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An Incomplete Factorization Preconditioner For LMS Adaptive Filter

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Abstract—In this paper an efficient incomplete factorization preconditioner is proposed for the Least Mean Squares (LMS) adaptive filter. The proposed preconditioner is approximated from a priori knowledge of the factors of input correlation matrix with an incomplete strategy, motivated by the sparsity patter of the upper triangular factor in the QRD-RLS algorithm. The convergence properties of IPLMS algorithm are comparable with those of transform domain LMS(TDLMS) algorithm. Simulation results show efficiency and robustness of the proposed algorithm with reduced computational complexity.

Keywords—Autocorrelation matrix, Cholesky's factor, eigenvalue spread, Markov input.

I. Introduction

THE least mean squares (LMS) algorithm , proposed by Widrow and Hoff in 1960, is the most widely used adaptive filtering algorithm in practice [1]. It has been extensively applied in adaptive signal processing and adaptive control because of its simplicity in implementation and its O(N)computational complexity[2]. The LMS algorithm and its variants are proven to be computationally simple and numerically robust, but have a drawback of converging slowly especially when the input autocorrelation matrix has high condition number. The eigenvalue spread of the autocorrelation matrix is a measure of the condition number [3], [4], and controls the convergence rate of the LMS based algorithms[1]. Preconditioning is a good remedy for to reduce the affect of correlated input signals on the performance of the algorithm[5].

A class of preconditioned adaptive filtering algorithms known as transform domain adaptive filters, introduced by [6], presented transform domain LMS (TDLMS) algorithm using data-independent orthogonal transforms. Afterwards Francoise Beaufays [3] designed an analytical demonstration to explain the effect of unitary data-independent transformation followed by power normalization on input autocorrelation matrix and then Farhang [1] presented thorough study of transform domain adaptive filters including its convergence behavior as well as efficient implementation, and described its resemblance with LMS-Newton algorithm. LMS- Newton and Normalized LMS (NLMS) are improved versions of conventional LMS algorithm and have better convergence properties than that of conventional algorithm. Mean square error and tap-weight error (misalignment) behavior of improved NLMS type algorithm is discussed in [7] by setting a trade off between fast convergence and low final misadjustment.

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In this paper we propose an incomplete factorization preconditioner for the LMS adaptive filtering algorithm. The preconditioner is formed by applying an incomplete strategy to the Cholesky's factor of an optimized autocorrelation matrix of the input signals. Our purpose is to design an efficient preconditioner at negligible computational cost, which should be able to take the convergence rate of conventional LMS adaptive filter to one of its modified variations, like TDLMS, or Newton-LMS etc. Analytical development of the preconditioner from first order Markov signals, and its application on the autocorrelation matrix is presented. The eigenvalue spread of transformed autocorrelation matrix, after application of preconditioner and power normalization, is an exact measure of the asymptotic eigenvalue spread of autocorrelation matrix transformed by DFT in TDLMS algorithm. Simulation results show better robustness of IPLMS adaptive filter at fairly low computational cost.

The paper is organized as follows: The LMS adaptive filter and its variants are presented in §II, incomplete factorization with preconditioning techniques and power normalization is described in §III. §IV shows the motivation and design of incomplete factorization preconditioner and the IPLMS algorithm followed by evaluation of exact eigenvalue spread of preconditioned autocorrelation matrix. Simulation results and comparison with TDLMS algorithm are given in §V, while concluding remarks are in last section.

II. LMS Adaptive Filter

Consider an FIR filter of length N with a tap-weight vector w_n, at instant n. The LMS algorithm minimizes the instantaneous objective function (MSE),

$$J(n) = e^2(n)$$

where $e(n) = s(n) - w_n^T a_n$. The vectors $a_i \in \mathcal{R}^N$ are formed by input signals $u(i)$ in such a way that

$$a_i = \begin{bmatrix} u(i) & u(i-1) & \cdots & u(i-N+1) \end{bmatrix}^T;\ 1 \le i \le n$$

Using input signals we can define the $n \times N$ data matrix A_n as:

$$A_n = \begin{pmatrix} u(1) & 0 & \cdots & 0 \\ u(2) & u(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ u(N) & u(N-1) & \cdots & u(1) \\ u(N+1) & u(N) & \cdots & u(2) \\ \vdots & \vdots & \ddots & \vdots \\ u(n-1) & u(n-2) & \cdots & u(n-N) \\ u(n) & u(n-1) & \cdots & u(n-N+1) \end{pmatrix}$$

Define $X = E\left[a_n a_n^T\right]$ as the autocorrelation matrix of input vector a_n, and $p = E\left[a_n s_n\right]$ as the crosscorrelation vector.

The mean square error $J(n)$ is minimized by continuously updating the weight vector w_n as each new input signal is received, according to the equation:

$$w_{n+1} = w_n + 2\mu e(n)\, a_n \tag{1}$$

where μ is a positive constant that controls the rate of convergence. To ensure the stability of the adaptive process, value of μ must satisfy the condition:

$$0 < \mu < \frac{1}{\lambda_{\max}} \tag{2}$$

$\lambda_{\max}$ is the largest eigenvalue of the autocorrelation matrix $X = E\left[a_n a_n^T\right]$ and is given by the maximum of the power spectrum of input signal a_n.

For stationary input and an appropriate choice of μ, the minimum value of $e(n)$ generates a Cauchy sequence $\{w_n\}_{n=1}^{\infty}$ from (1) in $\mathcal{R}^N$. But since $\mathcal{R}^N$ is a Banach space [8], there exists an optimum weight vector $w_o \in \mathcal{R}^N$, such that $w_n \to w_o$ as $n \to \infty$. Value of w_o, as given by Wiener- Hopf equation [1], is:

$$w_o = \mathrm{X}^{-1} p \tag{3}$$

Let us define the misalignment vector m_n [7] , as :

$$m_n = w_n - w_o$$

Then $m_n = \|m_n\|_2 = \|w_n - w_o\|_2 \to 0$ as $n \to \infty$.

It is not difficult to show that under independence assumption[1],

$$E[m_{n+1}] = (I - 2\mu X)\, E[m_n] \tag{4}$$

This relation is used in literature [1],[2] to show that convergence behavior of the LMS algorithm is directly linked to the eigenvalue spread of X. For highly correlated input, X has high eigenvalue spread, and convergence of the algorithm can be extremely slow. To improve the convergence speed , we need to reduce eigenvalue spread of X by using some decorrelation techniques. We may overcome this problem by employing preconditioning theory from numerical linear algebra . Here we briefly describe some algorithms which have been derived from conventional LMS algorithm by using techniques similar to that theory.

A. The LMS-Newton Algorithm

In this algorithm the input vector a_n, in error term $\mu\, e(n)\ a_n$ of (1), is preconditioned by an estimate X_n^{-1} of the inverse X^{-1} of input correlation matrix. The modified update equation is:

$$w_{n+1} = w_n + 2\mu e(n)\, X_n^{-1} a_n \tag{5}$$

It can be shown that the convergence characteristics of the LMS-Newton algorithm are independent of the eigenvalue spread of X. But it has an increased complexity of computing the inverse of input correlation matrix.

B. Normalized LMS Algorithm

The normalized LMS (NLMS) algorithm, which was developed as a constrained optimization problem [2], can be considered as a preconditioned LMS algorithm. The preconditioner $(\psi I + a_n a_n^T)^{-1}$ is a regularized inverse of $a_n a_n^T$, where $\psi \approx 0$. The update equation is given by:

$$w_{n+1} = w_n + \mu e(n)\, (\psi I + a_n a_n^T)^{-1} a_n$$

Using matrix inversion lemma, we have

$$w_{n+1} = w_n + \frac{\mu}{\psi + a_n^T a_n} e(n)\, a_n \tag{6}$$

where ψ is selected to be small enough when compared with $a_n^T a_n$. NLMS has fast convergence as compared with the conventional LMS algorithm, but has a drawback of increased misadjustment.

C. TD-LMS Algorithm

Transform domain LMS algorithms is a class of robust preconditioned algorithms having good tracking capabilities in non stationary environments. Application of an orthogonal transform , followed by a power normalization step, has the ability to reduce the eigenvalue spread of input correlation matrix, which results in an increase of convergence speed of the algorithm [1]. Here we give a brief description of the TD-LMS algorithm.

The input vector a_n, and weight vector w_n are transformed to $\hat{a}_n = \mathrm{T} a_n$ and $\hat{w}_n = \mathrm{T} w_n$ respectively, through an orthogonal transform T. With error estimate$e(n) = s(n) - \hat{w}_n^T \hat{a}_n$, and power normalization

$$\sigma_n^2(i) = \beta\, \sigma_{n-1}^2(i) + (1-\beta)\, \hat{a}_n^2(i) \quad ; \quad i = 0, 1, \cdots, N-1,$$

where $0 < \beta < 1$, the weight vector update equation is:

$$\hat{w}_{n+1} = \hat{w}_n + 2\mu D^{-1} e(n)\, \hat{a}_n \tag{7}$$

with $D = diag\left[\sigma_n^2(0)\, ,\, \sigma_n^2(1)\, ,\, \cdots\, ,\, \sigma_n^2(N-1)\right]$.

An analytical approach [3] has shown significant decrease in the eigenvalue spread of the input correlation matrix of a first order Markov signal after application of discrete fourier (DFT) and discrete cosine (DCT) transforms, followed by power normalization.

The preconditioning technique, presented in the next section, uses an approximate Cholesky's factor of input correlation matrix as a preconditioner for the conventional LMS algorithm.

III. Incomplete Factorization Preconditioner and Power Normalization

The input autocorrelation matrix X plays an important role in convergence of the LMS algorithm. For stationary input and small μ, the convergence rate of the algorithm depends on eigenvalue spread of X. To overcome the problem of slow convergence, we would make use of a factorization preconditioner. Clearly the best preconditioner is the inverse of X , but it is computationally expensive.

Computing a factorization of a matrix A and using the factors as preconditioners for linear system $Aw = b$ is a common practice [9]. When dealing with a large system, complete factorization is expensive and requires a large amount of storage. Incomplete factorization helps to reduce these problems.

Consider the linear least squares problem of the form:

$$\min_{w \in \mathcal{R}^N} \|b - Aw\|_2^2$$

where A is an $m \times n(m \geq n)$ full rank matrix. It is possible to find a solution of the above equation by implicitly solving the normal equation:

$$A^T A w = A^T b \tag{8}$$

Since the condition number of the correlation matrix $X = A^T A$ is the square of that of A, therefore a slight increase in condition number of A can make the normal equation highly ill-conditioned. In such a situation, preconditioning is necessary for robustness. Incomplete factorization provides a good factorization preconditioner to reduce the condition number and improve the convergence speed of the system.

Incomplete QR factorization of A is an approximation of complete QR factorization of the form:

$$A = QR + E$$

where $Q \in \mathcal{R}^{m \times n}$ may not have perfectly orthonormal columns, $R \in \mathcal{R}^{n \times n}$ is an upper triangular matrix and $E \in \mathcal{R}^{m \times n}$ is the error matrix that can be made as sparse as we please. If $A \approx QR$, then $A^T A \approx R^T R$ and matrix R becomes an incomplete Cholesky's factor of the correlation matrix X. This Incomplete Cholesky's factor R can be used as a preconditioning matrix for the normal equation(8), that is,

$$(R^{-T} A^T A R^{-1}) R w = R^{-T} A^T b$$

or

$$\tilde{X} \tilde{w} = \tilde{b} \tag{9}$$

Where

$$\tilde{X} = R^{-T} A^T A R^{-1} = (AR^{-1})^T (AR^{-1})$$

$$\tilde{w} = Rw$$

$$\tilde{b} = R^{-T} A^T b = (AR^{-1})^T b$$

The closer the incomplete Cholesky's factor of $X = A^T A$ to its complete Cholesky's, the closer the condition number of $\tilde{X}$ is to 1. Since preconditioned normal equation (9) is obtained by premultiplying equation (8) by R^{-T}, therefore complexity $O(n^2)$ of inversion of nonsingular upper triangular matrix R has great importance in the computation of the preconditioner.

A. *Power Normalization*

Power normalization is another tactic to reduce the eigenvalue spread of input data. This step is performed in TD-LMS algorithm to normalize the input signals to the power of unity . We perform this step in our preconditioned LMS algorithm for the same purpose. For power normalization step, let $D = diag(\tilde{X})$, and transform (9) into:

$$(D^{-1/2} \tilde{X} D^{-1/2}) D^{1/2} \tilde{w} = D^{-1/2} \tilde{b}$$

The associated autocorrelation matrix is:

$$\tilde{X}^{\mathcal{N}} = D^{-1/2} \tilde{X} D^{-1/2} \tag{10}$$

It can easily be verified that the mean-square values of the transformed signals as well as the diagonal values of $\tilde{X}^{\mathcal{N}}$ are all clustered to unity [1].

IV. Preconditioned LMS Algorithm

Starting with preconditioned Weiner Hopf equation

$$R^{-T} \left(\mathrm{X} R^{-1} \right) R w_o = R^{-T} p$$

The filter update equation for preconditioned system is:

$$\tilde{w}_{n+1} = \tilde{w}_n + 2\mu e\left(n\right) \tilde{a}_n$$

where

$$e\left(n\right) = s\left(n\right) - \tilde{w}_n^T \tilde{a}_n$$

with filter output

$$\tilde{w}_n^T \tilde{a}_n = \left(R w_n\right)^T \left(R^{-T} a_n\right) = w_n^T a_n = y\left(n\right) \tag{11}$$

showing invariance of filter output under transformation R^{-T}.

Applying power normalization step on (11),

$$D^{1/2} \tilde{w}_{n+1} = D^{1/2} \tilde{w}_n + 2\mu e\left(n\right) D^{-1/2} \tilde{a}_n$$

After simplification, it becomes

$$\tilde{w}_{n+1} = \tilde{w}_n + 2\mu e\left(n\right) D^{-1} \tilde{a}_n \tag{12}$$

Defining a transformed misalignment as $\tilde{m}_n = \tilde{w}_n - \tilde{w}_o$, the convergence behavior can be studied by examining

$$E\left[\tilde{m}_{n+1}\right] = \left(I - 2\mu \tilde{X}\right) E\left[\tilde{m}_n\right] = \left(I - 2\mu R^{-T} X R^{-1}\right) E\left[\tilde{m}_n\right]$$

Thus we can get fast convergence for the input vectors having correlation matrix X close to $R^T R$, in which case the eigenvalue spread of matrix $R^{-T} X R^{-1}$ clusters around 1. But for systems with large filter order, computation of complete Cholesky's factor become expensive, and is not a good choice for preconditioner. An incomplete factorization , however, provides a better option by using certain dropping strategies to have a trade off between convergence speed and computational cost of the preconditioner.

A. Design and Motivation of Preconditioner

The preconditioner , presented here, is obtained by using a priori knowledge of the autocorrelation properties of input signals. As discussed earlier the convergence speed of LMS algorithm depends on the eigenvalue spread of the autocorrelation matrix X. For the Preconditioned algorithm, presented here, the convergence speed is determined by the eigenvalue spread of $R^{-T}XR^{-1}$. Clustering of the eigenvalue spread closed to 1 is made without compromising too much on the computational cost of the preconditioner. We propose a fixed Cholesky's factor R, that is a fairly good approximation of the Cholesky's factor of all the autocorrelation matrices of input signals until time instant n. Moreover , to reduce computational cost, we split R in to sub-blocks and form an incomplete upper triangular matrix by zeroing out all its elements except those in the upper triangular sub-blocks along the main diagonal. The motivation of the strategy is explained below.

Motivation comes from the sparsity patter of the Cholesky's factor obtained in the QRD-RLS algorithm [2]. The Cholesky's factor, R_n at instant n, is such that diagonals are almost constant, except the first element in each diagonal band. Moreover as we go away from the main diagonal, the elements reduce in magnitude and don't contribute much in further computations. Our incomplete factorization preconditioner R is a fixed approximation of R_n with a dropping strategy. For our dropping strategy, partition the block diagonal of R_n into p upper triangular sub matrices $R_n^{(L_i)}; (1 \leq i \leq p)$, of size $(L_i \times L_i)$ such that

$$N = \sum_{i=1}^{p} L_i$$

and

$$R = \lim_{n\to\infty} \begin{pmatrix} R_n^{(L_1)} & & & \mathbf{O} \\ & R_n^{(L_2)} & & \\ & & \ddots & \\ \mathbf{O} & & & R_n^{(L_p)} \end{pmatrix}$$

Here not all L_i are equal in general. It is important to note that R is more sparse and require less computation for inversion as compared with R_n.

B. Eigenvalue and Eigenvalue spread

To understand the effect of our preconditioner R on the eigenvalue spread of input correlation matrix, we consider the first order Markov signals, which are a very broad but simple class of signals. A Markov-1 input signal $a_n = \begin{bmatrix} u(n) & u(n-1) & \cdots & u(n-N+1) \end{bmatrix}^T$ of parameter $\alpha \in [0,1]$ has an autocorrelation matrix equal to

$$M(N) = \begin{pmatrix} 1 & \alpha & \alpha^2 & \cdots & \alpha^{N-1} \\ \alpha & 1 & \alpha & \cdots & \alpha^{N-2} \\ \alpha^2 & \alpha & 1 & & \\ \vdots & \vdots & & \ddots & \vdots \\ \alpha^{N-1} & \alpha^{N-2} & & \cdots & 1 \end{pmatrix}$$

It is a symmetric toeplitz matrix having Cholesky's factor

$$R(N) = \begin{pmatrix} 1 & \alpha & \alpha^2 & \cdots & \alpha^{N-1} \\ 0 & \sqrt{1-\alpha^2} & \alpha\sqrt{1-\alpha^2} & \cdots & \alpha^{N-2}\sqrt{1-\alpha^2} \\ 0 & 0 & \sqrt{1-\alpha^2} & \ddots & \alpha^{N-3}\sqrt{1-\alpha^2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sqrt{1-\alpha^2} \end{pmatrix}$$

In order to obtain an incomplete Cholesky's factor $\tilde{R}(N)$, split $R(N)$ in two sub-blocks of sizes L_1 and L_2. In that case filter order N is L_1+L_2. Choosing triangular blocks of size L_1 and L_2 along the diagonal of $R(N)$, the 2-block incomplete Cholesky's factor becomes:

$$R(N) = \begin{pmatrix} R(L_1) & \mathbf{O}_{(L_1\times L_2)} \\ \mathbf{O}_{(L_2\times L_1)} & R(L_2) \end{pmatrix}$$

where

$$R(L_1) = \begin{pmatrix} 1 & \alpha & \alpha^2 & \cdots & \alpha^{L_1-1} \\ 0 & \sqrt{1-\alpha^2} & \alpha\sqrt{1-\alpha^2} & \cdots & \alpha^{L_1-2}\sqrt{1-\alpha^2} \\ 0 & 0 & \sqrt{1-\alpha^2} & \cdots & \alpha^{L_1-3}\sqrt{1-\alpha^2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sqrt{1-\alpha^2} \end{pmatrix}$$

$$R(L_2) = \begin{pmatrix} \sqrt{1-\alpha^2} & \alpha\sqrt{1-\alpha^2} & \alpha^2\sqrt{1-\alpha^2} & \cdots & \alpha^{L_2-1}\sqrt{1-\alpha^2} \\ 0 & \sqrt{1-\alpha^2} & \alpha\sqrt{1-\alpha^2} & \cdots & \alpha^{L_2-2}\sqrt{1-\alpha^2} \\ 0 & 0 & \sqrt{1-\alpha^2} & \cdots & \alpha^{L_2-3}\sqrt{1-\alpha^2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sqrt{1-\alpha^2} \end{pmatrix}$$

It can easily be verified that inverse of $R(N)$ is:

$$R^{-1}(N) = \begin{pmatrix} R^{-1}(L_1) & \mathbf{O}_{(L_1\times L_2)} \\ \mathbf{O}_{(L_2\times L_1)} & R^{-1}(L_2) \end{pmatrix}$$

where,

$$R^{-1}(L_1) = \begin{pmatrix} 1 & \frac{-\alpha}{\sqrt{1-\alpha^2}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sqrt{1-\alpha^2}} & \frac{-\alpha}{\sqrt{1-\alpha^2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\sqrt{1-\alpha^2}} & \frac{-\alpha}{\sqrt{1-\alpha^2}} \\ 0 & 0 & \cdots & & \frac{1}{\sqrt{1-\alpha^2}} \end{pmatrix}$$

$$R^{-1}(L_2) = \begin{pmatrix} \frac{1}{\sqrt{1-\alpha^2}} & \frac{-\alpha}{\sqrt{1-\alpha^2}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sqrt{1-\alpha^2}} & \frac{-\alpha}{\sqrt{1-\alpha^2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\sqrt{1-\alpha^2}} & \frac{-\alpha}{\sqrt{1-\alpha^2}} \\ 0 & 0 & \cdots & & \frac{1}{\sqrt{1-\alpha^2}} \end{pmatrix}$$

Clearly the $R^{-1}(N)$ is very sparse having nonzero entries in the main diagonal and upper band only. Now transform the toeplitz autocorrelation matrix $M(N)$ by its incomplete Cholesky's factor $R(N)$ to get the preconditioned matrix

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$\tilde{M}(N) = R^{-T}(N)M(N)R^{-1}(N).$

$$\tilde{M}(N) = \begin{pmatrix} & & & & \frac{\alpha^{L_1}}{\sqrt{1-\alpha^2}} & 0 & \cdots & 0 \\ & I_{L_1} & & & \alpha^{L_1-1} & 0 & \cdots & 0 \\ & & & & \vdots & \vdots & \ddots & \vdots \\ & & & & \alpha & 0 & \cdots & 0 \\ \frac{\alpha^{L_1}}{\sqrt{1-\alpha^2}} & \alpha^{L_1-1} & \cdots & \alpha & \frac{1}{1-\alpha^2} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & & & \\ \vdots & \vdots & \ddots & \vdots & \vdots & & I_{L_2-1} & \\ 0 & 0 & \cdots & 0 & 0 & & & \end{pmatrix}$$

Using $D = diag(\tilde{M}(N))$ in (10), the normalized autocorrelation matrix of the form

$$\tilde{M}^{\mathcal{N}}(N) = D^{-1/2}(\tilde{M}(N))D^{-1/2}$$

is obtained. This action normalizes the filter input as well as diagonal of $\tilde{M}^{\mathcal{N}}(N)$ to unity. Note that

$$Eigenvaluespreadof\tilde{M}^{\mathcal{N}}(N) = \frac{1+\alpha}{1-\alpha}$$

It is straightforward to see that for all N , eigenvalue spread of preconditioned and power normalized input correlation matrix $\tilde{M}^{\mathcal{N}}(N)$ remains equal to $\frac{1+\alpha}{1-\alpha}$. Exactly the same value results while computing the asymptotic eigenvalue spread after the application of DFT and power normalization on Markov-1 autocorrelation matrix in [3]. Hence we have the following result:

Theorem: The convergence behavior of 2-block incomplete factorization Preconditioned LMS (IPLMS) algorithm is similar to that of DFT based TDLMS algorithm.

V. Simulation Results

For simulation, consider an adaptive system identification experiment involving a finite impulse response (FIR) filter of order N. A white Gaussian input signal of variance $\sigma^2 = 1$ is passed through a coloring filter with frequency response [1]:

$$H(z) = \frac{\sqrt{1-\alpha^2}}{1-\alpha z^{-1}}$$

where $|\alpha| < 1$, α is the correlation parameter and controls the spectral condition number of the autocorrelation matrix. $\alpha = 0$ corresponds to the case when condition number is close to 1.

An output noise of SNR $30dB$ is added to the desired signal $s(n)$ of the unknown filter of length $N = 8$. Setting $L1 = 5$ and $L2 = 3$, we compute the learning curves of mean squares error and misalignment for our 2-block IPLMS algorithm and compare the results with that of DFT based TDLMS algorithm. Since transformed autocorrelation matrices exhibit same eigen behavior in both cases, therefore, we can have same value of stepsize parameter μ for the two algorithms, let it be 0.02 for both algorithms. Taking average of 100 independent runs, we find that both the algorithms exhibit almost same convergence behavior for correlated input signals with $\alpha = 0.75$ in figure-1, and $\alpha = 0.85$ in figure-2. But if we look at the learning curve of misalignment of the two algorithms in figure-3, we find better robustness of IPLMS algorithm as compared with TDLMS algorithm.

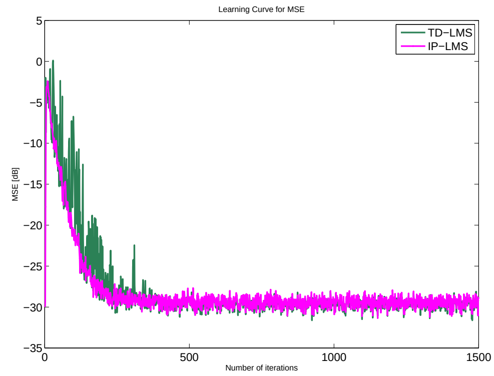

Fig. 1. Mean square error of IPLMS and TDLMS algorithm for $\alpha = 0.75$.

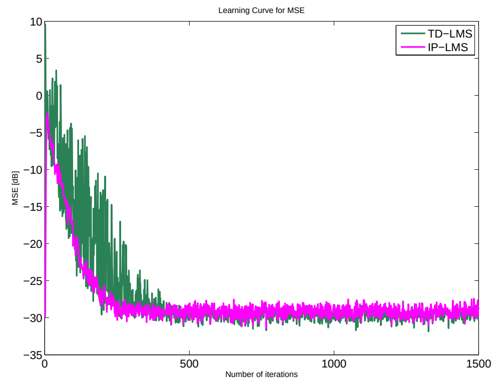

Fig. 2. Mean square error of IPLMS and TDLMS algorithm for $\alpha = 0.85$.

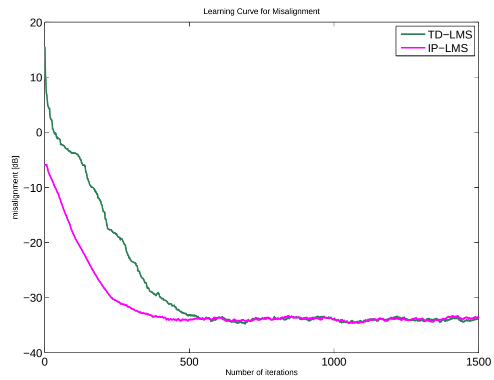

Fig. 3. Misalignment of IPLMS and TDLMS algorithm for $\alpha = 0.85$.

VI. Conclusion

We have developed an incomplete factorization preconditioner based LMS adaptive filtering algorithm. The preconditioner is formed by applying an incomplete strategy to the Cholesky's factor of an optimized autocorrelation matrix of the input signals. An efficient preconditioner is designed at negligible computational cost, which is able to take the convergence rate of conventional LMS adaptive filter to one of its modified variations, the TDLMS algorithm. Analytical development and application of preconditioner at the autocorrelation matrix of Markov-1 typed input signals gives exact eigenvalue spread of transformed autocorrelation matrix after power normalization, which is an exact measure of the asymptotic eigenvalue spread of autocorrelation matrix transformed by DFT and power normalization in TDLMS algorithm.

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