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New pth moment stable criteria of stochastic neural networks

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Abstract—In this paper, the issue of pth moment stability of a class of stochastic neural networks with mixed delays is investigated. By establishing two integro-differential inequalities, some new sufficient conditions ensuring pth moment exponential stability are obtained. Compared with some previous publications, our results generalize some earlier works reported in the literature, and remove some strict constraints of time delays and kernel functions. Two numerical examples are presented to illustrate the validity of the main results.

Keywords—neural networks, stochastic, pth moment stable, time-varying delays, distributed delays.

I. INTRODUCTION

DURING the past few decades, recurrent neural networks (see [1]-[6]), such as Hopfield neural networks, cellular neural networks and other networks have been extensively studied, and successfully applied in many areas such as combinatorial optimization, signal processing and pattern recognition. In particular, the stability problem of neural networks has received much research attention since, when applied, the neural network is sometimes assumed to have only one equilibrium that is globally stable. However, because of the finite switching speed of neurons and amplifiers, time delay is unavoidable. It may cause undesirable dynamic network behaviors such as oscillation and instability. Therefore, there has been a growing research interest on the stability analysis problems for delayed neural networks, and many excellent papers and monographs have been available. On the other hand, a real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random. As Haykin [7] pointed out that in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations form the release of neurotransmitters and other probabilistic causes, therefore, stochastic effects should be taken into account. Up to now, many sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the mean square asymptotic or exponential stability for stochastic delayed neural networks (see [8]-[12]), and the references cited therein.

As a generalized form of mean square exponential stability, pth moment exponential stability has been a growing research interest in recent years (see [13]-[19]). In Ref [15], by using the method of variation parameter and inequality technique, Sun and Cao generalized the results derived by Wan and Sun [9] from mean square exponential stability to pth moment exponential stability for a class of stochastic recurrent neural networks with time-varying delays. For discarding the strict constraint of time delays [15], Huang and He [16] established an improved criterion by using Halanay inequality. However, it is worth noting that some of the constraint conditions for time delays or kernel functions in previous literature such as [1], [3], [15] are strict, and few authors have considered the problem of pth moment exponential stability of stochastic neural networks with mixed delays.

Motivated by the above discussions, the main aim of this paper is to study pth moment exponential stability of a class of stochastic neural networks with mixed delays. By establishing two new integro-differential inequalities, two new sufficient conditions ensuring pth moment exponential or asymptotical stability are obtained, respectively. These results obtained in our paper extend some earlier results reported in the literature, and remove some strict constraints of time delays and kernel functions. Two simulation examples are provided to show the validity of the main results.

II. PRELIMINARIES

In this paper, we will study the generalized stochastically perturbed neural network model with mixed delays as follows

$$\begin{cases} dx_i(t) = [-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t))) \\ \qquad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} k_{ij}(t-s) f_j(x_j(s))ds]dt \\ \qquad + \sum_{j=1}^{n} \sigma_{ij}(t, x_j(t), x_j(t-\tau(t)))d\omega_j(t), t > 0 \\ x_i(t) = \eta_i(t), t \le 0, i = 1, 2, \cdots, n, \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is the state vector associated with neurons; $c_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and the external stochastic perturbations; a_{ij}, b_{ij} and d_{ij} represent the connection weight and the delayed connection weight respectively; f_i is activation function, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$, $0 < \tau(t) \le \tau$ is transmission delay. Moreover, $\omega_j(t)$ is a standard Brown motion defined on a complete probability space $(\Omega, \mathscr{F}, P)$ with a

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natural filtration $\mathscr{F}_{t\geq 0}$(i.e.,$\mathscr{F}_t = \sigma\{\omega(x(s)) : -\infty < s \leq t\}$), and σ_{ij} is the diffusion coefficient. Kernel functions $k_{ij}(t) > 0$ for $i, j = 1, 2, \ldots, n$, are real-valued nonnegative continuous functions defined on $[0, \infty)$. The initial conditions for system (1) are given in the form

$$x(t) = \eta(t), -\infty < t \leq 0, \tag{2}$$

where $\eta \in L^p_{\mathscr{F}_0}((-\infty, 0], \mathscr{R}^n)$, here $L^p_{\mathscr{F}_0}((-\infty, 0], \mathscr{R}^n)$ is the family of all $\mathscr{F}_0$ measurable $C((-\infty, 0], \mathscr{R}^n)-$valued random variable satisfying that $\sup_{-\infty<t\leq 0} E|\eta(t)|^p < \infty$, $C((-\infty, 0], \mathscr{R}^n)$ denote the family of all continuous $\mathscr{R}^n-$valued functions $\phi(t)$ on $(-\infty, 0]$ with the norm $\|\eta\|^p_\Delta = \sup_{-\infty<t\leq 0} |\eta(t)|^p$. Throughout this paper, the following standard hypothesis are needed

(H_1) Function $f_i(x)$ satisfies the Lipschitz condition. That is, for each $i = 1, 2, \ldots, n$, there exists constant $l_i > 0$, such that $|f_i(x) - f_i(y)| \leq l_i|x - y|, \forall x, y \in \mathscr{R}$, where $l_i (i = 1, 2, \cdots, n)$ is Lipschitz constant.

(H_2) There are nonnegative constants $\mu_i, \nu_i (i = 1, 2, \cdots, n)$ such that $trace[\sigma^T(t; x, y)\sigma(t; x, y)] \leq \sum_{i=1}^n (\mu_i x_i^2 + \nu_i y_i^2)$ $\forall (t; x, y) \in \mathscr{R}^+ \times \mathscr{R}^n \times \mathscr{R}^n$.

(H_3) Assume that $f(0) \equiv 0, \sigma(t, 0, 0) \equiv 0$.

(H_4) $\int_0^\infty k_{ij}(t)dt = 1$, $i, j = 1, 2, \cdots, n$.

(H_5) There exists a positive number ε such that $\int_0^\infty e^{\varepsilon t} k_{ij}(t)dt \triangleq \overline{k}_{ij} < \infty$.

Definition 2.1: ([20]). The trivial solution of system (1) is said to be pth moment exponentially stable if there exists a pair of positive constants λ and α such that

$$E\|x(t, t_0, \eta)\|^p \leq \alpha E\|\eta\|^p_\Delta e^{-\lambda(t-t_0)}, t \geq t_0$$

holds for any t_0 and $\eta \in L^p_{\mathscr{F}_{t_0}}((-\infty, 0], \mathscr{R}^n)$. Especially, when $p = 2$, it is called to be exponentially stable in mean square.

Lemma 2.1: ([21]) Let $p \geq 2$ and $a > 0, b > 0$, then

$$a^{p-1}b \leq \frac{(p-1)a^p}{p} + \frac{b^p}{p},$$

and

$$a^{p-2}b^2 \leq \frac{(p-2)a^p}{p} + \frac{2b^p}{p}.$$

III. Main results

In this section, we will discuss the global stability of the trivial solution of system (1). To proceed, we first generalize two important inequalities as follows.

Lemma 3.1: Assume that positive scalars γ, l, h satisfy $l + h < \gamma$, $y(t)$ is a nonnegative continuous function on $(-\infty, +\infty)$ and satisfies the following inequality on interval $[t_0, +\infty)$

$$D^+y(t) \leq -\gamma y(t) + hy(t - \tau(t)) + l\int_{-\infty}^t k(t - s)y(s)ds,$$

where $0 < \tau(t) \leq \tau$, $t_0 \geq 0, \int_0^\infty k(s)ds = 1, k(s) > 0$, τ is a constant, then as $t \geq t_0$,we have

$$y(t) \leq \sup_{-\infty<\theta\leq 0} y(t_0 + \theta) \triangleq y_{t_0}, and \lim_{t\to+\infty} y(t) = 0.$$

Proof. We will complete the proof in two steps. In step 1, we will prove that $y(t) \leq y_{t_0}$. In step 2, we'll prove that $\lim_{t\to+\infty} y(t) = 0$.

Step 1: we first prove that for any positive constant $d > 1$, the following inequality holds

$$y(t) < dy_{t_0}, t \geq t_0. \tag{3}$$

Since for any $t \in (-\infty, t_0)$, $y(t) \leq \sup_{-\infty<\theta\leq 0} y(t_0 + \theta) = y_{t_0}$. If $y_{t_0} = 0$, then we get $0 \leq y(t) \leq 0$, namely $y(t) \equiv 0$. Thus, we always assume that $y_{t_0} > 0$. When $t \leq t_0$, we have $y(t) \leq y_{t_0} < dy_{t_0}$. If inequality (3) does not hold, there must exist $t_1 > t_0$ such that

$$y(t_1) = dy_{t_0}, y(t) < dy_{t_0}, \forall t < t_1,$$

which implies that $D^+y(t_1) \geq 0$. On the other hand

$$\begin{aligned} D^+y(t_1) &\leq -\gamma dy_{t_0} + hdy_{t_0} + l\int_{-\infty}^{t_1} dk(t_1 - s)y_{t_0}ds \\ &= [-\gamma + h + l\int_{-\infty}^{t_1} k(t_1 - s)ds]dy_{t_0} \\ &= [-\gamma + h + l\int_0^{+\infty} k(s)ds]dy_{t_0} \\ &= [-\gamma + h + l]dy_{t_0} < 0. \end{aligned} \tag{4}$$

This contradicts to $D^+y(t_1) \geq 0$, so inequality (3) holds. According to the arbitrary property of positive constant d, we obtain

$$y(t) \leq \sup_{-\infty<\theta\leq 0} y(t_0 + \theta) \triangleq y_{t_0}, \forall t \geq t_0. \tag{5}$$

Step 2: In what follows, we will prove $\lim_{t\to+\infty} y(t) = 0$.

From inequality (5), we know that $y(t)$ is a bounded continuous function, thus when $t \to +\infty$, the upper limit(noted by p) of $y(t)$ exists, namely

$$\overline{\lim}_{t\to+\infty} y(t) = p, p \geq 0. \tag{6}$$

The remaining proof is to prove $p = 0$.

If it's not true, there must exist arbitrary positive constant $\varepsilon > 0$, and constant $T_1 > t_0$ such that

$$y(t - \tau(t)) < p + \varepsilon, \quad y(t) < p + \varepsilon, \quad \forall t > T_1.$$

On the other hand, since $\int_0^\infty k(s)ds = 1$, there must exist $T_2 > t_0$ such that

$$\int_t^{+\infty} k(s)ds < \varepsilon, \forall t \geq T_2.$$

Set $T = \max\{T_1, T_2\}$, when $t \geq T$, we have

$$\begin{aligned} D^+y(t) &\leq -\gamma y(t) + hy(t - \tau(t)) + l\int_{-\infty}^t k(t - s)y(s)ds \\ &< -\gamma y(t) + h(p + \varepsilon) + l\varepsilon y_{t_0} + (p + \varepsilon)l. \end{aligned} \tag{7}$$

By direct calculation, we get

$$y(t) \leq y(t_0)\exp\{-\gamma(t - t_0)\} + \frac{1}{\gamma}[(p + \varepsilon)h + \varepsilon l y_{t_0} + pl + \varepsilon l].$$

According to inequality (6), we can obtain

$$p \leq \frac{1}{\gamma}[h\varepsilon + \varepsilon l y_{t_0} + pl + ph + \varepsilon l].$$

In views of the arbitrary property of ε, we have $p \le \frac{pl+ph}{\gamma}$, namely $\gamma \le h+l$, which contradict to the assumption $\gamma > h+l$, thus $\lim_{t\to+\infty} y(t)=0$ holds. The proof is completed.

Lemma 3.2: Assume that positive scalars γ, l, h satisfy $kl+h<\gamma$, $y(t)$ is a nonnegative continuous function on $(-\infty,+\infty)$ and satisfies the following inequality on interval $[t_0,+\infty)$

$$D^+y(t) \le -\gamma y(t) + hy(t-\tau(t)) + l\int_{-\infty}^{t} k(t-s)y(s)ds,$$

where $0 \le \tau(t) \le \tau$, $t_0 \ge 0, k=\int_0^\infty e^{\varepsilon s}k(s)ds, k(s)>0$, τ is a constant, then as $t \ge t_0$, we have

$$y(t) \le y_{t_0} e^{-\varepsilon(t-t_0)}, \tag{8}$$

where $y_{t_0} \triangleq \sup_{-\infty<\theta\le 0} y(t_0+\theta)$, ε is the unique positive solution of the following equation

$$\varepsilon = \gamma - lk - he^{\varepsilon\tau}.$$

Proof. Set $\widetilde{y}(t) = d \cdot y_{t_0} e^{-\varepsilon(t-t_0)}$, $d>1$. If inequality (8) is not true, then we can have $y(t) > y_{t_0} e^{-\varepsilon(t-t_0)}$. On the other hand, since $y(t_0) \le y_{t_0} < \widetilde{y}(t)$, thus there must exist t^* such that

$$y(t) < \widetilde{y}(t), \forall t<t^*; \qquad y(t^*) = \widetilde{y}(t^*).$$

Namely

$$D^+y(t^*) - D^+\widetilde{y}(t^*) \ge 0, \forall t<t^*. \tag{9}$$

In views of the condition of Lemma 3.2, we get

$$\begin{aligned} D^+y(t^*) &\le -\gamma y(t^*) + hy(t^*-\tau(t^*)) + l\int_{-\infty}^{t^*} k(t^*-s)y(s)ds \\ &< -\gamma\widetilde{y}(t^*) + hy(t^*-\tau(t^*)) + l\int_{-\infty}^{t^*} k(t^*-s)\widetilde{y}(s)ds \\ &< -\gamma\widetilde{y}(t^*) + h\widetilde{y}(t^*-\tau(t^*)) + l\int_{-\infty}^{t^*} k(t^*-s)\widetilde{y}(s)ds \\ &\le -\gamma y_{t_0} e^{-\varepsilon(t^*-t_0)} \\ &\quad + hy_{t_0} e^{-\varepsilon(t^*-\tau-t_0)} + ly_{t_0} e^{-\varepsilon(t^*-t_0)} \int_0^{+\infty} e^{\varepsilon s}k(s)ds \\ &= [-\gamma + lk + he^{\varepsilon\tau}] y_{t_0} e^{-\varepsilon(t^*-t_0)} \\ &= -\varepsilon y_{t_0} e^{-\varepsilon(t^*-t_0)} = D^+\widetilde{y}(t^*). \end{aligned} \tag{10}$$

This contradict to inequality (9). From the arbitrary of d, one can see that inequality (8) holds.

Remark 1. If $l=0$ and replacing $y(t-\tau(t))$ as $\overline{y(t-\tau(t))} \triangleq \sup_{t-\tau\le s\le t} y(s)$, Lemma 3.2 becomes the classical Halanay inequality, thus, this lemma can be regard as a generalized form of Halanay inequality.

Theorem 3.1: Under the assumptions $(H_1)-(H_3)$ and (H_5), the trivial solution of system (1) is pth moment exponentially stable($p \ge 2$), if there exist positive scalars $\lambda_i>0, i=1,2,\cdots,n$ such that

$$-A_1 + A_2 + k'A_3 < 0, \tag{11}$$

where

$$\begin{aligned} A_1 = \min_{1\le i\le n}\{& c_i p - \sum_{j=1}^n (p-1)|a_{ij}l_j| - \sum_{j=1}^n (p-1)|b_{ij}l_j| \\ &- \sum_{j=1}^n (p-1)|d_{ij}| - \sum_{j=1}^n \mu_j \frac{(p-1)(p-2)}{2} \\ &- \sum_{j=1}^n \nu_j \frac{(p-1)(p-2)}{2} - \sum_{j=1}^n \frac{\lambda_j}{\lambda_i}(|a_{ji}l_i| - \mu_i(p-1))\}, \end{aligned}$$

$$A_2 = \max_{1\le i\le n}\{\sum_{j=1}^n \frac{\lambda_j}{\lambda_i}[|b_{ji}l_i| + \nu_i(p-1)\},$$

$$A_3 = \max_{1\le i\le n}\{\sum_{j=1}^n \frac{\lambda_j}{\lambda_i}|d_{ji}||l_i|^p\}, k' = \max_{1\le i,j\le n}\{\overline{k}_{ij}\}.$$

Proof. Constructing Lyapunov functional for system (1) by

$$V(x(t)) = \sum_{i=1}^n \lambda_i |x_i(t)|^p.$$

Similar to the disposal route in [1], [16], by It$\hat{o}$'s formula, we have

$$\begin{aligned} &\mathscr{L}V(x(t)) \\ &= \sum_{i=1}^n \lambda_i\{p|x_i(t)|^{p-1} sgn\{x_i(t)\}[-c_i x_i(t) \\ &+ \sum_{j=1}^n a_{ij}f_j(x_j(t)) + \sum_{j=1}^n b_{ij}f_j(x_j(t-\tau)) \\ &+ \sum_{j=1}^n d_{ij}\int_{-\infty}^t k_{ij}(t-s)f_j(x_j(s))ds]\} \\ &+ \frac{1}{2}p(p-1)\sum_{i=1}^n \lambda_i|x_i(t)|^{p-2}\sum_{j=1}^n \sigma_{ij}^2 sgn\{x_i(t)\} \\ &\le \sum_{i=1}^n \lambda_i\{-c_i p|x_i(t)|^p + \sum_{j=1}^n p|a_{ij}l_j||x_i(t)|^{p-1}|x_j(t)| \\ &+ \sum_{j=1}^n p|b_{ij}l_j||x_i(t)|^{p-1}|x_j(t-\tau)| \\ &+ \sum_{j=1}^n |d_{ij}|\int_{-\infty}^t k_{ij}(t-s)p|x_i(t)|^{p-1}|f_j(x_j(s))|ds\} \\ &+ \frac{1}{2}p(p-1)\sum_{i=1}^n \lambda_i|x_i(t)|^{p-2}\sum_{j=1}^n(\mu_j x_j^2(t) + \nu_j x_j^2(t-\tau)) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{n} \lambda_i \{-c_i p|x_i(t)|^p + \sum_{j=1}^{n} p|a_{ij}l_j||x_i(t)|^{p-1}|x_j(t)| \\
&+ \sum_{j=1}^{n} p|b_{ij}l_j||x_i(t)|^{p-1}|x_j(t-\tau)| \\
&+ \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)p|x_i(t)|^{p-1}|f_j(x_j(s))|ds \\
&+ \frac{p(p-1)}{2} \sum_{j=1}^{n} \mu_j |x_i(t)|^{p-2} x_j^2(t) \\
&+ \frac{p(p-1)}{2} \sum_{j=1}^{n} \nu_j |x_i(t)|^{p-2} x_j^2(t-\tau)\} \\
&\leq \sum_{i=1}^{n} \lambda_i \{-c_i p|x_i(t)|^p + \sum_{j=1}^{n} |a_{ij}l_j|[(p-1)|x_i(t)|^p \\
&+ |x_j(t)|^p] + \sum_{j=1}^{n} |b_{ij}l_j|[(p-1)|x_i(t)|^p + |x_j(t-\tau)|^p] \\
&+ \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)[(p-1)|x_i(t)|^p + |f_j(x_j(s))|^p]ds \\
&+ \frac{(p-1)}{2} \sum_{j=1}^{n} \mu_j [(p-2)|x_i(t)|^p + 2|x_j(t)|^p] \\
&+ \frac{(p-1)}{2} \sum_{j=1}^{n} \nu_j [(p-2)|x_i(t)|^p + 2|x_j(t-\tau)|^p]\} \\
&= \sum_{i=1}^{n} \lambda_i \{-c_i p|x_i(t)|^p + \sum_{j=1}^{n} (p-1)|a_{ij}l_j||x_i(t)|^p \\
&+ \sum_{j=1}^{n} |a_{ij}l_j||x_j(t)|^p + \sum_{j=1}^{n} (p-1)|b_{ij}l_j||x_i(t)|^p \\
&+ \sum_{j=1}^{n} |b_{ij}l_j||x_j(t-\tau)|^p \\
&+ \sum_{j=1}^{n} (p-1)|d_{ij}||x_i(t)|^p \int_{-\infty}^{t} k_{ij}(t-s)ds \\
&+ \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)|f_j(x_j(s))|^p ds \\
&+ \sum_{j=1}^{n} \mu_j \frac{(p-1)(p-2)}{2} |x_i(t)|^p + \sum_{j=1}^{n} \mu_j (p-1)|x_j(t)|^p \\
&+ \sum_{j=1}^{n} \nu_j \frac{(p-1)(p-2)}{2} |x_i(t)|^p + \sum_{j=1}^{n} \nu_j (p-1)|x_j(t-\tau)|^p\} \\
&= \sum_{i=1}^{n} \lambda_i \{[-c_i p + \sum_{j=1}^{n} (p-1)|a_{ij}l_j| + \sum_{j=1}^{n} (p-1)|b_{ij}l_j| \\
&+ \sum_{j=1}^{n} (p-1)|d_{ij}| + \sum_{j=1}^{n} \mu_j \frac{(p-1)(p-2)}{2} \\
&+ \sum_{j=1}^{n} \nu_j \frac{(p-1)(p-2)}{2}] \cdot |x_i(t)|^p \\
&+ \sum_{j=1}^{n} [|a_{ij}l_j| + \mu_j(p-1)] \cdot |x_j(t)|^p \\
&+ \sum_{j=1}^{n} [|b_{ij}l_j| + \nu_j(p-1)] \cdot |x_j(t-\tau)|^p \\
&+ \sum_{j=1}^{n} |d_{ij}| \int_{-\infty}^{t} k_{ij}(t-s)|f_j(x_j(s))|^p ds\} \\
&= \sum_{i=1}^{n} \lambda_i \{[-c_i p + \sum_{j=1}^{n} (p-1)|a_{ij}l_j| \\
&+ \sum_{j=1}^{n} (p-1)|d_{ij}| + \sum_{j=1}^{n} \mu_j \frac{(p-1)(p-2)}{2} \\
&+ \sum_{j=1}^{n} \nu_j \frac{(p-1)(p-2)}{2} + \sum_{j=1}^{n} (p-1)|b_{ij}l_j| \\
&+ \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i} (|a_{ji}l_i| + \mu_i(p-1))] \cdot |x_i(t)|^p \\
&+ \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i} [|b_{ji}l_i| + \nu_i(p-1)] \cdot |x_i(t-\tau)|^p \\
&+ \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i} |d_{ji}||l_i|^p \int_{-\infty}^{t} k_{ji}(t-s)|x_i(s)|^p ds\} \\
&\leq -A_1 V(x(t)) + A_2 V(x(t-\tau)) \\
&+ A_3 \int_{-\infty}^{t} k'(t-s)V(x(s))ds,
\end{aligned} \tag{12}
$$

where $k'(t-s) \triangleq \max_{1\leq i,j\leq n} k_{ji}(t-s)$

On the other hand, by Itô's formula, for all $t > 0$, we have

$$
\begin{aligned}
D^+V(t, x(t)) &= \mathscr{L}V(t, x(t))dt \\
&+ \frac{\partial V(t, x(t))}{\partial x(t)} \sigma(t, x(t), x(t-\tau))d\omega(t).
\end{aligned} \tag{13}
$$

Taking mathematical expectation of the both side of equation (13), we have

$$
\begin{aligned}
D^+EV(t, x(t)) &\leq -A_1 EV(x(t)) + A_2 EV(x(t-\tau)) \\
&+ A_3 \int_{-\infty}^{t} k'(t-s)EV(x(s))ds.
\end{aligned} \tag{14}
$$

In views of Lemma 3.2, we can get

$$
E\|x(t)\|^p \leq \lambda^{-1} E\|\eta\|_{\triangle}^p e^{-\varepsilon t}, \tag{15}
$$

where $\lambda = \max_{1\leq i\leq n}\{\lambda_i\}$, which complete the proof.

Remark 2. In previous publications such as [1], [3], the kernel functions are usually assumed to satisfy $(H_4), (H_5)$ and $\int_0^\infty s e^{\varepsilon s} k(s)ds < \infty$. Time-variance delays are usually required to satisfy $0 < \dot{\tau}(t) < 1$ or $0 < \dot{\tau}(t) \leq \mu$. In Theorem 3.1, we only require time delays are bounded, the derivative of time-variance delays can be unbounded, or even non-differentiable. Kernel functions only need to satisfy $\int_0^\infty e^{\varepsilon t} k_{ij}(t)dt \triangleq \overline{k}_{ij} < \infty$. These requirements enlarge the selections of time delays and kernel functions.

Remark 3. When $l = 0$, Theorem 3.1 becomes the Theorem 3.3 and Corollary 3.4 in [16]. Thus, the criteria in [16] can be regarded as special case of ours.

Theorem 3.2: Under the assumptions $(H_1) - (H_4)$, the trivial solution of system (1) is pth moment uniformly and asymptotically stable($p \geq 2$), if there exist positive scalars $\lambda_i > 0, i = 1, 2, \cdots, n$ such that

$$-A_1 + A_2 + A_3 < 0, \tag{16}$$

where

$$\begin{aligned} A_1 = \min_{1\leq i\leq n}\{ & c_i p - \sum_{j=1}^{n}(p-1)|a_{ij}l_j| - \sum_{j=1}^{n}(p-1)|b_{ij}l_j| \\ & - \sum_{j=1}^{n}(p-1)|d_{ij}| - \sum_{j=1}^{n}\mu_j\frac{(p-1)(p-2)}{2} \\ & - \sum_{j=1}^{n}\nu_j\frac{(p-1)(p-2)}{2} - \sum_{j=1}^{n}\frac{\lambda_j}{\lambda_i}(|a_{ji}l_i| - \mu_i(p-1))\}, \end{aligned}$$

$$A_2 = \max_{1\leq i\leq n}\{\sum_{j=1}^{n}\frac{\lambda_j}{\lambda_i}[|b_{ji}l_i| + \nu_i(p-1)\},$$

$$A_3 = \max_{1\leq i\leq n}\{\sum_{j=1}^{n}\frac{\lambda_j}{\lambda_i}|d_{ji}||l_i|^p\}.$$

Proof. In views of inequality (14) and Lemma 3.1, we can easily obtain this conclusion, which complete the proof.

Remark 4. Theorem 3.1 is invalid, when time-variance delays are only bounded and the kernel functions only satisfy (H_4). In this case, Theorem 3.2 provides a useful complement criterion.

IV. Numerical Examples

In this section, two numerical examples will be presented to show the validity of the main results derived above.

Example 1. Consider the following stochastic neural network with mixed delays:

$$\begin{cases} d\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = -[\begin{bmatrix} 2.3 & 0 \\ 0 & 2.3 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ + \begin{bmatrix} 0.21 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}\begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix} \\ + \begin{bmatrix} -0.31 & 0.11 \\ 0.21 & -0.31 \end{bmatrix}\begin{bmatrix} f_{11} \\ f_{12} \end{bmatrix} \\ + \int_{-\infty}^{t}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} \tanh(x_1(s)) \\ \tanh(x_2(s)) \end{bmatrix} ds]dt \\ + \begin{bmatrix} \sqrt{0.2}x_1(t) & b_{12} \\ b_{21} & \sqrt{0.2}x_2(t) \end{bmatrix}\begin{bmatrix} d\omega_1(t) \\ d\omega_2(t) \end{bmatrix}, t > 0 \\ x(t) = [1.01, -1.15]^T, t \leq 0. \end{cases} \tag{17}$$

where

$$f_{11} = \tanh(x_1(t - (2 + 0.01|\sin t|))),$$

$$f_{12} = \tanh(x_2(t - (2 + 0.01|\sin t|))),$$

$$a_{11} = -0.51e^{-2(t-s)}, a_{12} = 0.31e^{-2(t-s)},$$

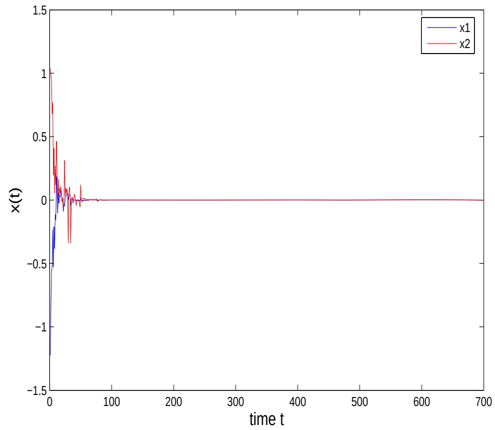

Fig. 1. State trajectories of the stochastic system (17)

$$a_{21} = 0.3e^{-2(t-s)}, a_{22} = 0.25e^{-2(t-s)},$$

$$b_{12} = \sqrt{0.3}x_2(t - (2 + 0.01|\sin t|)),$$

$$b_{21} = \sqrt{0.3}x_1(t - (2 + 0.01|\sin t|)).$$

We can verify that the point $(0, 0)^T$ is an equilibrium point of system (17). By simple calculation, we get that $l_1, l_2 = 1$. Set $\varepsilon = 0.1$, then $\overline{k}_{ij} \approx 2.2, i, j = 1, 2$. Let $\lambda_1, \lambda_2 = 1, p = 2$ then we have

$$A_1 = 4.09, A_2 = 0.52, A_3 = 0.81,$$

$$A_2 + kA_3 < A_1.$$

In views of Theorem 3.1, the equilibrium point $(0, 0)^T$ of the given stochastic system (17) is mean square exponentially stable. Fig.1 shows that the trajectories of the stochastic system (17) converge to zero in mean square exponentially.

Remark 5. From system 17, we can see that kernel function $k(t) = e^{-2t}$. It obviously satisfies (H_5), but does not satisfy (H_4). However, in previous publications, we always assume that (H_4) is held. Thus, our criteria enlarge the selection of kernel functions.

Example 2. Consider stochastic neural network with mixed delays as follows

$$\begin{cases} d\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = [-\begin{bmatrix} 1.72 & 0 \\ 0 & 1.72 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ + \begin{bmatrix} 0.11 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}\begin{bmatrix} \arctan(x_1(t)) \\ \arctan(x_2(t)) \end{bmatrix} \\ + \begin{bmatrix} -0.21 & 0.11 \\ 0.21 & -0.11 \end{bmatrix}\begin{bmatrix} f_{11} \\ f_{12} \end{bmatrix} \\ + \int_{-\infty}^{t}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} \arctan(x_1(s)) \\ \arctan(x_2(s)) \end{bmatrix} ds]dt \\ + \begin{bmatrix} \sqrt{0.2}x_1(t) & b_{12} \\ b_{21} & \sqrt{0.2}x_2(t) \end{bmatrix}\begin{bmatrix} d\omega_1(t) \\ d\omega_2(t) \end{bmatrix}, t > 0 \\ x(t) = [2.8, -2.3]^T, t \leq 0. \end{cases} \tag{18}$$

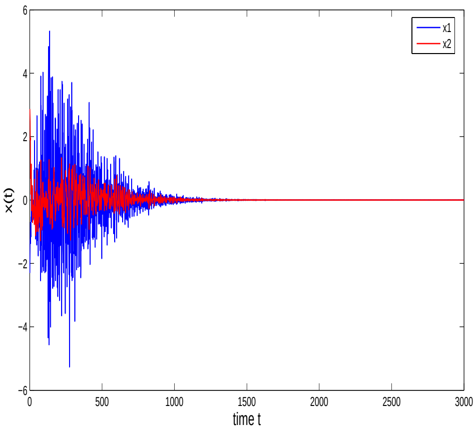

Fig. 2. State trajectories of the stochastic system (17)

where

$$f_{11} == \arctan(x_1(t - (2 + 0.01|\sin t|))),$$

$$f_{12} = \arctan(x_2(t - (2 + 0.01|\sin t|))),$$

$$a_{11} = \frac{-0.21\varepsilon}{2} e^{-\frac{0.11\varepsilon}{2}(t-s)}, a_{12} = \frac{\varepsilon}{2} e^{-\frac{\varepsilon}{2}(t-s)},$$

$$a_{21} = \frac{0.2\varepsilon}{2} e^{-\frac{0.25\varepsilon}{2}(t-s)}, a_{22} = \frac{\varepsilon}{2} e^{-\frac{\varepsilon}{2}(t-s)},$$

$$b_{12} = \sqrt{0.3}x_2(t - (2 + 0.01|\sin t|)),$$

$$b_{21} = \sqrt{0.3}x_1(t - (2 + 0.01|\sin t|)),$$

ε is the same as defined in (H_5). We can verify that the point $(0, 0)^T$ is an equilibrium point of system (18). By simply calculating, we can get $l_1, l_2 = 1$. Set $\varepsilon = 0.1$, $\lambda_1, \lambda_2 = 1, p = 3$ then we have

$$A_1 = 5.16, A_2 = 0.42, A_3 = 0.41,$$

$$A_2 + A_3 < A_1.$$

In views of Theorem 3.2, the equilibrium point $(0, 0)^T$ of the given stochastic system (18) is 3th moment asymptotically stable. Fig.2 shows that the trajectories of the stochastic system (18) converge to zero asymptotically.

Remark 6. From system (18), one can see that kernel function $k(t) = \frac{\varepsilon}{2}e^{-\frac{\varepsilon}{2}t}$. It obviously satisfies (H_4), but does not satisfy (H_5). In this case, Theorem 3.1 lost its validity, but Theorem 3.2 is still valid.

V. Conclusions

By establishing two new integro-differential inequalities, some novel sufficient conditions ensuring pth moment stability for a class of stochastic neural networks are obtained. These new results discard some strict constraints of time delays and kernel functions. Simulation examples show that the results obtained in this paper are valid.

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