

# Fuzzy Controller Design for Ball and Beam System with an Improved Ant Colony Optimization

Yeong-Hwa Chang, Chia-Wen Chang, Hung-Wei Lin, and C.W. Tao

**Abstract**—In this paper, an improved ant colony optimization (ACO) algorithm is proposed to enhance the performance of global optimum search. The strategy of the proposed algorithm has the capability of fuzzy pheromone updating, adaptive parameter tuning, and mechanism resetting. The proposed method is utilized to tune the parameters of the fuzzy controller for a real beam and ball system. Simulation and experimental results indicate that better performance can be achieved compared to the conventional ACO algorithms in the aspect of convergence speed and accuracy.

**Keywords**—Ant colony algorithm, Fuzzy control, ball and beam system.

## I. INTRODUCTION

ANT colony optimization (ACO) algorithm is a population-based evolutionary computation method, inspired to mathematically simulate the social behaviors of ants forage. Similar to genetic algorithms (GAs), ACO is also initiated with a population of candidates that are randomly moved in a multidimensional search space [1]-[3]. However, GA saves only the better generations, thus it may lead to local optima rather than the global optimum. The ACO is a multi-agent approach for solving difficult combinatorial optimization problems, such as the traveling salesman problem (TSP) [4][5]. ACO algorithms have been successfully applied to versatile combinatorial optimization problems, namely vehicle routing [6], quadratic assignment problem (QAP) [8] [9], and job-shop scheduling [7].

Fuzzy rule-based systems (FRBSs) have been extensively applied in many areas of interest, including the controller design, cluster analysis, and image processing. However, the derivation of fuzzy rules is often difficult and requires expert knowledge. Many researchers have proposed optimization methods for fuzzy systems using meta-heuristic algorithms such as tabu search, genetic algorithms, simulated annealing (SA), and evolutionary algorithms (EAs) to overcome this

problem [10].

In the literatures, some ACO-based fuzzy control strategies have been proposed [11][12], however, only the conventional ACO is utilized. In this paper we first propose an improved ACO algorithm with fuzzy pheromone updating. Then the proposed improved optimization algorithm is applied to a real ball and beam system for the position balance control. Compared with other ACO related methods, the superiority of the proposed work can be evaluated from the simulation and experimental results.

This paper is organized as follows. Section II the common concepts of ACO algorithm are presented. The proposed improved ACO algorithm is described in Section III. As for the ball and beam system, the simulation results, based on the proposed algorithm, are addressed in Section IV. Experimental set-up and measurement results are provided in Section V. The concluding remarks are given in Section VI.

## II. ANT COLONY OPTIMIZATION

The ant colony optimization was developed in early 1990s by Dorigo et al. [1]. The ACO technique is one of the meta-heuristic optimization methods and is inspired by the capability of real ants to establish the shortest path from a food source to their nest. Ants lay the chemical substance or the trails of pheromone, on the ground when they move along paths. Each individual ant makes a decision of the moving direction based on the strength of the pheromone trails. The better path is that has higher amount of the pheromone trails on the ground. While more and more ants track on the food source, the shorter path accumulates the more pheromone trails. Thus, most of the ants are attracted to the shorter path, and this behavior of the path selection encourages the positive feedback effect. It is noted that the ants finally will find the shortest path. The ACO algorithm is inspired by the aforementioned observation and can be named artificial ant. Although the ACO imitates the behavior of real ant, the property of artificial ants is distinct from real ants, such as:

- Artificial ants have the ability of memory; they can remember the nodes on the path such that the artificial ants can avoid selecting the same path.
- Artificial ants are not completely blind; they have information of the heuristic function except the pheromone trail.

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- An environment is the discrete time; they lay the pheromone trail on the sample time.

The ant system (AS) was the first algorithm within the ACO algorithms framework [2]. The AS algorithm was successfully applied to the traveling salesman problem, and it had satisfactory results compared with traditional methods such as GA, EA, and SA. The TSP is typically a combinatorial optimization problem, which can be modeled as  $G = (V, E)$ , where  $V = \{1, 2, \dots, N\}$  is a set of nodes and  $E = \{(i, j) | (i, j) \in V \times V\}$  is a set of arcs. The object of the TSP is to find the minimum length Hamiltonian circle on  $G$ . The cost function is defined as the distance of edge  $(i, j)$ . The distance between each pair of nodes  $(x_i, y_i)$  and  $(x_j, y_j)$  is represented by  $d(i, j)$  that is the Euclidean distance between node  $i$  and  $j$ , where  $d(i, j) = d(j, i)$ . Let  $\tau_{ij}$  be the value of pheromone trails. In AS, the probability that an ant  $k$ , currently located at the city  $i$ , chooses the city  $j$  as the next city is given by

$$P_{ij}^k = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, & \text{if } j \in N_i^k \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where  $\eta_{ij} = 1/d_{ij}$  is the heuristic value of moving from city  $i$  to city  $j$ ,  $N_i^k$  is the set of cities remaining to be visited by the ants,  $\alpha$  and  $\beta$  are the relative weights of pheromone and visibility, respectively. The pheromone level of the selected element is updated by

$$\tau_{ij} = \rho\tau_{ij} + \Delta\tau_{ij} \quad (2)$$

where  $\rho$  is a parameter such that  $1-\rho$  represents the evaporation coefficient,  $0 < 1-\rho < 1$ , and  $\Delta\tau_{ij}$  is related to the performance of each ant.

### III. THE PROPOSED APPROACH

For the complicated combinatorial problems, the requirement of long convergence time and trapped in the local optima are the typical drawbacks with conventional ACOs. In this paper, an improved ACO algorithm, including the adaptive  $\alpha$  parameter, the fuzzy pheromone and the clear mechanism, is proposed to improve the convergence performance. In the rest of this paper, the proposed optimization algorithm is named as FACO.

#### A. The adaptive $\alpha$ parameter

The parameters  $\alpha$  and  $\beta$  are the weighing values of the pheromone trails  $\tau_{ij}$ , and the visibility  $\eta_{ij}$ , respectively. In the early iterations, the pheromone trails on the path are not significant, thus the path choice probability can be simply influenced by the visibility. In the later iterations, the trails of

pheromone are significantly accumulated for the best path, and the choosing probability is influenced by the pheromone trails. In order to increase the range of solution space and decrease the number of iteration, the  $\alpha$  can be undated by

$$\alpha = \alpha + \Delta\alpha \quad (3)$$

where  $\Delta\alpha$  is a heuristic value.

#### B. The Fuzzy Pheromone

The pheromone level is updated by (2) and  $\Delta\tau_{ij} = 1/L_{gb}$ ,  $L_{gb}$  is the length of the optimal global tour from the beginning of the trail. In this paper, a modified pheromone updating is given as follows

$$\tau_{ij} = \rho\tau_{ij} + f(L_{gb}) \quad (4)$$

where  $f(\cdot)$  is a fuzzy function. The proposed fuzzy inference system is represented as

$$R_i : \text{IF } X = A_{i1} \text{ THEN } Y = B_i \quad (5)$$

where  $R_i$  is the  $i^{\text{th}}$  fuzzy relation,  $X$ , is the input variable,  $Y$  is the output variable,  $A_{i1}$  is the fuzzy set in antecedent part, and  $B_i$  is the fuzzy set in the consequent part. To calculate the output  $Y$ , the centroid method is used for defuzzification. With the fuzzy pheromone mechanism, the input and output membership functions are shown in Fig. 1, and the fuzzy inference rules are developed in Table 1 with the fuzzy sets VS (very small), S (small), M (medium), L (large), and VL (very large).

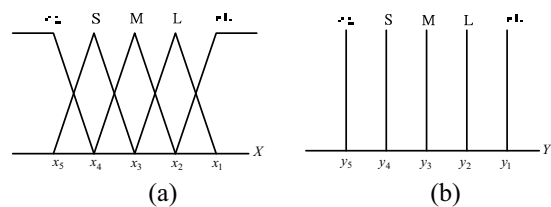


Fig.1. Membership functions of the fuzzy pheromone (a) Input and (b) output membership functions.

Table 1. Rule table of the fuzzy pheromone

$X$	VS	S	M	L	VL
$Y$	VL	L	M	S	VS

#### C. The clear mechanism

Commonly, most of the optimization methods intend to find the global optimal solution sooner. The behaviors to find the optimal solution include exploration and exploitation. The task of exploration is to search unknown regions of objective space and the purpose of exploitation is to find best solution in attractive areas of objective space. In practice, the exploration process can increase the probability to obtain the global optimum and the convergent speed can be improved through

the exploitation iterations. However, it is not easy to simultaneously perform the exploration and exploitation procedures. In conventional ACO methods, the pheromone trails arise quickly on the best path that can lead to most ants toward this path. The pheromone of the best path is much larger than other paths and the selection probability of the other paths could be much smaller. Therefore, the phenomenon of stagnation will be occurred. In this paper, a clear mechanism is proposed as follows

$$\tau_{ij}^{\max} = \begin{cases} 0, & \text{if } P_{\text{random}} < P_{\text{clear}} \\ \tau_{ij}^{\max}, & \text{otherwise} \end{cases} \quad (6)$$

where  $\tau_{ij}^{\max}$  is the pheromone of the best path,  $P_{\text{random}}$  is randomly number,  $0 < P_{\text{random}} < 1$ , and  $P_{\text{clear}}$  is parameter of the clear mechanism. When the clear mechanism is performed, the pheromone trails of the best path will be reset to zero. Consequently, the ants can select other paths and deviate from the local optimal solution.

#### IV. SIMULATION RESULTS

In this section, the proposed FACO is utilized for a ball and beam control system. The configuration of the end-point driven ball and beam system is shown in Fig. 2, where  $o_0$  is the small gear that is mounted on a DC motor which provides the necessary torque of interest,  $o_1$  represents the big gear that can control the angle of the beam,  $o_2$ , and  $o_3$  denote the link and beam, respectively, and  $o_4$  is the ball which rolls on the beam. This system is an underactuated model, and the control objective is to move the ball to the desired position on the beam. The state vector of the ball and beam system is  $x = [x_1 \ x_2 \ x_3 \ x_4]^T$ , where  $x_1$  is the ball position,  $x_2$  is the velocity of the ball,  $x_3$  is the beam angle, and  $x_4$  represents the angular velocity of the beam. According to the Euler-Lagrange method, the mathematical model of a ball and beam system can be represented as follows (Detailed derivations are discussed in the Appendix):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= Ax_1x_4^2 - Ag \sin x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= B(x_1)[C \cos x_3 \cos(ld^{-1}x_3)u - Dx_4 \cos x_3 \cos(ld^{-1}x_3) \\ &\quad - E \cos x_3 - Fx_1 \cos x_3 - Gx_1x_2x_4] \end{aligned} \quad (7)$$

where  $A = (1 + m_B^{-1}J_B R^{-2})^{-1}$ ,  $B(x_1) = (J_B + J_b + m_B x_1^2)^{-1}$ ,  $C = 4.2K_b l (R_a d)^{-1}$ ,  $D = (K_b l)^2 (R_a d^2)^{-1}$ ,  $E = l m_b g / 2$ ,  $F = m_b g$ ,  $G = 2m_B$ , and  $u$  is the input voltage of the DC motor. The parameters of the system are given in Table 2.

The schematic diagram of the ball and beam control system with FACO is shown in Fig. 3, where  $x_k^*$  is the command of

$x_k$  and  $e_k = x_k^* - x_k$  is the error of state variable,  $k = 1, \dots, 4$ . From the state equations in (1), it is indicated that there are two dynamic objects, i.e. beam and ball. Therefore, the control of the ball and beam system is decoupled into two subsystems, the position control of ball and the balance control of beam. Two unique fuzzy control strategies are utilized to balance the beam and to keep the ball in the designated position. The proposed FACO optimized control scheme contains a fuzzy beam-balance controller, a fuzzy ball-position controller, and the FACO tuning mechanism.

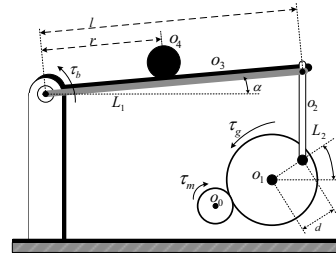


Fig. 2. Scheme diagram of ball and beam system.

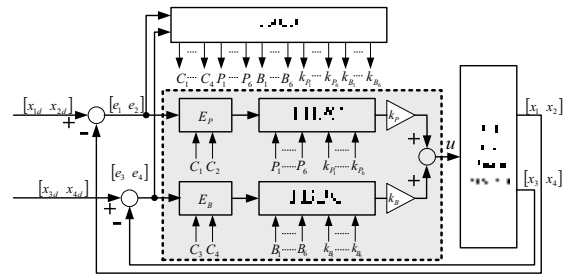


Fig. 3. The control scheme of the ball and beam system with FACO

Table 2. Parameters of ball and beam system.

Symbol	Definition	Value
$m_B$	mass of the ball	0.029 kg
$m_b$	mass of the beam	0.334 kg
$l$	beam length	0.4 m
$d$		0.04 m
$r_B$	radius of the ball	0.0095 m
$J_B$	pendulum length	0.11 m
$J_b$	payload length	0.02 m
$K_b$	gear stand length	0.1491 V/(rad/sec)
$K_t$	gravity acceleration	0.1491 Nm/A
$R_a$	radius of planet gear	18.91 $\Omega$
$r_1$	radius of sun gear	0.013 m

In this paper, to reduce the design complexity, a single-input FLC (SFLC) is adopted. With this control scheme,  $E_P = c_1 e_1 + c_2 e_2$  and  $E_B = c_3 e_3 + c_4 e_4$  are designed as the inputs of the fuzzy beam-balance controller and the fuzzy ball-position controller, respectively, where  $c_1 \sim c_4$  are the error constant. The input and output membership functions of FBBC and FBPC are indicated in the Fig. 4, where  $E_P$  and  $E_B$  represent the input vectors of FBPC and FBBC, respectively.

The  $F_{FBPC}$  and  $F_{FBBC}$  are denoted as output vectors of FBPC and FBBC, respectively. For the output fuzzy sets, the membership functions are defined to be fuzzy singleton functions. The input variables ( $E_p$  and  $E_B$ ) and output variables ( $F_{FBPC}$  and  $F_{FBBC}$ ) be fuzzily partitioned into nine fuzzy sets, negative very big (NV), negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM), positive big (PB), positive very big (PV). The fuzzy IF-THEN rules are expressed as

$$R_{Bi} : \text{IF } E_B \text{ is } MF_{Bi}, \text{ THEN } F_{FBBC} \text{ is } O_{Bi} \quad (8)$$

$$R_{Pi} : \text{IF } E_p \text{ is } MF_{Pi}, \text{ THEN } F_{FBPC} \text{ is } O_{Pi} \quad (9)$$

where the fuzzy sets  $MF_{Bi}$ ,  $MF_{Pi}$ ,  $O_{Bi}$ ,  $O_{Pi}$ ,  $i=1,2,\dots,9$  are defined in Table 3.

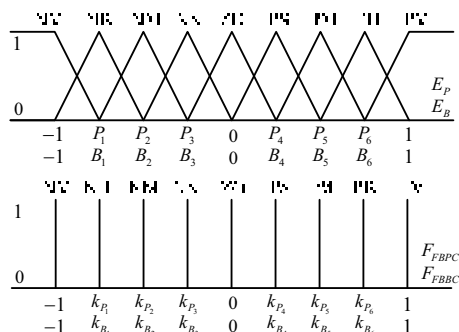


Fig. 4. Membership functions for the FBPC and the FBBC.

Table 3. Fuzzy rule base

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		$i$	1	2	3	4	5	6	7	8	9
input	$E_p(MF_{Pi})$		NV	NB	NM	NS	ZO	PS	PM	PB	PV
	$E_B(MF_{Bi})$		NV	NB	NM	NS	ZO	PS	PM	PB	PV
output	$F_{FBPC}(O_{Pi})$		PV	PB	PM	PS	ZO	NS	NM	NB	NV
	$F_{FBBC}(O_{Bi})$		NV	NB	NM	NS	ZO	PS	PM	PB	PV

To show the effectiveness of the proposed method, the FACO is proposed to adjust the parameters of the mentioned fuzzy controllers. The parameters of controllers contain the error constant,  $c_1 \sim c_4$ , the parameters of the input membership functions,  $B_1 \sim B_6$ ,  $P_1 \sim P_6$ , and the parameters of the output membership functions,  $k_{B_1} \sim k_{B_6}$ ,  $k_{P_1} \sim k_{P_6}$ . In this case, the aim of optimization is that minimizes the cost function. The cost function is the root mean square error (RMSE), and it is defined as follows

$$Cost = \sqrt{\frac{1}{N} \sum_{t=1}^N ((x_1^*(t) - x_1(t))^2 + (x_3^*(t) - x_3(t))^2)} \quad (10)$$

where  $N$  is the number of samples. The optimized parameters are represented in Table 4. For evaluating the proposed method and other algorithms, some criteria are selected as

Integral of Square Error (ISE)

$$ISE = \sum_{t=1}^N ((x_1^*(t) - x_1(t))^2) \quad (11)$$

Integral of the Absolute value of the Error (IAE)

$$IAE = \sum_{t=1}^N |x_1^*(t) - x_1(t)| \quad (12)$$

Integral of the time multiplied by the Absolute value of the Error (ITAE)

$$ITAE = \sum_{t=1}^N t |x_1^*(t) - x_1(t)| \quad (13)$$

The initial states of the ball and beam system are set to be  $x = [0 \ \pi \ 0 \ 0]$  and the disturbance is added at 10 sec. Fig. 5 indicates the responses of the ball and beam system with conventional ACO and FACO fuzzy controllers. It can be seen that, from Fig. 5(a) and Fig. 5(b), before 10 sec, the ball is stopped as desired and the beam is also at the required angle, respectively. In Fig. 5(a), the response of the proposed method has no overshoot and quickly converges to the desired position. In particular, with the disturbance added at 10 sec, the proposed method still has the best performance among other conventional ACOs. The Table 5 presents the criteria with all methods. It can be shown that the proposed algorithm is better than other conventional ACO optimization methods.

Table 4. Parameter table

Method	MF	Membership Function Value, $P_i, B_i, k_{P_i}, k_{B_i}$						Error Constant, $C_j$			
		$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$j=1$	$j=2$	$j=3$	$j=4$
AS [2]	in $E_p$	-0.75	-0.57	-0.04	0.04	0.57	0.75	13	8	12	0.85
	in $E_B$	-0.76	-0.70	-0.25	0.25	0.70	0.76				
	out $F_{FBPC}$	-0.75	-0.54	-0.25	0.25	0.54	0.75				
	out $F_{FBBC}$	-0.75	-0.50	-0.21	0.21	0.50	0.75				
ACS [1]	in $E_p$	-0.55	-0.54	-0.05	0.05	0.54	0.55	7	4	13	0.7
	in $E_B$	-0.70	-0.54	-0.23	0.23	0.54	0.70				
	out $F_{FBPC}$	-0.75	-0.54	-0.25	0.25	0.54	0.75				
	out $F_{FBBC}$	-0.76	-0.50	-0.22	0.22	0.50	0.76				
MMAS [13]	in $E_p$	-0.92	-0.80	-0.10	0.10	0.80	0.92	14	7	11	0.6
	in $E_B$	-0.85	-0.50	-0.15	0.15	0.50	0.85				
	out $F_{FBPC}$	-0.75	-0.50	-0.15	0.15	0.50	0.75				
	out $F_{FBBC}$	-0.75	-0.5	-0.25	0.25	0.5	0.75				
FACO	in $E_p$	-0.76	-0.60	-0.05	0.05	0.60	0.76	10	7	8	1
	in $E_B$	-0.97	-0.60	-0.25	0.25	0.60	0.97				
	out $F_{FBPC}$	-0.95	-0.50	-0.20	0.20	0.50	0.95				
	out $F_{FBBC}$	-0.75	-0.55	-0.25	0.25	0.55	0.75				

Table 5. Simulation performance criterion with disturbance

	IAE	ISE	ITSE
AS	$4.05 \times 10^4$	$4.53 \times 10^5$	$7.99 \times 10^8$
ACS	$3.66 \times 10^4$	$3.97 \times 10^5$	$7.00 \times 10^8$
MMAS	$2.96 \times 10^4$	$3.39 \times 10^5$	$3.67 \times 10^8$
FACO	$2.79 \times 10^4$	$3.14 \times 10^5$	$2.67 \times 10^8$

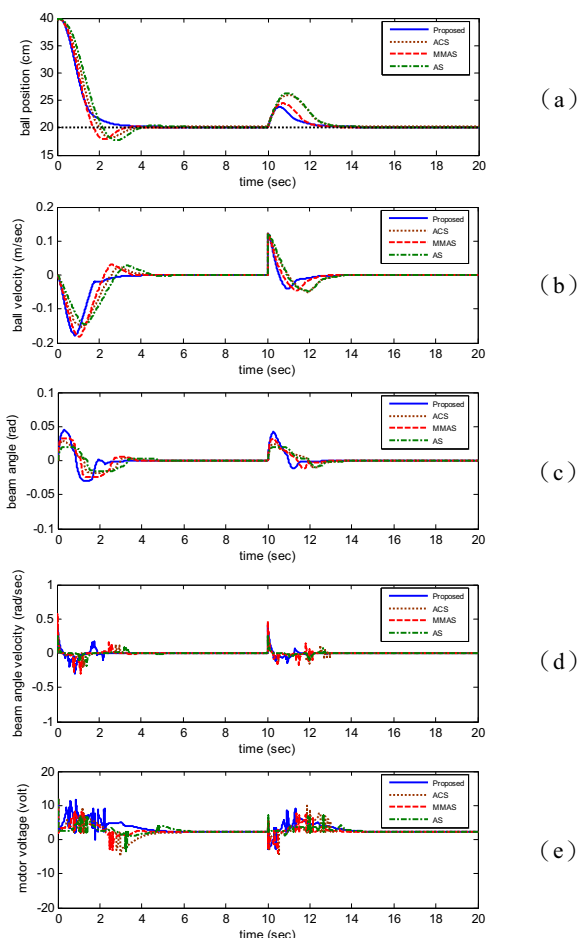


Fig. 5. Simulation response: (a) ball position, (b) ball velocity, (c) beam angle, (d) beam angle velocity, (e) controller output.

### V. EXPERIMENT SETUP AND RESULTS

The experimental setup of a beam and ball control system is shown in Fig. 6 and Fig. 7, where the control kernel is embedded in a digital control platform that combines a DSP (TMS320C6713) and FPGA (Flex EPF10k70) development boards. The sampling time of the experiment is selected to be 1ms. Without loss of generality, two initial conditions are considered,  $x_3(0) = 0^\circ$  and  $x_3(0) = -1^\circ$ . Fig. 8 shows the experiment responses of the beam and ball system with the initial ball position,  $x_1(0) = 38cm$ , and the initial beam angle,  $x_3(0) = -1^\circ$ . The beam and ball system successfully balances the beam to  $0^\circ$  and positions the ball to desired position,  $x_{1d} = 20cm$ . The results of experiment performance criterion are showed in Table 6 and Table 7. It can be shown that the proposed method has better performance than other methods.

### VI. CONCLUSION

This paper presents an ACO-optimized fuzzy controller for a beam and ball system. The proposed fuzzy-based ACO algorithm has the enhanced capability of pheromone updating. For the fuzzy controller design of a beam and ball system, the

proposed improved ACO algorithm is applied to optimize the parameter settings of the input and output membership functions. Simulation and experimental results illustrate that the improved ACO algorithm can provide better control performance subject to disturbance.

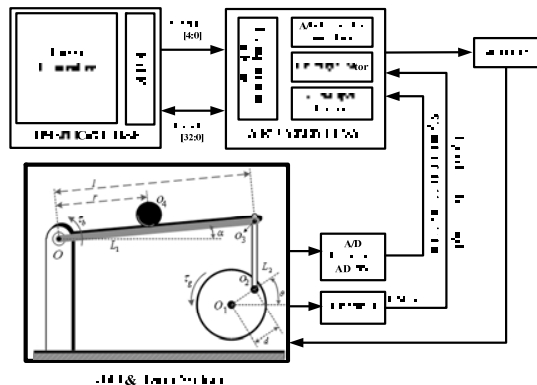


Fig. 6. Control scheme of the beam and ball control system.



Fig. 7. Experiment setup of the ball and beam system

Table 6. Performance criterion ( $x_1(0) = 38cm, x_3(0) = 0^\circ$ )

	IAE	ISE	ITSE
AS	$1.53 \times 10^4$	$3.06 \times 10^5$	$5.49 \times 10^7$
ACS	$1.25 \times 10^4$	$2.52 \times 10^5$	$3.28 \times 10^7$
MMAS	$1.18 \times 10^4$	$1.91 \times 10^5$	$3.07 \times 10^7$
FACO	$1.12 \times 10^4$	$1.42 \times 10^5$	$2.30 \times 10^7$

Table 7. Performance criterion ( $x_1(0) = 38cm, x_3(0) = -1^\circ$ )

	IAE	ISE	ITSE
AS	$1.48 \times 10^4$	$3.04 \times 10^5$	$5.26 \times 10^7$
ACS	$1.26 \times 10^4$	$2.30 \times 10^5$	$3.30 \times 10^7$
MMAS	$1.17 \times 10^4$	$1.85 \times 10^5$	$2.97 \times 10^7$
FACO	$1.08 \times 10^4$	$1.36 \times 10^5$	$2.19 \times 10^7$

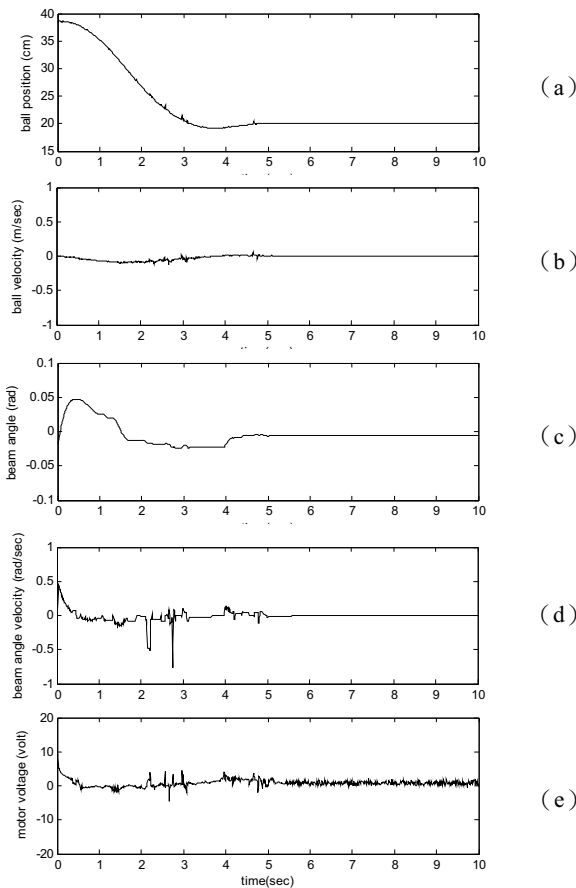


Fig. 8. Experiment response ( $x_1(0) = 38\text{cm}, x_3(0) = -1^\circ$ ):  
(a) ball position, (b) ball velocity, (c) beam angle, (d) beam angle velocity, (e) controller output.

#### APPENDIX

The ball and beam system is a classic underactuated nonlinear system. The system parameters for the ball and beam system model are listed in Table 2. To obtain the possible mathematical model of a ball and beam system, the associated Euler-Lagrange dynamic equation is first addressed in (A.1).

$$\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{q}} \right] - \frac{\partial L}{\partial q} = Q \quad (\text{A.1})$$

where  $q = [r \ \alpha]^T$ ,  $Q = [\tau_B \ \tau_b]^T$ ,  $L = (\text{the kinetic energy, } K) - (\text{the potential energy, } P)$ ,  $r$  is the position of the ball,  $\alpha$  is the beam angle,  $\tau_b$  is a torque provided by a DC motor to the beam via gear and linker, and  $\tau_B$  is an exogenous torque to the Ball.

It is noted that  $\tau_B$  is physically considered as the disturbance to the ball. As show in Fig. 1, to derive the dynamic equation of the ball and beam system, the kinetic energy  $K$  and  $P$  can be represented as follows:

$$K = \frac{1}{2} m_B \dot{r}^2 + \frac{1}{2} J_B \left( \frac{\dot{r}}{R} \right)^2 + \frac{1}{2} (J_B + m_B r^2) \dot{\alpha}^2 + \frac{1}{2} J_b \dot{\alpha}^2 \quad (\text{A.2})$$

$$P = \frac{l}{2} m_b g \sin \alpha + m_B g r \sin \alpha \quad (\text{A.3})$$

Substituting (A.2) and (A.3) into Lagrange dynamic

equation (A.1) yields

$$\tau_B = \frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{r}} \right] - \frac{\partial L}{\partial r} = \ddot{r} \left( m_B + \frac{J_B}{R^2} \right) - m_B r \dot{\alpha}^2 + m_B g \sin \alpha \quad (\text{A.4})$$

$$\begin{aligned} \tau_b &= \frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{\alpha}} \right] - \frac{\partial L}{\partial \alpha} \\ &= (J_b + J_B + m_B r^2) \ddot{\alpha} + 2m_B r \dot{r} \dot{\alpha} + (m_B g r + \frac{l}{2} m_b g) \cos \alpha \end{aligned} \quad (\text{A.5})$$

Due to  $\tau_b$  is not directly provided by DC motor, the relationship between  $\tau_b$  and the DC motor can be derived as follows:

$$\tau_b = n \left( \frac{K_b}{R_a} V_a - \frac{K_b K_t}{R_a} \dot{\theta}_m \right) \frac{l}{d} \cos \alpha \cos \theta \quad (\text{A.6})$$

In (A.4), there is no external force, i.e.  $\tau_B = 0$ . From (A.4)-(A.6), the state equation of the ball and beam system can be represented as (7)

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