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Existence and exponential stability of almost periodic solution for recurrent neural networks on time scales

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Abstract—In this paper, a class of recurrent neural networks (RNNs) with variable delays are studied on almost periodic time scales, some sufficient conditions are established for the existence and global exponential stability of the almost periodic solution. These results have important leading significance in designs and applications of RNNs. Finally, two examples and numerical simulations are presented to illustrate the feasibility and effectiveness of the results.

Keywords—Recurrent neural network; Almost periodic solution; Global exponential stability; Time scale.

I. Introduction

It is well known that recurrent neural networks(RNNs) include a lot of famous neural networks such as cellular neural networks (CNNs), Hopfield neural networks (HNNs), bidirectional associative memory (BAM) networks, etc. In past few years, different classes of RNNs have been extensively studied due to their promising potential for applications in the areas of signal and image processing, associative memories and pattern classification, parallel computation and optimization problems, see [1-6] and references therein.

As is well known, the properties of periodic oscillatory solutions are of great interest in many applications. For instance, the human brain is in periodic oscillatory or chaos. Hence, it is of fundamental importance to study periodic oscillatory and chaos phenomena of neural networks. However, upon considering long-term dynamical behaviors, the periodic parameters often turn out to experience certain perturbations, that is, parameters become periodic up to a small error. Thus, almost periodic oscillatory behavior is considered to be more accordant with reality.

The theory of calculus on time scales (see [7] and references cited therein) was initiated by Stefan Hilger in 1988 [8] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications and has recently received much attention since his foundational work [9-12]. Therefore, it is practicable to study that on time scales which can unify the continuous and discrete situations.

Motivated by the above, in this paper, we consider the following RNNs with variable delays on time scales:

$$\begin{cases} y_i^{\Delta}(t) &= -a_i(t)y_i(t) + \sum\limits_{j=1}^{n} c_{ij}(t)g_j(y_j(t)) \\ & \quad + \sum\limits_{j=1}^{n} d_{ij}(t)f_j(y_j(t-\tau_{ij}(t))) \\ & \quad +I_i(t),\ t\in\mathbb{T}^+, \\ y_i(s) &= \phi_i(s), \quad s\in[-\hat{\tau},0]_{\mathbb{T}},\ i\in\Lambda, \end{cases} \tag{1}$$

where $\mathbb{T}$ is an almost periodic time scale, $\mathbb{T}^+ = \mathbb{T}\cap(0,+\infty)$, $\Lambda=\{1,2,\ldots,n\}$, the integer n corresponds to the number of units in (1); $y_i(t)$ corresponds to the state of the ith unit at time t; $a_i(t)>0$ represents the passive decay rate; c_{ij} and d_{ij} weight the strength of jth unit on the ith unit at time t; $I_i(t)$ is the input to the ith unit at time t from outside the networks; g_i and f_i denote activation functions of transmission; $\tau_{ij}(t)$ corresponds to the signal transmission delay along the axon of the jth unit which is nonnegative and bounded, i.e., $0\le\tau_{ij}(t)\le\hat{\tau}$.

II. Preliminaries

In this section, we shall first recall some basic definitions, lemmas which will be used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma,\rho:\mathbb{T}\to\mathbb{T}$ and the graininess $\mu:\mathbb{T}\to\mathbb{R}^+$ are defined, respectively, by $\sigma(t)=\inf\{s\in\mathbb{T}:s>t\},\rho(t)=\sup\{s\in\mathbb{T}:s<t\},\mu(t)=\sigma(t)-t$.

The basic theories of dynamic systems on time scales and almost periodic differential equations, one can see [7,13,14].

Definition 2.1 (see [13]) Let $x\in\mathbb{R}^n$, and $A(t)$ be an $n\times n$ rd-continuous matrix on $\mathbb{T}$, the linear system

$$x^{\Delta}(t)=A(t)x(t),\ t\in\mathbb{T} \tag{2}$$

is said to admit an exponential dichotomy on $\mathbb{T}$ if there exist positive constants k,α, projection P and the fundamental solution matrix $X(t)$ of (2), satisfying

$$|X(t)PX^{-1}(\sigma(s))|_0\le ke_{\ominus\alpha}(t,\sigma(s)),$$
$$s,t\in\mathbb{T},t\ge\sigma(s),$$
$$|X(t)(I-P)X^{-1}(\sigma(s))|_0\le ke_{\ominus\alpha}(\sigma(s),t),$$
$$s,t\in\mathbb{T},t\le\sigma(s),$$

where $|\cdot|_0$ is a matrix norm on $\mathbb{T}$.

Lemma 2.1 (see [14]) If the linear system (2) admits an exponential dichotomy, $-\bar{A}$ is an M-matrix, then system

$$x^{\Delta}(t)=A(t)x(t)+f(t),\ t\in\mathbb{T}, \tag{3}$$

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has a unique almost periodic solution $x(t)$, and

$$x(t) = \int_{-\infty}^{t} X(t)PX^{-1}(\sigma(s))f(s)\Delta s - \int_{t}^{+\infty} X(t)(I-P)X^{-1}(\sigma(s))f(s)\Delta s,$$

where $X(t)$ is the fundamental solution matrix of (2), and $\bar{A} = \big(\sup(a_{ij}(t))\big)_{n\times n},\ 1 \le i,j \le n,\ t \in \mathbb{T}$.

Lemma 2.2 (see [13]) Let $c_i(t)$ be an almost periodic function on $\mathbb{T}$, where $c_i(t) > 0$, $-c_i(t) \in \mathcal{R}^+$, $\forall\ t \in \mathbb{T}$ and

$$\min_{1\le i\le n}\big\{\inf_{t\in\mathbb{T}} c_i(t)\big\} = \widetilde{m} > 0,$$

then the linear system

$$x^{\Delta}(t) = \mathrm{diag}\big(-c_1(t), -c_2(t), \cdots, -c_n(t)\big)x(t)$$

admits an exponential dichotomy on $\mathbb{T}$.

Lemma 2.3 (see [14]) If the following conditions satisfy:

(1) $D^+x_i^{\Delta}(t) \le \sum\limits_{j=1}^{n} a_{ij}x_j(t) + \sum\limits_{j=1}^{n} b_{ij}\bar{x}_j(t),\ t \in [t_0, +\infty)_{\mathbb{T}}$, $i,j = 1,2,\cdots,n$, where $a_{ij} \ge 0 (i \ne j)$, $b_{ij} \ge 0$, $\sum\limits_{i=1}^{n} \bar{x}_i(t_0) > 0$, $\bar{x}_i(t) = \sup\limits_{s\in[t-\tau_0,t]_{\mathbb{T}}} x_i(s)$, and $\tau_0 > 0$ is a constant;

(2) $\widetilde{M} := -(a_{ij} + b_{ij})_{n\times n}$ is an M-matrix;

then there exists constants $\gamma_i > 0$ and $a > 0$, such that the solutions of inequality (1) satisfies

$$x_i(t) \le \gamma_i\bigg(\sum_{j=1}^{n} \bar{x}_j(t_0)\bigg)e_{\ominus a}(t, t_0),\ \forall t \in (t_0, +\infty)_{\mathbb{T}},$$

where $i = 1,2,\cdots,n$.

III. Existence and exponential stability

In this section, we will study the existence and exponential stability of almost periodic solution of (1). Hereafter, we will use the norm $\|z\| = \max\limits_{i\in\Lambda}\big\{\sup\limits_{t\in\mathbb{T}}|z_i(t)|\big\}$, and let $AP(\mathbb{T})$ as a set constructed by all almost periodic functions on an almost time scale $\mathbb{T}$.

Firstly, we make the following assumptions:

(H_1) $a_i(t), c_{ij}(t), d_{ij}(t), \tau_{ij}(t),\ I_i(t)$ are all almost periodic functions defined on $\mathbb{T}$, $i,j \in \Lambda$.

(H_2) The activation functions $f_j, g_j \in C(\mathbb{R},\mathbb{R})$ and satisfy $f_j(0) = 0,\ g_j(0) = 0$, respectively. Moreover, there exists positive numbers L_j^f, L_j^g such that $|f_j(x) - f_j(y)| \le L_j^f|x-y|,\ \ |g_j(x) - g_j(y)| \le L_j^g|x-y|,\ j \in \Lambda$.

(H_3) $\min\limits_{i\in\Lambda}\big\{\inf\limits_{t\in\mathbb{T}} a_i(t)\big\} > 0$, and $1 - \mu(t)a_i(t) > 0,\ \forall\ t \in \mathbb{T},\ i \in \Lambda$.

We know that all almost periodic functions are bounded. For convenience, we denote $\overline{h} = \sup\limits_{t\in\mathbb{T}}|h(t)|,\ \underline{h} = \inf\limits_{t\in\mathbb{T}}|h(t)|$ for any $h(t) \in AP(\mathbb{T})$.

Theorem 3.1 Assume that $(H_1) - (H_3)$ hold, then system (1) has exactly one almost periodic solution in the region $\|z - z_0\| \le \frac{QW}{1-Q}$, if the following condition holds

$$Q = \max_{i\in\Lambda}\bigg\{\frac{1}{\underline{a}_i}\bigg[\sum_{j=1}^{n}\bar{d}_{ij}L_j^f + \sum_{j=1}^{n}\bar{c}_{ij}L_j^g\bigg]\bigg\} < 1,$$

where
$z_0 = \big\{\int_{-\infty}^{t} e_{-a_1}(t,\sigma(s))I_1(s)\Delta s, \cdots, \int_{-\infty}^{t} e_{-a_n}(t,\sigma(s)) \times I_n(s)\Delta s\big\}$, $W = \max\limits_{i\in\Lambda}\Big\{\frac{\overline{I}_i}{\underline{a}_i}\Big\}$.

Proof: Let $\mathbb{B} = \big\{z | z = \big(\psi_1, \psi_2, \cdots, \psi_n\big)^T\big\}$, where z is a continuous almost periodic function on $\mathbb{T}$ with the norm

$$\|z\| = \max_{i\in\Lambda}\big\{\sup_{t\in\mathbb{T}}|\psi_i(t)|\big\},$$

then, $\mathbb{B}$ is a Banach space.

For any $z \in \mathbb{B}$, we consider the almost periodic solution $y_z(t)$ of the nonlinear almost periodic differential equation

$$y_i^{\Delta}(t) = -a_i(t)y_i(t) + \sum_{j=1}^{n} d_{ij}(t)f_j(\psi_j(t - \tau_{ij}(t))) + \sum_{j=1}^{n} c_{ij}(t)g_j(\psi_j(t)) + I_i(t),\ i \in \Lambda. \tag{4}$$

Since $\min\limits_{i\in\Lambda}\big\{\inf\limits_{t\in\mathbb{T}} a_i(t)\big\} > 0$, by Lemma 2.2, the linear system

$$y^{\Delta}(t) = \mathrm{diag}(-a_1(t), -a_2(t), \cdots, -a_n(t))y(t)$$

admits an exponential dichotomy. Then, together with Lemma 2.1, the uniqueness solution of system (4) can be expressed as the following form:

$$\begin{aligned} y_z(t) = \bigg\{ &\int_{-\infty}^{t} e_{-a_1}(t,\sigma(s))\bigg[\sum_{j=1}^{n} d_{1j}(s)f_j(\psi_j(s - \tau_{1j}(s))) \\ &+ \sum_{j=1}^{n} c_{1j}(s)g_j(\psi_j(s)) + I_1(s)\bigg]\Delta s, \cdots, \\ &\int_{-\infty}^{t} e_{-a_n}(t,\sigma(s))\bigg[\sum_{j=1}^{n} d_{nj}(s)f_j(\psi_j(s - \tau_{nj}(s))) \\ &+ \sum_{j=1}^{n} c_{nj}(s)g_j(\psi_j(s)) + I_n(s)\bigg]\Delta s\bigg\}. \end{aligned} \tag{5}$$

Define a mapping $\Phi : \mathbb{B} \to \mathbb{B}$ by setting

$$\Phi(z)(t) = y_z(t),\ \forall\ z \in \mathbb{B}.$$

Set

$$\mathbb{B}^* = \bigg\{z | z \in \mathbb{B}, \|z - z_0\| \le \frac{QW}{1-Q}\bigg\}.$$

Then $\mathbb{B}^*$ is a closed convex subset of $\mathbb{B}$. According to the definition of the norm of the Banach space $\mathbb{B}$, we have

$$\|z_0\| = \max_{i\in\Lambda}\bigg\{\sup_{t\in\mathbb{T}}\bigg|\int_{-\infty}^{t} e_{-a_i}(t,\sigma(s))I_i(s)\Delta s\bigg|\bigg\} \le \max_{i\in\Lambda}\bigg\{\frac{\overline{I}_i}{\underline{a}_i}\bigg\} = W.$$

Therefore,

$$\|z\| \le \|z - z_0\| + \|z_0\| = \frac{W}{1-Q}.$$

First, we prove that the mapping Φ is a self-mapping from $\mathbb{B}^*$ to $\mathbb{B}^*$, In fact, for any $z \in \mathbb{B}^*$, we have

$$\begin{aligned}
&\|\Phi(z) - z_0\| \\
&= \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \Bigg| \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} d_{ij}(s) f_j(\psi_j(s-\tau_{ij}(s))) \\
&\quad + \sum_{j=1}^{n} c_{ij}(s) g_j(\psi_j(s)) \Bigg] \Delta s \Bigg| \Bigg\} \\
&\le \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} |d_{ij}(s)||f_j(\psi_j(s-\tau_{ij}(s)))| \\
&\quad + \sum_{j=1}^{n} |c_{ij}(s)||g_j(\psi_j(s))| \Bigg] \Delta s \Bigg\} \\
&\le \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} \bar{d}_{ij} L_j^f |\psi_j(s-\tau_{ij}(s))| \\
&\quad + \sum_{j=1}^{n} \bar{c}_{ij} L_j^g |\psi_j(s)| \Bigg] \Delta s \Bigg\} \\
&\le \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} \bar{d}_{ij} L_j^f + \sum_{j=1}^{n} \bar{c}_{ij} L_j^g \Bigg] \Delta s \Bigg\} \|z\| \\
&\le \max_{i\in\Lambda} \Bigg\{ \frac{1}{\underline{a}_i} \Bigg[\sum_{j=1}^{n} \bar{d}_{ij} L_j^f + \sum_{j=1}^{n} \bar{c}_{ij} L_j^g \Bigg] \Bigg\} \|z\| \\
&= Q\|z\| \le \frac{QW}{1-Q},
\end{aligned}$$

which implies that $\Phi(z)(t) \in \mathbb{B}^*$. Therefore, the mapping Φ is a self-mapping from $\mathbb{B}^*$ to $\mathbb{B}^*$.

Next, we prove that the mapping Φ is a contraction mapping of $\mathbb{B}^*$. In fact, in view of $(H_1)-(H_3)$, for any $z, \bar{z} \in \mathbb{B}$, where

$$z = (\psi_1, \psi_2, \cdots, \psi_n)^T, \quad \bar{z} = (\bar{\psi}_1, \bar{\psi}_2, \cdots, \bar{\psi}_n)^T,$$

we have

$$\begin{aligned}
&\|\Phi(z) - \Phi(\bar{z})\| \\
&= \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \Bigg| \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} d_{ij}(s) [f_j(\psi_j(s-\tau_{ij}(s))) \\
&\quad - f_j(\bar{\psi}_j(s-\tau_{ij}(s)))] \\
&\quad + \sum_{j=1}^{n} c_{ij}(s) [g_j(\psi_j(s)) - g_j(\bar{\psi}_j(s))] \Bigg] \Delta s \Bigg| \Bigg\} \\
&\le \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} |d_{ij}(s)||f_j(\psi_j(s-\tau_{ij}(s))) \\
&\quad - f_j(\bar{\psi}_j(s-\tau_{ij}(s)))| \\
&\quad + \sum_{j=1}^{n} |c_{ij}(s)||g_j(\psi_j(s)) - g_j(\bar{\psi}_j(s))| \Bigg] \Delta s \Bigg\} \\
&\le \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} \bar{d}_{ij} L_j^f |\psi_j(s-\tau_{ij}(s)) - \bar{\psi}_j(s-\tau_{ij}(s))| \\
&\quad + \sum_{j=1}^{n} \bar{c}_{ij} L_j^g |\psi_j(s) - \bar{\psi}_j(s)| \Bigg] \Delta s \Bigg\} \\
&\le \max_{i\in\Lambda} \sup_{t\in\mathbb{T}} \Bigg\{ \int_{-\infty}^{t} e_{-a_i}(t,\sigma(s)) \\
&\quad \times \Bigg[\sum_{j=1}^{n} \bar{d}_{ij} L_j^f + \sum_{j=1}^{n} \bar{c}_{ij} L_j^g \Bigg] \Delta s \Bigg\} \|z - \bar{z}\| \\
&\le \max_{i\in\Lambda} \Bigg\{ \frac{1}{\underline{a}_i} \Bigg[\sum_{j=1}^{n} \bar{d}_{ij} L_j^f + \sum_{j=1}^{n} \bar{c}_{ij} L_j^g \Bigg] \Bigg\} \|z - \bar{z}\| \\
&= Q\|z - \bar{z}\|
\end{aligned}$$

This implies that the mapping Φ is a contraction mapping since $Q < 1$. Hence, Φ has exactly one fixed point z^* in $\mathbb{B}^*$ such that $\Phi(z^*) = z^*$. Otherwise, it is easy to verify that z^* satisfies system (1). Thus, system (1) has a unique almost periodic solution in $\mathbb{B}^*$. This completes the proof. ■

Theorem 3.2 Assume that $(H_1) - (H_3)$ and conditions of Theorem 3.1 hold. Suppose that $A - (CL^g + DL^f)$ is an M-matrix, where $A = \mathrm{diag}(\underline{a}_1, \underline{a}_2, \cdots, \underline{a}_n)_{n\times n}$, $C = (\bar{c}_{ij})_{n\times n}, D = (\bar{d}_{ij})_{n\times n}$, $L^g = \mathrm{diag}(L_1^g, L_2^g, \cdots, L_n^g), L^f = \mathrm{diag}(L_1^f, L_2^f, \cdots, L_n^f)$, then the almost periodic solution of system (1) is globally exponentially stable.

Proof: From Theorem 3.1, we know that (1) has an almost periodic solution $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$. Suppose that $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ be an arbitrary solution of (1).

Let $u(t) = x(t) - x^*(t)$, then for $i \in \Lambda$, system (1) can be written as

$$\begin{aligned}
u_i^{\Delta}(t) &= -a_i(t)u_i(t) + \sum_{j=1}^{n} c_{ij}(t) p_j(u_j(t)) \\
&\quad + \sum_{j=1}^{n} d_{ij}(t) q_j(u_j(t-\tau_{ij}(t))), t \in \mathbb{T}^+, \quad (6)
\end{aligned}$$

where $p_j(u_j(t)) = g_j(x_j(t)) - g_j(x_j^*(t))$, $q_j(u_j(t-\tau_{ij}(t))) = f_j(x_j(t-\tau_{ij}(t))) - f_j(x_j^*(t-\tau_{ij}(t)))$. The initial condition of system (6) is $\Psi(s) = \psi(s) - x^*(s),\ s \in [-\hat{\tau}, 0]_{\mathbb{T}}$.

From (H_2), we can get

$$|p_j(u_j)| \le L_j^g |u_j|, \ \ |q_j(u_j)| \le L_j^f |u_j|, \ j \in \Lambda.$$

Let $V_i(t) = |u_i(t)|$, then the upper right derivative

$D^+V^\Delta(t)$ along the solutions of system (6) is as follows:

$$\begin{aligned} D^+V_i^\Delta(t) &= \text{sign}(u_i(t))u_i^\Delta(t) \\ &\le -\underline{a}_i|u_i(t)| + \sum_{j=1}^n \bar{c}_{ij}L_j^g|u_j(t)| + \sum_{j=1}^n \bar{d}_{ij}L_j^f|\bar{u}_j(t)| \\ &\le -\underline{a}_iV_i(t) + \sum_{j=1}^n \bar{c}_{ij}L_j^gV_j(t) + \sum_{j=1}^n \bar{d}_{ij}L_j^f\overline{V}_j(t), \end{aligned}$$

that is

$$D^+V^\Delta(t) \le (-A + CL^g)V(t) + DL^f\overline{V}(t),\ t \in \mathbb{T}^+.$$

For $A - (CL^g + DL^f)$ is an M-matrix, by Lemma 2.3, there exist constants $\alpha > 0,\ r > 0$, such that

$$V_i(t) = |u_i(t)| \le r \sup_{\delta\in[-\hat{\tau},0]_{\mathbb{T}}} |\psi_i(\delta) - x^*(\delta)|e_{\ominus\alpha}(t, 0),\ i \in \Lambda,$$

that is

$$\begin{aligned} |x_i(t) - x_i^*(t)| &\le r \sup_{\delta\in[-\hat{\tau},0]_{\mathbb{T}}} |\psi_i(\delta) - x^*(\delta)|e_{\ominus\alpha}(t, 0) \\ &\le \frac{r}{e_{\ominus\alpha}(0, \delta)}\|\psi - x^*\|e_{\ominus\alpha}(t, \delta),\ i \in \Lambda. \end{aligned}$$

Let $N = N(\delta) = \frac{r}{e_{\ominus\alpha}(0,\delta)}$, then

$$\|x - x^*\| \le N\|\psi - x^*\|e_{\ominus\alpha}(t, \delta),\ t \in \mathbb{T}^+.$$

Therefore, the almost periodic solution $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ of system (1) is globally exponentially stable. This completes the proof. ■

IV. Examples and Simulations

Example 1. Assume that $\mathbb{T} = \mathbb{R}$, $\Lambda = \{1, 2\}$. Take

$$a_i(t) = \begin{bmatrix} 2 + \sin t & 0 \\ 0 & 2 - \cos(t) \end{bmatrix},$$

$$c_{ij}(t) = \begin{bmatrix} 0 & 0.5\sin\sqrt{2}t \\ 0.3\sin t & 0 \end{bmatrix},$$

$$d_{ij}(t) = \begin{bmatrix} 0.4\sin 2t & 0 \\ 0 & 0.5\sin\sqrt{2}t \end{bmatrix},$$

$$I_i(t) = \begin{bmatrix} 3\sin\sqrt{5}t \\ 3\cos 2t \end{bmatrix},\quad \tau(t) = \cos^2 t,$$

$$f_j(y_j(t - \tau_{ij}(t))) = \tanh(y_i - \tau(t))),$$

$$g_j(y_j) = \frac{1}{2}(|y_j + 1| - |y_j - 1|).$$

in system (1). By a direct calculation, one can derive that $\underline{a}_i = 1,\ \lambda_i = 1,\ L_j^f = L_j^g = 1,\ i, j \in \Lambda,\ Q = 0.9 < 1$, and

$$A - (CL^g + DL^f) = \begin{bmatrix} 0.6 & -0.5 \\ -0.3 & 0.5 \end{bmatrix}$$

is an M-matrix.

According to Theorems 3.1 and 3.2, system (1) has an almost periodic solution, which is exponential stability, see Fig.1.

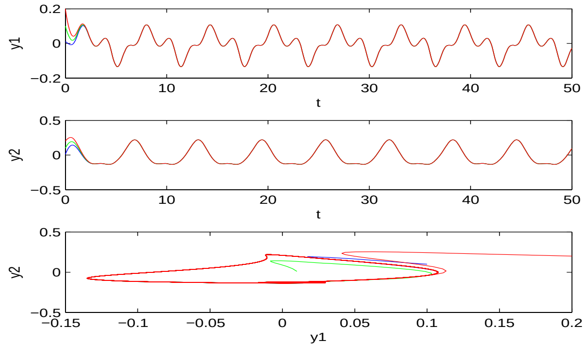

Fig. 1. Transient response of states y_1, y_2 in example 1 with initial values $(0.01, 0.01), (0.1, 0.1)$ and $(0.2, 0.2)$.

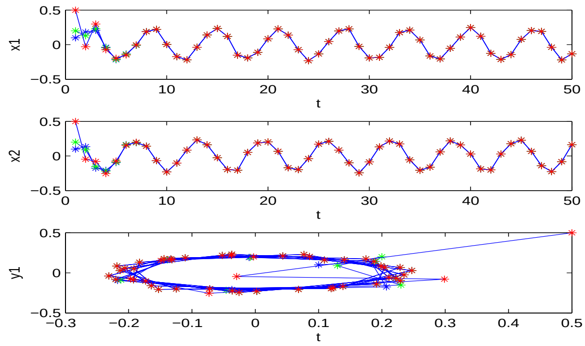

Fig. 2. Transient response of states y_1, y_2 in example 2 with initial values $(0.1, 0.1), (0.2, 0.2)$ and $(0.5, 0.5)$.

Example 2. Assume that $\mathbb{T} = \mathbb{Z}$, $\Lambda = \{1, 2\}$. Take

$$a_i(t) = \begin{bmatrix} 0.5 + 0.1\sin\frac{\pi}{2}t & 0 \\ 0 & 0.5 - 0.1\cos\frac{\pi}{2}t \end{bmatrix},$$

$$c_{ij}(t) = \begin{bmatrix} 0 & 0.05\sin\sqrt{2}t \\ 0.03\sin t & 0 \end{bmatrix},$$

$$d_{ij}(t) = \begin{bmatrix} 0.04\sin 2t & 0 \\ 0 & 0.05\sin\sqrt{2}t \end{bmatrix},$$

$$I_i(t) = \begin{bmatrix} 0.3\sin\sqrt{5}t \\ 0.3\cos 2t \end{bmatrix},\quad \tau(t) = \sin t$$

$$f_j(y_j(t - \tau_{ij}(t))) = \tanh(y_i - \tau(t))),$$

$$g_j(y_j) = \tanh(y_j).$$

in system (1). By a direct calculation, one can derive that $\underline{a}_i = 0.4,\ \lambda_i = 1,\ L_j^f = L_j^g = 1,\ i, j \in \Lambda,\ Q = 0.225 < 1$, and

$$A - (CL^g + DL^f) = \begin{bmatrix} 0.36 & -0.05 \\ -0.03 & 0.35 \end{bmatrix}$$

is an M-matrix.

According to Theorems 3.1 and 3.2, system (1) has an almost periodic solution, which is exponential stability, see Fig.2.

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