

# On the Coupled Electromechanical Behavior of Artificial Materials with Chiral-Shell Elements

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**Abstract**—In the present work we investigate both the elastic and electric properties of a chiral material. We consider a composite structure made from a polymer matrix and anisotropic inclusions of GaAs taking into account piezoelectric and dielectric properties of the composite material. The principal task of the work is the estimation of the functional properties of the composite material.

**Keywords**—Coupled electromechanical behavior, Composite structure, Chiral metamaterial.

## I. INTRODUCTION

**M**ETAMATERIALS are composite materials with properties, which are dependent both on the physical properties of individual components and the macrostructure. Usually the individual components are the reason for the effective macroscopic behavior of a structure [2, 3].

The complex materials can be synthesized by an insertion in a matrix of various periodic structures with different geometric shapes, which modify the functional properties of the composite material. An example of such structures is a periodic matrix with shells having a helical geometry, and which are sealed in a polymer matrix [11]-[14].

By the synthesis of such complex structures a variation of different parameters of the material is possible (e.g. dimensions of the structure, shape, frequency, etc.). That makes possible to obtain significantly different properties of the resulting material and to find various areas of applications.

In recent years significant progress in synthesis of metamaterials with perspective and unusual functional properties is observed. In particular, the helical shell structures have found various applications, for example, in microelectromechanical and nanoelectromechanical systems (MEMS/NEMS), optics and medicine, see e.g. [2-4].

The research of artificial complex materials requires interdisciplinary knowledge and involves various fields of application areas as development of MEMS/NEMS, solid state physics, optoelectronics, material science, theory of composite structures, nanoscience, etc., [3, 4, 7]. In this case the

question of the micro-macro behaviour of metamaterials is connected with the prediction of functional properties of such structures, [19].

In the present work we investigate both the elastic and the electric properties of a chiral material and consider the composite structure made from a polymer matrix and anisotropic inclusions of GaAs with taking into account the piezoelectric and dielectric properties of the composite material. The principal task of the present work is the estimation of the functional properties of the composite.

## II. PROBLEM STATEMENT

For the determination of functional properties of the artificial composite material we consider a characteristic unit cell of the material. Within the frame of electroelasticity one can make a problem statement which requires the unique solution of the following system of equation. The basic equations of electroelastic bodies, with the geometry as is depicted on Fig. 1, in the case of quasielectrostatics and absence of external loads take the form [9]:

- Equation of equilibrium (no body forces)

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0} \quad (1)$$

- Maxwell's equations (the magnetic component is ignored)

$$\nabla \cdot \mathbf{D} = 0, \quad \mathbf{E} = \nabla \varphi \quad (2)$$

- Constitutive equations

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon} - \mathbf{e} \cdot \mathbf{E}, \quad \mathbf{D} = \mathbf{e}^T \cdot \boldsymbol{\varepsilon} - \mathbf{d} \cdot \mathbf{E}, \quad (3)$$

- Strain tensor in case of the small deformations

$$\boldsymbol{\varepsilon} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \quad (4)$$

- Boundary conditions

$$\begin{aligned} \varphi|_{\Gamma_\varphi^1} &= \varphi_1, & \varphi|_{\Gamma_\varphi^2} &= \varphi_2, \\ -\mathbf{n} \cdot \mathbf{D}|_{\Gamma_q} &= q, & \mathbf{u}|_{\Gamma_u} &= \mathbf{u}_0, \\ \mathbf{n} \cdot \boldsymbol{\sigma}|_{\Gamma_\sigma} &= \mathbf{p} \end{aligned} \quad (5)$$

In Eqs. (1)-(5) the following notation is used:  $\mathbf{n}$  is the normal vector to the inclusion boundary

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$\Gamma = \Gamma_u \cup \Gamma_\sigma = \Gamma_\varphi^1 \cup \Gamma_\varphi^2 \cup \Gamma_q$  (on the boundary  $\Gamma$  can be applied both the mechanical and electrical types of the boundary conditions respective to the type of piezoelectric task, Figs. 3a) and 3b)),  $\mathbf{u}$  is the vector of displacements,  $\mathbf{E}$  is the vector of the electric field expressed by the electric potential  $\varphi$ ,  $\boldsymbol{\sigma}$  is the stress tensor,  $\mathbf{D}$  is the vector of the electric induction (also called electrical flux vector, electric displacement vector),  $\boldsymbol{\varepsilon}$  is the strain tensor,  $\mathbf{C}$ ,  $\mathbf{e}$ ,  $\mathbf{d}$  are the elasticity tensor, the tensors of the piezoelectric and the dielectric parameters, respectively.  $\mathbf{p}$  and  $\mathbf{q}$  are the external load on the surface element with the normal vector  $\mathbf{n}$  and surface charge, [10]. Here  $\Gamma_q = \Gamma_\sigma$ ,  $\Gamma_u = \Gamma_\varphi^1 \cup \Gamma_\varphi^2$ , according to the type of piezoelectric task, see Fig. 3. The difference of the electric potential applied along  $\Gamma_\varphi^1$  and  $\Gamma_\varphi^2$  generates the electrical flux vector. This gradient of the electrical potential is the reason of the mechanical response of internal force generated by the inverse piezoelectric effect.

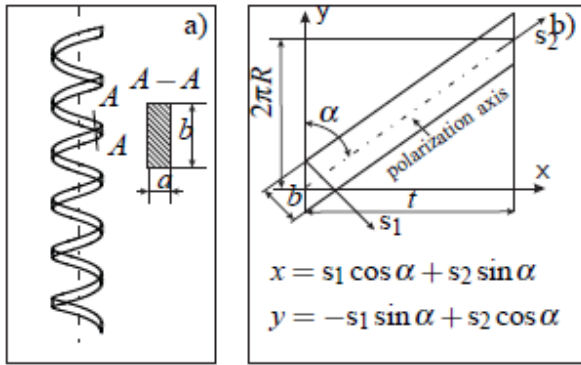


Fig. 1 Geometric description of helices. Cylindrical shell with screw cuts: a) Helical spring shell; b) Loft of one coil of a median shell surface

The constitutive equations (3) and the boundary conditions (5) one can call the  $[\mathbf{u} - \varphi]$  form. This type of piezoelectric problem statement depends on the constitutive equations and boundary conditions in present form (3) and (5).

In addition, we take into account that the normal component of the electric flux vector  $\mathbf{D}$  is continuous on the matrix-inclusion interface. Consequently, according to the superposition principle for the linear physical systems (additive property), one can have the continuity of the others field variables [17].

### III. MATERIAL OVERVIEW

In the present work we consider a composite, which consists of a polyimide matrix (PA) with a periodic located array of helical shell-like structures. The question of the choice of the alternative master in the metamaterial is connected with the applications and the relative simple and cheap possibilities of a synthesis of such structures.

The process of a creation of artificial composite structures with helical inclusions. The standard process of nanohelix

creation consist in the epitaxial deposition of a layer on a substrate. Then by the internal stresses this layer separates from the substrate and roll in a helix relative anisotropic properties and result bending moment in this layer. It is also possible to laminate shells to each other. A result of such lamination is a multilayer composite with helical shape, Fig. 1.

After forming of an array of helices this array is sealed in the polymer matrix. The polymer matrix allows to keep the form, size and location of helices in the structure. The principal task of a polymer matrix is the common work of inclusions, uniformly of stress distribution and failure protection, see e.g. [11-13].

For a more precise understanding of the behaviour of the composite structure briefly consider the components material in a particularly and process of a such metamaterial composition.

In the capacity of polymer matrix in a present work the polyimide structure is used. These polymers have particular thermal and mechanical properties [18]. The dielectric properties of these polymers can be improved by reduction of the values of dielectric parameters [15, 16, 18]. The use of fluorinated polyimides can reduce the dielectric parameter value from 3.4 to 2.8.

We use the following properties of the PA-material: Young's modulus  $E=2.30$  (MPa), Poisson's ratio  $\nu=0.28$ , dielectric constant  $d=0.28$ .

The inclusions in the polymer matrix are piezoelectric helical one-layer structures. The material of the shells is GaAs, which has a cubic symmetry. More specifically GaAs is an instance of a non-centrosymmetrical classes according to piezoelectric constants distribution. It is the 43m and 23 classes by the Hermann - Mauguin notation [8]. This type of symmetry is close to the orthorhombic class as 222, but in case of a cubic symmetry all of the piezoelectric moduli are equal, [21].

Hereby in case of the cubic symmetry for the classes 43m and 23 one can see the following matrix representation of the piezoelectric moduli:

$$[\mathbf{e}]^T = \begin{pmatrix} \cdot & \cdot & \cdot & e_{14} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & e_{14} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & e_{14} \end{pmatrix} \quad (6)$$

For the orthorhombic symmetry, class 222, the piezoelectric matrix is:

$$[\mathbf{e}]^T = \begin{pmatrix} \cdot & \cdot & \cdot & e_{14} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & e_{25} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & e_{36} \end{pmatrix} \quad (7)$$

where  $\cdot$  denotes zero values.

We taking into account the following properties of the GaAs shell: dielectric constant  $d=12.9$ , piezoelectric modulus  $e_{14} = -2.69 \cdot 10^{-12}$  (m/V), and coefficients of the second

order stiffness tensor  $C_{11} = 11.9 \cdot 10^4$  (MPa),  
 $C_{12} = 5.34 \cdot 10^4$  (MPa),  $C_{44} = 5.96 \cdot 10^4$  (MPa) (others  
 moduli in the stiffness matrix are equal zero), see [5], [6].

It is obvious, that the effect of inclusion deformation under applied electrical field is the reason to get deformation of the composite structure. Hereby, one can see the presence of a direct and inverse piezoelectric effect, this means that one can understand the result artificial composite material as a piezoelectric one. The deformation behavior of the composite structure, which formed from described above materials, differ from the ordinary electroelastic systems, because the source of intentional stresses is the deformation of a piezoelectric shell. Consider an unit cell of the metamaterial, which consist of isotropic polyimide matrix and anisotropic one-layer helical inclusion of GaAs.

For account of a piezoelectric effect an additional disparity of a lattice spacing in a contact matrix/inclusion. The value of the resultant stresses of the unit cell by deformation of an inclusion, is from the conditions for mechanical equilibrium of the system and boundary conditions defined. The result composite material one can consider as a local orthotropic composite with a symmetry axis directed at tangential to the surfaces of the helixes, see Fig. 2 (a).

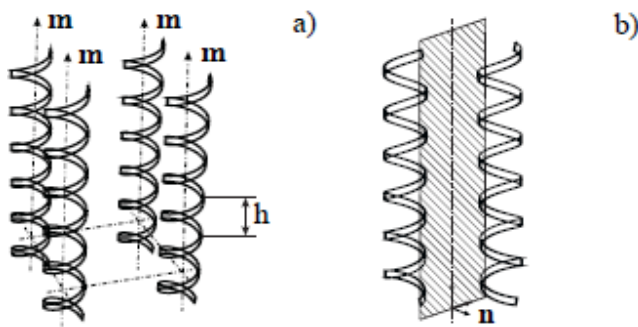


Fig. 2 Representation of shells, **m** is the plane of symmetry (a) Lattice arrangement; (b) Right/left-hand twist of the helixes, **n** is the normal vector according to the plane of the mirror reflection

For the definition of effective values, which are material characterized, one can use the homogeneous methods [20]. Hereby it is necessary to define 7 independent constants, which form a stiffness matrix of the resulting chiral material.

Hereby one should define 5 independent constants of a piezoelectric tensor and 3 independent constants of a dielectric matrix. For the orthorhombic class with the symmetry mm2 in case of (001) and (100) orientation the piezoelectric tensor in a matrix representation takes the form (8) and (9), respectively.

$$[\mathbf{e}]^T = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & e_{15} & \cdot \\ \cdot & \cdot & \cdot & e_{24} & \cdot & \cdot \\ e_{31} & e_{32} & e_{33} & \cdot & \cdot & \cdot \end{pmatrix} \quad (8)$$

$$[\mathbf{e}]^T = \begin{pmatrix} e_{11} & e_{12} & e_{13} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & e_{24} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & e_{35} & \cdot \end{pmatrix} \quad (9)$$

#### IV. COMPUTATIONAL ASPECTS

Consider the physical meaning of the constants, which is necessary to the describing of the matamaterial behaviour. The physical meaning of the piezoelectric constants according to the matrix representation as one can see above and with taking into account the (100)-material orientation is following, [21]:

- Modulus  $e_{11}$  is induced polarization of the material in direction 1 per unit stress applied in direction 1 (directions 1-2-3 form the orthogonal triad or basis of space). By the inverse piezoelectric effect induced strains in direction 1 per unit electric field applied in the direction 1.
- Modulus  $e_{12}$  is induced polarization of the material in direction 1 per unit stress applied in direction 2. By the inverse piezoelectric effect induced strains in direction 2 per unit electric field applied in the direction 1.
- Modulus  $e_{13}$  is induced polarization of the material in direction 1 per unit stress applied in direction 3. By the inverse piezoelectric effect induced strains in direction 3 per unit electric field applied in the direction 1.
- Modulus  $e_{24}$  is induced polarization of the material in direction 2 per unit shear stress applied in the 12-plane. By the inverse piezoelectric effect induced shear strain in the 12-plane per unit electric field applied in the direction 2.
- Modulus  $e_{35}$  is induced polarization of the material in direction 3 per unit shear stress applied in the 13-plane. By the inverse piezoelectric effect induced shear strain in the 13-plane per unit electric field applied in the direction 3.

If we consider the permittivity of a chiral material one can get the following definitions, [21]:

$$[\mathbf{d}] = \begin{pmatrix} d_{11} & \cdot & \cdot \\ \cdot & d_{22} & \cdot \\ \cdot & \cdot & d_{33} \end{pmatrix} \quad (10)$$

where for the (100) orientation  $d_{13}$  is the permittivity for a dielectric displacement in the direction 1 and electric field in the direction 3 under constant stress,  $d_{22}$  is the permittivity for a dielectric displacement in the direction 2 and electric field in the direction 2 under constant stress,  $d_{11}$  is the permittivity for a dielectric displacement in the direction 3 and electric field in the direction 1 under constant stress. By consideration of the first type of a constitutive equation (3) and respective boundary condition (5) one can present the task of the

determination of the functional elastic and electric constants by the solving the following subtasks, see Table I.

TABLE I  
 COMPUTATIONAL ASPECT  
 $[\mathbf{u} - \varphi]$  RELATIONS,  $\langle \cdot \rangle$  IS A AVERAGED VALUES BY VOLUME OF  
 FINITE ELEMENTS

Condition	Elastic properties	Electric properties
$\varepsilon_{11} \neq 0$	$C_{11}^{\text{eff}} = \frac{\langle \sigma_{11} \rangle}{\langle \varepsilon_{11} \rangle}$	$e_{11}^{\text{eff}} = \frac{\langle D_1 \rangle}{\langle \varepsilon_{11} \rangle}$
$\varepsilon_{22} \neq 0$	$C_{22}^{\text{eff}} = \frac{\langle \sigma_{22} \rangle}{\langle \varepsilon_{22} \rangle}$	$e_{12}^{\text{eff}} = \frac{\langle D_2 \rangle}{\langle \varepsilon_{22} \rangle}$
$\varepsilon_{33} \neq 0$	$C_{33}^{\text{eff}} = \frac{\langle \sigma_{33} \rangle}{\langle \varepsilon_{33} \rangle}$	$e_{13}^{\text{eff}} = \frac{\langle D_3 \rangle}{\langle \varepsilon_{33} \rangle}$
$\varepsilon_{12} \neq 0$	$C_{44}^{\text{eff}} = \frac{\langle \sigma_{12} \rangle}{\langle \varepsilon_{12} \rangle}$	$e_{24}^{\text{eff}} = \frac{\langle D_2 \rangle}{\langle \varepsilon_{12} \rangle}$
$\varepsilon_{13} \neq 0$	$C_{55}^{\text{eff}} = \frac{\langle \sigma_{13} \rangle}{\langle \varepsilon_{13} \rangle}$	$e_{35}^{\text{eff}} = \frac{\langle D_3 \rangle}{\langle \varepsilon_{13} \rangle}$
$E_1 \neq 0$		$d_{31}^{\text{eff}} = -\frac{\langle D_3 \rangle}{\langle E_1 \rangle}$
$E_2 \neq 0$		$d_{22}^{\text{eff}} = \frac{\langle D_2 \rangle}{\langle E_2 \rangle}$
$E_3 \neq 0$		$d_{13}^{\text{eff}} = \frac{\langle D_1 \rangle}{\langle E_3 \rangle}$

Here we accept as a nonzero components of strain tensor and vector of the electrical field only which are indicated in Table I. The subtasks with nonzero components of the strain reflect the DPE-analysis and the rest tasks is connected with the IPE-problem. It should be noted, that the determination of the engineering constants is possible. And the coefficients of a stiffness matrix can be expressed in terms of generalized Young's moduli and Poisson's ratios (which have the same significance as Young's modulus and Poisson's ratio for uniaxial loading along the three basis vectors 1-2-3). Hereby one can retrieve the stiffness matrix by the determination Young's moduli  $E_i$  and Poisson's ratios  $\nu_{ij}$ ,  $i, j = 1 \div 3$ . Then the missing in the in Table I coefficients of the stiffness matrix can be following [20]:

$$C_{ij}^{\text{eff}} = E_i(\nu_{ji} + \nu_{ki}\nu_{jk})\gamma, \quad (12)$$

where  $\gamma = 1/(1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13})$ ,  
 $\nu_{ij}/E_i = \nu_{ji}/E_j$ , no  $\sum : i \neq j \neq k = 1, 2, 3$ , see [1].

### V. MODELING SCHEME

By determination of the functional properties we consider the unit cell of the chiral material which consist from the one-layer piezoelectric inclusion of GaAs and isotropic matrix of polyimide Fig. 3. By model formulation of the unit cell in a

finite element package Simulia ABAQUS we defined the material of the matrix as a piezoelectric material with the zero piezoelectric matrix, because it is necessary for this task to have the electric degrees of freedom in a every finite element. This is not upset the physical meaning of using only dielectric master with a piezoelectric inclusion, because we have not the piezoelectric contribution to the electro-elastic behaviour of a unit cell matrix. The presence both displacement and electrical potential as nodal variables makes it possible to use all necessary types of boundary condition for a matrix and on the interface region of different materials.

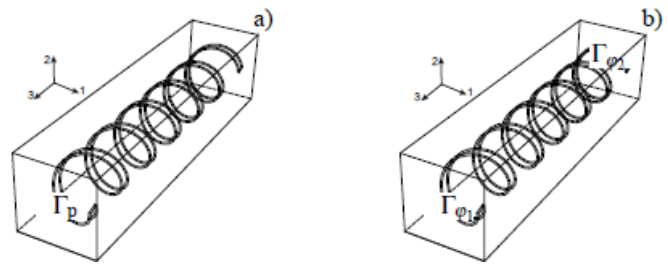


Fig. 3 Boundary conditions: (a) Direct piezoelectric effect (b) Inverse piezoelectric effect

For example the difference of a electric potentials assignment for a task, in which we define the effective dielectric constants by the process of inverse piezoelectric process Fig. 3 (b).

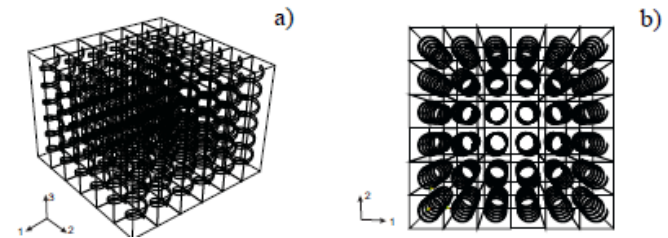


Fig. 4 Assembly of the composite material, right-hand twist of helices (a) 123 view (b) 12 view



Fig. 5 Finite element triangulation (general quantity of elements 137738) (a) Finite element mesh of matrix, element type C3D4E, element quantity - 128873 (b) Finite element mesh of inclusion, element type C3D4E, element quantity - 8865 (Simulia Abaqus triangulation)

It should be noted, that matrix and inclusion are meshed with the same type of the finite elements, as a 4-node linear piezoelectric tetrahedron Fig. 5. For the meshing of the general structure we used increasing of the size of interior

elements with a moderate growth. Where is possibly to appropriate was used the technique of the mapped tri meshing on the bounding faces [22].

The assembly of the matrix and inclusion is the one part, which construct by the merge of the both initial parts. This solve the problem of a complication aspect of the conductivity between standard "tie"-contact on the matrix/inclusion boundary interface [22] with taking into account the continuity condition of the electrical flux vector through matrix/inclusion interface.

Hereby for the definition all of the functional moulduli, which described the material behaviour of a chiral material, is necessary to implement 6 tests with mechanical boundary conditions and 3 tasks for a inverse piezoelectric effect, see as example Fig. 6.

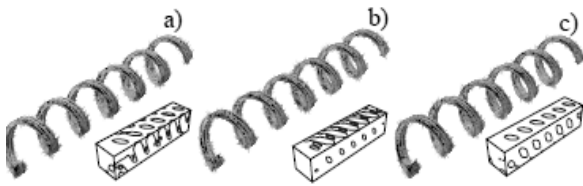


Fig. 6 Result example by the DPE (a) Determination of a  $e_{11}^{eff}$  and  $C_{11}^{eff}$  by non zero  $\varepsilon_{11}$  component (b) Determination of a  $e_{12}^{eff}$  and  $C_{22}^{eff}$  by non zero  $\varepsilon_{22}$  component (c) Determination of a  $e_{13}^{eff}$  and  $C_{33}^{eff}$  by non zero  $\varepsilon_{33}$  component

By the implementations a few tasks on the direct and inverse piezoelectric effect the material properties of the artificial composite material were estimated.

With taking into account the general form of the tensors the coefficient of the stiffness matrix in units (MPa) in case of transversal anisotropy and are:

$$C_{11} = 2.307 \cdot 10^4, C_{12} = 0.937 \cdot 10^4, C_{13} = 1.169 \cdot 10^4$$

$$C_{22} = 2.299 \cdot 10^4, C_{23} = 1.172 \cdot 10^4, C_{33} = 2.289 \cdot 10^4$$

$$C_{44} = 0.906 \cdot 10^4, C_{55} = 0.906 \cdot 10^4, C_{66} = 0.905 \cdot 10^4$$

Consider also the obtained piezoelectric coefficients in units (m/V) and dielectric constants. It should be noted, that the absent coefficients are equal zero.

$$e_{11} = 6.26 \cdot 10^{-17}, e_{12} = 6.81 \cdot 10^{-18}, e_{13} = 4.04 \cdot 10^{-17}$$

$$e_{24} = 1.14 \cdot 10^{-16}, e_{24} = 1.65 \cdot 10^{-16}$$

$$d_{13} = 2.8, d_{22} = 2.8, d_{31} = -2.79$$

Rather for an understanding of a result difference between the structures with a various materials. We also conducted in the case described above the analysis with a more harder matrix and the same inclusions. The averaged functional properties for a matrix of  $Al_2O_3$  and GaAs inclusions one can see below with taking into account the following properties

for the  $Al_2O_3$ : Young modulus  $E=3.00$  (MPa), Poisson ratio  $\nu = 0.20$ , dielectric constant  $d=9.10$ .

Let us consider the coefficients of the stiffness matrix in units (MPa) in case of transversal anisotropy:

$$C_{11} = 2.997 \cdot 10^4, C_{12} = 1.027 \cdot 10^4, C_{13} = 0.971 \cdot 10^4$$

$$C_{22} = 2.987 \cdot 10^4, C_{23} = 0.950 \cdot 10^4, C_{33} = 2.982 \cdot 10^4$$

$$C_{44} = 1.250 \cdot 10^4, C_{55} = 1.249 \cdot 10^4, C_{66} = 1.250 \cdot 10^4$$

Let us consider also the obtained piezoelectric coefficients in units (m/V) and dielectric constants:

$$e_{11} = 1.50 \cdot 10^{-17}, e_{12} = 5.47 \cdot 10^{-18}, e_{13} = 7.44 \cdot 10^{-17}$$

$$e_{24} = 2.36 \cdot 10^{-16}, e_{24} = 1.70 \cdot 10^{-16}$$

$$d_{13} = 9.1, d_{22} = 9.1, d_{31} = -9.1$$

The engineering constants which described the elastic properties for the structure polyimide/GaAs one can see in Table II and for the  $Al_2O_3$ /GaAs in Table III.

TABLE II  
 RESULT VALUES FOR THE POLYIMIDE/GAAS: ENGINEERING CONSTANTS

Young's modulus		Poisson's ratio	
$E_1$	3.075e+4	$\nu_{12}$	0.22
		$\nu_{21}$	0.23
$E_2$	2.99e+4	$\nu_{23}$	0.20
		$\nu_{32}$	0.20
$E_3$	2.98e+4	$\nu_{13}$	0.20
		$\nu_{31}$	0.21

TABLE III  
 RESULT VALUES FOR THE  $Al_2O_3$ /GAAS: ENGINEERING CONSTANTS

Young's modulus		Poisson's ratio	
$E_1$	2.395e+4	$\nu_{12}$	0.126
		$\nu_{21}$	0.127
$E_2$	2.388e+4	$\nu_{23}$	0.285
		$\nu_{32}$	0.456
$E_3$	1.491e+4	$\nu_{13}$	0.283
		$\nu_{31}$	0.454

Consider the similar model of the chiral material with the left twisting of the inclusions. It should be noted that the inclusion volume fraction is not changed. We are using the polyamide/GaAs structure. By the consideration of the resultant functional properties (in units (MPa)) which are presented below one can make a remark about the difference of the behaviour chiral material with the left and right twisting of the inclusions.

$$C_{11} = 2.30 \cdot 10^4, C_{12} = 1.18 \cdot 10^4, C_{13} = 0.971 \cdot 10^4$$

$$C_{22} = 2.29 \cdot 10^4, C_{23} = 0.965 \cdot 10^4, C_{33} = 2.30 \cdot 10^4$$

$$C_{44} = 0.907 \cdot 10^4, C_{55} = 0.907 \cdot 10^4, C_{66} = 0.906 \cdot 10^4$$

Let us consider also the obtained piezoelectric coefficients in units (m/V) and dielectric constants:

$$e_{11} = -6.261 \cdot 10^{-17}, e_{12} = -6.84 \cdot 10^{-18}, e_{13} = -4.12 \cdot 10^{-17}$$

$$e_{24} = -1.15 \cdot 10^{-16}, e_{24} = -1.67 \cdot 10^{-16}$$

$$d_{13} = 2.80, d_{22} = 2.80, d_{31} = -2.82$$

One can see, that elastic behaviour of the material was not strongly change, as and the dielectric properties. The dielectric properties as a first case present the isotropic behaviour. It is note, that changing of the twisting of the inclusions does not reflect the symmetry of the composite. And all isotropic properties will be saved. But can reflect the difference in the polarization of the resulting material. It is clearly one can see by consideration of the matrix of the piezoelectric constants, which is a reflection of the piezoelectric tensor by applying of the reflection in a plane with the normal vector  $\mathbf{n}$ . This reflection operator can be written in form  $\mathbf{Q} = \mathbf{I} - 2\mathbf{n} \otimes \mathbf{n}$ , where  $\mathbf{I}$  is the identity tensor.

On the supposition of the orthorhombic class of properties symmetry in case of the piezoelectric modulus one can see, that polarization of the resulting composite was changed. This is the principal result of this research because this makes possibly to construct the particular polarization of the artificial chiral materials.

## VI. CONCLUSION

In the present work a chiral composite is considered. The finite element simulation of a unit cell of the chiral composite is implemented. The difference of the results of the chiral material behaviour by variation of the functional properties of basic materials and twisting of the helixes is noted. The presence of anisotropy according to mechanical and electrical behaviour in a composite is established.

By consideration of averaging results in the case of the polymeric matrix and matrix of  $\text{Al}_2\text{O}_3$ , which is more stiff than polymer one can note the appearance of the equal anisotropy as have the matrix of the chiral material. On this evidence one can draw the conclusion, that a conservation of the chirality of the composite material is possible with a more compliant dielectric matrix. According to this and to the case of a fabrication of the artificial composite structure with the polymer matrix the using of a polymer chiral composite can find broad area of the real applications [12].

The twisting of the helixes reflects the considerable changes only in case of piezoelectric properties of the material.

By consideration of a packing variations of the helical inclusions one can have a supposition, that geometry of the helical inclusions as a shell width, packing density, quantity of the spiral turns, etc. can have a principal significance by the

averaging of the electrical properties. As an example, by the increasing helix length and quantity of the spiral turns (in the case of polarization described above) one can received more evident piezoelectric effect in a chiral material, but the case of the increasing only shell width not improve the piezoelectric characteristics of the composite structure, of course a question of the shape optimization for this structure is more complicated and should be considered according to the optimization problem, [15].

The results of this work can be used for the designing of a metamaterial structure with a chiral properties, for a prediction of them mechanical and electrical response.

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