanced reduction methods, although producing very good approximation of transient response, have the drawback of high reduction errors at low frequencies, due to the intrinsic mismatch between the DC gains of the full and reduced order models. Therefore, these methods are not suitable to steady –state applications.

The applied modal based reduction techniques make the discarded dynamic behavior static, so that the DC gain of the reduced model is equal to the original model one. Therefore, this method, which actually neglects the fast dynamics phenomena, is thought to be the best suited to steady-state applications, but a good performance in what concerns the transient behavior is not expected. Moreover due to its inherent lack of a clear criterion for determining the proper order of the reduced –order model, it becomes strongly dependent on the specific system to which it is applied. In fact, when the issue of neglecting the so called fast poles is to be assessed one has to be extremely cautious and a pre-analysis of the characteristics of the system is required.

Singular perturbations seem to be a more prominent reduction technique. Due to the fact that the effects of fast dynamics are reintroduced in the reduced system, this technique seems to be more general, namely in what concerns the prediction of steady – state and transient behavior of the systems. In addition to the agreement between full and reduced order results, the following advantages should be mentioned:

The reduction order methodology retains the physical meaning of the variables.

The method self contains the procedures to systematically improve the answers of the simplified mode.

The integral manifolds methods has all of advantages are mentioned for singular perturbation method, in addition accuracy for this method is very excellent. As integral manifolds have shown to be the most promising reduction-order technique, it has been selected as the basis for the development of the dynamic equivalent .Finally accuracy for both methods is compared in paper.

IV. INTEGRAL MANIFOLDS BACKGROUND [10]

A smooth s-dimensional surface S in the n-dimensional space R^n is defined by $m=n-s$ independent algebraic or transcendental scalar equations. In their simplest form , these equations express certain m coordinates z as m explicit functions of the remaining s coordinates x , that is they define S by its graph:

$$S : z=h(x) , z \in R^m ; x \in R^s ; m+s=n \tag{4}$$

It is assumed that, for all x in a domain of practical interest, $\partial h/\partial x$ exists and has full rank m. For approximated constructions of $h(x)$ pursued in this paper it will also be assumed that higher order derivatives of $h(x)$ exist and are continuous. In a more general situation the surface S may vary with time t , then

$$S_t: z=h(x,t) \quad , z \in R^m ; x \in R^s ; m+s=n \tag{5}$$

It will be assumed that $\partial h/\partial t$ exists and is continuous over an interval of interest $t \in (t_0,t_1)$, preferably infinite:
$t_1 \to \infty$

Let us now use the same coordinate's z and x to describe a dynamic system D_t in R^n:

$$z^\bullet = g(x,z,t),.........z \in R^m \tag{6}$$

$$x^\bullet = f(x,z,t),.........z \in R^s , m+s=n \tag{7}$$

Where appropriate differentiability assumptions are made about g and f. The surface S_t, and the system D_t have thus been introduced as two entities unrelated to each other. In this paper we explore a particularly useful relationship of s_t and D_t : when S_t is an integral manifold of D_t. The term invariant manifold will be used when such an integral manifold is time-invariant, that is when $\partial h/\partial t = 0$ and $S_t=S$ as in (4).

Manifold Definition: Surface S_t is an integral manifold of D_t if every solution $z(t)$, $x(t)$, of (6) – (7) which is in S_t at $t=t_0$.

$$Z(t_0)= h(x(t_0) , t_0) \tag{8}$$

Remains in S_t for all $t \in (t_0, t_1)$, That is

$$z(t) = h(x(t) , t) , t \in (t_0, t_1) \tag{9}$$

This definition furnishes a condition which can be used to verify whether $h(x,t)$ in (5) defines an integral manifold of(6)-(7).

Manifold condition: If $h(x,t)$ satisfies the partial differential equation:

$$\frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} f(x,h(x,t),t) = g(x,h(x,t)t) \tag{10}$$

The surface S_t given by (5) is an integral manifold of the dynamic system (6)-(7).
This condition is simply obtained by differentiating (9) with respect to t,:

$$\varepsilon z^\bullet = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x} x \tag{11}$$

and then substituting $z^\bullet$ and $x^\bullet$ from (6) to (7) . Once the existence of an integral manifold S_t of D_t has been established

World Academy of Science, Engineering and Technology
International Journal of Energy and Power Engineering
Vol:2, No:1, 2008

and its defining function $h(x,t)$ has been found , then the restriction of D_t to the manifold S_t is given by the s th-order system

$$x^{\bullet} = f(x,h(x,t),t), \qquad x \in R^s \tag{12}$$

which is obtained by the substation of $z=h(x,t)$ into (7) .

In addition to being a tool for reduced order modeling, the concept of an integral manifold is also a decomposition tool. A reduced order model (12) is a correct description of the dynamic D_t only when the intial state is in S_t, as in (8). When the initial state of D_t is not in S_t, the knowledge of the manifold function $h(x,t)$ continues to be useful by allowing us to replace the z-coordinates by the " off-manifold" coordinates η.

$$\eta = z - h(x,t) \quad \eta \in R^m \tag{13}$$

In terms of the new coordinates η and x the original system (6) – (7) becomes:

$$\eta^{\bullet} = g(x,\eta + h(x,t),t) - \frac{\partial h}{\partial x} f(x,\eta + h(x,t).t) - \frac{\partial h}{\partial x} \tag{14}$$

$$x^{\bullet} = f(x,h(x,t),t), \tag{15}$$

An advantage of this full order description of D_t over (6) – (7) is that now the manifold condition is simply η=0. The decomposition is achieved in the sense that on the surface S_t the subsystem (14) is at equilibrium. $\eta(t_0)=o$ implies $\eta(t)=0$ for all $t \in (t_0,t_1)$ and all x. the " off –manifold " / " in –manifold" description (14) –(15) is particularly helpful when the in-manifold behavior of D_t is of primary interest and the off-manifold variable is evaluated separately as a correction term. The analysis presented in the subsequent sections illustrates both conceptual and computational advantages of this nonlinear decomposition approach.

V. APPLICATION TO THE WIND PARK MODEL

The integral manifolds theory outlined in the previous section was applied to the case of the wind park detailed model.

The first step consists in the separation of the time variables in slow variables and fast variables, in order to be able to solve them in the appropriate time scales. The variables stator flux linkage, $\overline{\phi}_s$, first rotor circuit flux linkage, $\overline{\phi}_{r1}$,and speed rotor were considered as slow variables , the remaining ones (variables second rotor circuit flux linkage, $\overline{\phi}_{r2}$) were assumed as fast variable .

Another important issue is the definition of the small parameters. It is possible to prove [8] that the physical phenomena which allows the separation of the variables into slow and fast variables is the second rotor circuit constant time , $\varepsilon = L_{r2} / R_2$.

Under these conditions, to obtain the slow sub-system the wind park model has been rewritten to comply with the formulation given by (6) and (7). It should mentioned that

states x and z do represent the real variables, but actually they are associated with a different system in which $\varepsilon \neq 0$.

Finally, the approximations for x and z were obtained from equations (14) and (15), respectively.

VI. VALIDATION RESULTS

In order to evaluate the performance of the integral manifolds reduced order model (13) –(14) in describing the wind park transient behavior, some simulations have been carried on and the results compared with singular perturbations reduced order model[8] and full order non-linear detailed model.

The selected case-study targets the simulation of a fault in the a.c. system which causes the voltage dip 60% in generator terminal at t=50 msec and normal operation occurring 200 msec after .It has been assumed a two units wind park subject to correlated wind input.

Fig. 3 display the rotor speed, stator current, terminal voltage and electromagnetic torque for a given generator, using both the full detailed model and the slow sub-system (13) -(14) derived from the application of the integral manifolds method.

Following the variables separation criterion outlined above, the rotor speed has been considered as a slow variable, whereas both the stator current and terminal voltage has been considered as fast variables.
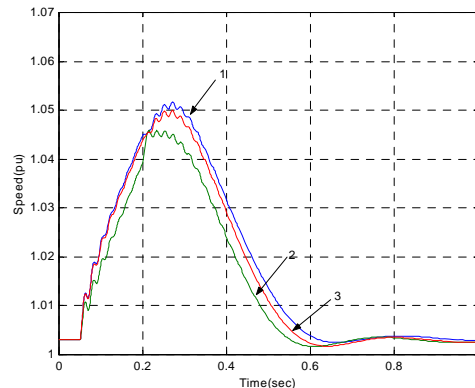
Fig. 3a Induction generator 2 rotor speed, (1) detailed model,

(2) Singular perturbation model, (3) integral manifolds model

World Academy of Science, Engineering and Technology
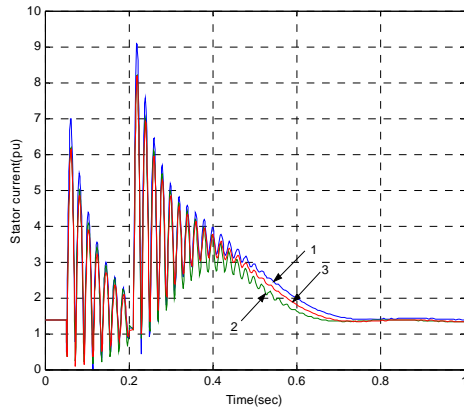International Journal of Energy and Power Engineering
Vol:2, No:1, 2008

Fig. 3b Induction generator 2 stator current, (1) detailed model,

(2) Singular perturbation model, (3) integral manifolds model
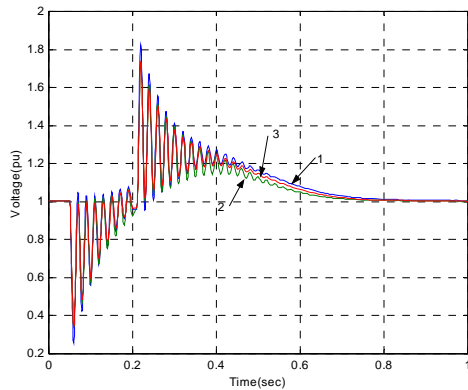

Fig. 3c Terminal voltage, (1) detailed model, (2) singular
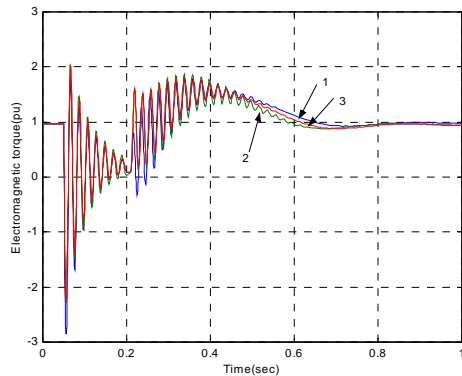perturbation model, (3) integral manifolds model

Fig. 3d Electromagnetic Torque (1) detailed model, (2) singular
perturbation model, (3) integral manifolds model.

VII. CONCLUSION

This paper presents an application of integral manifolds theory to reduce the order of a detailed wind park model. The final aim of this task is to obtain a dynamic equivalent of the wind park that retains the relevant dynamics of the park with respect to the utility grid.

The results achieved allow the conclusion that integral manifolds method is an excellent tool to derive dynamic equivalents of wind parks. The reduced order model reproduces with a high degree of accuracy the transient behavior results provided by the full order model.

Moreover, the computing time issue is satisfactory addressed. The slow and fast dynamics are analyzed in separated time scales, which allows the use of different time steps for the integration of the two sub-systems .Computing time for integral manifold method and singular perturbation method is 50% and 62% in contrast with detailed model respectively.

APPENDIX

A. List of Symbol

L_m, L – Magnetizing inductance and dynamic inductance

R, L – Resistance and inductance

R_c – Rotor end ring common resistance

U, i, ϕ – Instantaneous values of voltages, currents and flux linkages

ω_a, ω_r – Angular velocities of reference   frame and rotor (electrical)

H – Inertia coefficient

T_m, T_e – Mechanical torque and machine torque

s – Slip

ω – Stator angular frequency

γ – Leakage inductance and flux

$1,2$- subscripts for rotor upper and lower cage

B. Model Parameters

$V_{base} = 6.6KV, S_{base} = 4.173MVA, f_{base} = 50Hz$

Stator resistance (R_s): 7.4*e-3 Pu

Stator leakage inductance (L_s): 3.47*e-4 Pu

Rotor resistance (R_{r1}): 7.27*e-3 Pu

Rotor leakage inductance (L_{r1}): 3.58*e-4 Pu

Double cage resistance (R_{r2}): 1.36*e-1 Pu

Double cage inductance (L_{r2}): 2.63*e-3 pu

Magnetizing inductance (L_m): 0.0115 Pu

Lumped inertia coefficient (H):2 sec

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World Academy of Science, Engineering and Technology
International Journal of Energy and Power Engineering
Vol:2, No:1, 2008

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