World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:4, No:8, 2010

Optimal Control of a Linear Distributed Parameter System via Shifted Legendre Polynomials

Sanjeeb Kumar Kar

Abstract—The optimal control problem of a linear distributed parameter system is studied via shifted Legendre polynomials (SLPs) in this paper. The partial differential equation, representing the linear distributed parameter system, is decomposed into an *n* - set of ordinary differential equations, the optimal control problem is transformed into a two-point boundary value problem, and the two-point boundary value problem is reduced to an initial value problem by using SLPs. A recursive algorithm for evaluating optimal control input and output trajectory is developed. The proposed algorithm is computationally simple. An illustrative example is given to show the simplicity of the proposed approach.

Keywords—Optimal control, linear systems, distributed parameter systems, Legendre polynomials.

I. Introduction

OPTIMAL control of linear distributed parameter systems has been studied by many researchers. First this problem was solved by finite difference technique [1] to establish a state space model. But as it appears from the literature, [2] is the first person to study the optimal control problem of linear distributed parameter systems using orthogonal functions. He solved this problem by using Walsh functions and obtained piecewise constant solution. The optimal control problem was transformed into a two-point boundary value problem in [3] and [5], and obtained the solution. In [4], they have reduced the optimal control problem of distributed parameter systems to the optimal control problem of linear time-invariant lumped parameter systems. This problem was studied by employing block-pulse functions in [6]. In [8] and [9], they have also investigated this problem using Legendre polynomials and orthogonal polynomials, respectively. All these approaches are noniterative and algebraic, and consequently they are convenient for computation.

In this paper, using SLPs a simple and recursive algorithm is proposed for the optimal control of linear distributed parameter systems. Though the basic approach followed here is similar to the one in [3], the manner in which SLPs are defined, the way various operational matrices are introduced, and the way the recursive algorithm is developed are different. The paper is organized as follows: The next section briefly deals with SLPs and their properties. Section 3 discusses optimal control of linear distributed parameter systems via SLPs, and presents a recursive algorithm to solve the control problem. One example is considered in Section 4 to demonstrate and compare the performances of proposed method and the methods in [1] and [3]. Finally Section 5 concludes the paper.

Sanjeeb Kumar Kar is with the Department of Electrical Engineering, Institute of Technical Education and Research, SOA University, Bhubaneswar-751030, INDIA e-mail: (skkariter@rediffmail.com).

II. SLPs and Their Properties [7]

A set of SLPs, denoted by $\{\phi_i(t)\}$ for $i = 0, 1, 2, \ldots, m-1$, is orthogonal with respect to the weighting function $w(t) = 1$ over the interval $[t_0, t_f]$, i.e.

$$\int_{t_0}^{t_f} \phi_i(t)\phi_j(t)dt = \begin{cases} 0 & \text{if } \quad i \neq j \\ \frac{(t_f - t_0)}{(2i+1)} & \text{if } \quad i = j \end{cases} \tag{1}$$

These polynomials satisfy the recurrence relation

$$\phi_{i+1}(t) = \frac{(2i+1)}{(i+1)}\,\varphi(t)\,\phi_i(t) - \frac{i}{(i+1)}\,\phi_{i-1}(t) \tag{2}$$

for $i = 1, 2, 3, \ldots\ldots$

with

$$\varphi(t) = \frac{2(t - t_0)}{(t_f - t_0)} - 1 \tag{3}$$

$$\phi_0(t) = 1, \text{ and } \phi_1(t) = \varphi(t) \tag{4}$$

A function $f(t)$ that is square integrable on $t \in [t_0, t_f]$ can be represented in terms of SLP as

$$f(t) \approx \sum_{i=0}^{m-1} f_i\phi_i(t) = \mathbf{f}^T\boldsymbol{\phi}(t) \tag{5}$$

where

$$\mathbf{f} = \left[\begin{array}{cccc} f_0, & f_1, & \ldots, & f_{m-1} \end{array}\right]^T \tag{6}$$

is called Legendre spectrum of $f(t)$, and

$$\boldsymbol{\phi}(t) = \left[\begin{array}{cccc} \phi_0(t), & \phi_1(t), & \ldots, & \phi_{m-1}(t) \end{array}\right]^T \tag{7}$$

is called SLP vector. f_i in Eq. (5) is given by

$$f_i = \frac{(2i+1)}{(t_f - t_0)}\int_{t_0}^{t_f} f(t)\phi_i(t)dt \tag{8}$$

SLPs satisfy the relation

$$\phi_i(t) = \frac{(t_f - t_0)}{2\,(2i+1)}\left(\frac{d}{dt}\phi_{i+1}(t) - \frac{d}{dt}\phi_{i-1}(t)\right) \tag{9}$$

for $i = 1, 2, 3, \ldots\ldots$

Integrating $\phi_0(t)$ once with respect to t and expressing the result in terms of SLPs, we have

$$\int_{t_0}^{t} \phi_0(\tau)d\tau = \frac{(t_f - t_0)}{2}\left[\phi_0(t) + \phi_1(t)\right] \tag{10}$$

Integrating Eq. (9) once with respect to *t*, we obtain

$$\int_{t_0}^{t} \phi_i(\tau)d\tau = \frac{(t_f - t_0)}{2\,(2i+1)}\left[\phi_{i+1}(t) - \phi_{i-1}(t)\right] \tag{11}$$

Eqs. (10) and (11) can be written in the form of

$$\int_{t_0}^{t} \boldsymbol{\phi}(\tau)d\tau \quad \approx \quad P\boldsymbol{\phi}(t) \tag{12}$$

where

$$P = \frac{(t_f - t_0)}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ \frac{-1}{3} & 0 & \frac{1}{3} & 0 & \dots & 0 & 0 \\ 0 & \frac{-1}{5} & 0 & \frac{1}{5} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & \frac{1}{2m-3} \\ 0 & 0 & 0 & 0 & \dots & \frac{-1}{2m-1} & 0 \end{bmatrix} \tag{13}$$

which is called integration operational matrix of SLP. As can be seen from Eq. (13), it is a tridiagonal matrix of order $m \times m$, and it plays an important role in deriving a recursive algorithm in the next section.

SLPs satisfy the relation

$$\frac{d\boldsymbol{\phi}(t)}{dt} \quad = \quad D\boldsymbol{\phi}(t) \tag{14}$$

where

$$D = \frac{2}{(t_f - t_0)}\begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 3 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 5 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 3 & 0 & \dots & 2m-5 & 0 & 0 \\ 1 & 0 & 5 & \dots & 0 & 2m-3 & 0 \end{bmatrix} \tag{15}$$

which is called differentiation operational matrix of SLP.

III. Optimal Control of Linear Distributed Parameter Systems

Consider the one dimensional diffusion equation [1]

$$\frac{\partial y(x,t)}{\partial t} \quad = \quad \frac{\partial^2 y(x,t)}{\partial x^2} + u(x,t) \tag{16}$$

with initial condition (IC)

$$y(x,0) \quad = \quad f(x) \tag{17}$$

and boundary conditions (BCs)

$$\frac{\partial y(x,t)}{\partial t} \quad = \quad 0 \quad \text{at } x \;=\; 0 \text{ and } x \;=\; x_f \tag{18}$$

The problem is to find the optimal control $u(x,t)$ which minimizes the cost function

$$J \quad = \quad \frac{1}{2}\int_0^{t_f}\int_0^{x_f}\left[qy^2(x,t) + ru^2(x,t)\right]dx\,dt \tag{19}$$

where $q \geq 0$ and $r > 0$. Expressing $u(x,t)$ and $y(x,t)$ in terms of SLPs,

$$u(x,t) \quad \simeq \quad \sum_{i=0}^{n-1} u_i(t)\phi_i(x) \;=\; \mathbf{u}^T(t)\boldsymbol{\phi}(x) \tag{20}$$

$$y(x,t) \quad \simeq \quad \sum_{i=0}^{n-1} y_i(t)\phi_i(x) \;=\; \mathbf{y}^T(t)\boldsymbol{\phi}(x) \tag{21}$$

where

$$\mathbf{u}(t) \quad = \quad \left[u_0(t), u_1(t), \dots, u_{n-1}(t)\right]^T \tag{22}$$

$$\mathbf{y}(t) \quad = \quad \left[y_0(t), y_1(t), \dots, y_{n-1}(t)\right]^T \tag{23}$$

and

$$\boldsymbol{\phi}(x) \quad = \quad \left[\phi_0(x), \phi_1(x), \dots, \phi_{n-1}(x)\right]^T \tag{24}$$

The SLPs $\{\phi_i(x)\},\ i = 0, 1, \dots, n-1$ are defined over $0 \leq x \leq x_f$. Multiplying Eq. (16) by $\boldsymbol{\phi}^T(x)$, using Eqs. (20), (21), and integrating with respect to *x*, we have

$$\begin{aligned} \int_0^{x_f} \dot{y}(t)\boldsymbol{\phi}(x)\boldsymbol{\phi}^T(x)dx \quad &= \quad \int_0^{x_f} \mathbf{y}^T(t)\ddot{\boldsymbol{\phi}}(x)\boldsymbol{\phi}^T(x)dx \\ &\quad + \int_0^{x_f} \mathbf{u}^T(t)\boldsymbol{\phi}(x)\boldsymbol{\phi}^T(x)dx \\ &= \quad -\mathbf{y}^T(t)\int_0^{x_f} \dot{\boldsymbol{\phi}}(x)\dot{\boldsymbol{\phi}}^T(x)dx \\ &\quad + \int_0^{x_f} \mathbf{u}^T(t)\boldsymbol{\phi}(x)\boldsymbol{\phi}^T(x)dx \end{aligned} \tag{25}$$

after substituting the BCs. Using Eq. (1) we can write

$$\int_0^{x_f} \boldsymbol{\phi}(x)\boldsymbol{\phi}^T(x)dx = x_f\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & \frac{1}{3} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \frac{1}{2n-1} \end{bmatrix} = Q' \tag{26}$$

Let

$$d_{ij} \quad = \quad \int_0^{x_f} \dot{\phi}_i(x)\dot{\phi}_j(x)dx.$$

Then

$$\int_0^{x_f} \dot{\boldsymbol{\phi}}(x)\dot{\boldsymbol{\phi}}^T(x)dx \quad = \quad \int_0^{x_f} D\boldsymbol{\phi}(x)\boldsymbol{\phi}^T(x)D^T dx$$

$$= \begin{bmatrix} d_{00} & d_{01} & \cdots & d_{0,\,n-1} \\ d_{10} & d_{11} & \cdots & d_{1,\,n-1} \\ \vdots & \vdots & & \vdots \\ d_{n-1,\,0} & d_{n-1,1} & \dots & d_{n-1,\,n-1} \end{bmatrix} \tag{27}$$

where

$$d_{ij} = d_{j\,i} = \begin{cases} \frac{2i(i+1)}{x_f} & \text{if} \quad i \leq j \text{ and } i+j \text{ is zero or even.} \\ 0 & \text{if} \quad i < j \text{ and } i+j \text{ is odd.} \end{cases} \tag{28}$$

for $i, j = 0, 1, 2, \dots, n-1$. Substituting Eqs. (26) and (27) into Eq. (25), and simplifying, we have

$$\dot{\mathbf{y}}(t) \quad = \quad B\mathbf{y}(t) + \mathbf{u}(t) \tag{29}$$

where

$$B = -\frac{1}{x_f}\begin{bmatrix} d_{00} & d_{01} & \cdots & d_{0,\,n-1} \\ 3d_{10} & 3d_{11} & \cdots & 3d_{1,\,n-1} \\ 5d_{20} & 5d_{21} & \cdots & 5d_{2,\,n-1} \\ \vdots & \vdots & & \vdots \\ (2n-1)d_{n-1,\,0} & (2n-1)d_{n-1,1} & \cdots & (2n-1)d_{n-1,\,n-1} \end{bmatrix} \tag{30}$$

Now substituting Eqs. (20), (21) and (26) into Eq. (19), the performance index is given by

$$J = \frac{1}{2}\int_0^{t_f}\left[\mathbf{y}^T(t)Q\mathbf{y}(t) + \mathbf{u}^T(t)R\mathbf{u}(t)\right]dt \tag{31}$$

where

$$\frac{Q}{q} = \frac{R}{r} = Q' \tag{32}$$

The optimal control problem of distributed parameter system is now reduced to the optimal control problem of lumped parameter systems. The adjoint equation to solve this problem is given by

$$-\dot{\boldsymbol{\lambda}}(t) = Q\mathbf{y}(t) + B^T\boldsymbol{\lambda}(t) \quad \text{with } \boldsymbol{\lambda}(t_f) = \mathbf{0} \tag{33}$$

and the optimal control law is given by

$$\mathbf{u}(t) = -R^{-1}\boldsymbol{\lambda}(t) \tag{34}$$

Eqs. (29), (33) and (34) can be compactly written as

$$\begin{bmatrix} \dot{\mathbf{y}}(t) \\ \dot{\boldsymbol{\lambda}}(t) \end{bmatrix} = \begin{bmatrix} B & -R^{-1} \\ -Q & -B^T \end{bmatrix}\begin{bmatrix} \mathbf{y}(t) \\ \boldsymbol{\lambda}(t) \end{bmatrix} \tag{35}$$

with specified $\mathbf{y}(0)$ and $\boldsymbol{\lambda}(t_f)$. Or, alternatively

$$\dot{\mathbf{z}}(t) = F\mathbf{z}(t) \tag{36}$$

where

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{y}(t) \\ \boldsymbol{\lambda}(t) \end{bmatrix} \tag{37}$$

and

$$F = \begin{bmatrix} B & -R^{-1} \\ -Q & -B^T \end{bmatrix} \tag{38}$$

A. Recursive algorithm via SLP

Integrating Eq. (36) once with respect to *t*, we obtain

$$\mathbf{z}(t) - \mathbf{z}(0) = F\int_0^t \mathbf{z}(\tau)d\tau \tag{39}$$

Expressing $\mathbf{z}(t)$ and $\mathbf{z}(0)$ in terms of SLP and utilizing the integration operational property in Eq. (12), we have

$$Z = Z_0 + FZP \tag{40}$$

where

$$Z = [\mathbf{z}_0, \mathbf{z}_1, \ldots, \mathbf{z}_{m-1}] \tag{41}$$

and

$$Z_0 = [\mathbf{z}(0), \mathbf{0}, \ldots, \mathbf{0}] \tag{42}$$

which are $2n \times m$ matrices. Substituting matrix *P* of SLP in Eq. (40) and rearranging the terms we have

The following recursive algorithm can be obtained from Eqs. (43)-(45) :

$$\mathbf{z}_0 = M_{00}\mathbf{v}_0 \tag{46}$$

$$\bar{R}_{i,i-1} = -M_{ii}W_{i,i-1} \text{for} \quad i = m-1, \ldots, 2, 1. \tag{47}$$

$$M_{ii} = \begin{cases} W_{ii}^{-1} & \text{if } i = m-1 \\ (W_{ii} + W_{i,\,i+1}\bar{R}_{i+1,\,i})^{-1} & \text{if } i = m-2, \\ & m-3, \ldots, 1, 0. \end{cases} \tag{48}$$

$$\mathbf{z}_i = \bar{R}_{i,\,i-1}\mathbf{z}_{i-1} \quad \text{for } i = 1, 2, \ldots, m-1. \tag{49}$$

B. Algorithm for finding z(0)

As $\mathbf{z}(0) = \begin{bmatrix} \mathbf{y}(0) \\ \boldsymbol{\lambda}(0) \end{bmatrix}$ and $\boldsymbol{\lambda}(0)$ is unknown, $\mathbf{z}(0)$ is obviously unknown. Here we present an algorithm to find $\mathbf{z}(0)$. For $t = t_f$ Eq. (39) reduces to

$$\mathbf{z}(t_f) - \mathbf{z}(0) = t_f F\mathbf{z}_0 \tag{50}$$

Substituting Eqs. (45) and (46) into Eq. (50), we obtain

$$\mathbf{z}(t_f) = (I_{2n} + 2FM_{00})\,\mathbf{z}(0) = K\mathbf{z}(0) \tag{51}$$

$$\Rightarrow \begin{bmatrix} \mathbf{y}(t_f) \\ \boldsymbol{\lambda}(t_f) \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} \mathbf{y}(0) \\ \boldsymbol{\lambda}(0) \end{bmatrix}$$

Since $\boldsymbol{\lambda}(t_f) = K_{21}\mathbf{y}(0) + K_{22}\boldsymbol{\lambda}(0) = \mathbf{0}$, we can write

$$\boldsymbol{\lambda}(0) = -K_{22}^{-1}K_{21}\mathbf{y}(0) \tag{52}$$

Thus $\mathbf{z}(0)$ can be found.

C. Algorithm for calculating J

Expressing $\mathbf{y}(t)$ and $\mathbf{u}(t)$ in terms of SLPs, we have

$$\mathbf{y}(t) = Y\boldsymbol{\phi}(t) \tag{53}$$

$$\mathbf{u}(t) = U\boldsymbol{\phi}(t) \tag{54}$$

where

$$Y = \begin{bmatrix} y_{00} & y_{01} & \cdots & y_{0,\,m-1} \\ y_{10} & y_{11} & \cdots & y_{1,\,m-1} \\ \vdots & \vdots & & \vdots \\ y_{n-1,\,0} & y_{n-1,1} & \cdots & y_{n-1,\,m-1} \end{bmatrix} \tag{55}$$

and

$$U = \begin{bmatrix} u_{00} & u_{01} & \cdots & u_{0,\,m-1} \\ u_{10} & u_{11} & \cdots & u_{1,\,m-1} \\ \vdots & \vdots & & \vdots \\ u_{n-1,\,0} & u_{n-1,1} & \cdots & u_{n-1,\,m-1} \end{bmatrix} \tag{56}$$

$$\begin{bmatrix} W_{00} & W_{01} & \bigcirc & \bigcirc & \dots & \bigcirc & \bigcirc \\ W_{10} & W_{11} & W_{12} & W_{13} & \dots & \bigcirc & \bigcirc \\ \bigcirc & W_{21} & W_{22} & W_{23} & \dots & \bigcirc & \bigcirc \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \bigcirc & \bigcirc & \bigcirc & \bigcirc & \dots & W_{m-2,m-2} & W_{m-2,m-1} \\ \bigcirc & \bigcirc & \bigcirc & \bigcirc & \dots & W_{m-1,m-2} & W_{m-1,m-1} \end{bmatrix} \begin{bmatrix} \mathbf{z}_0 \\ \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_{m-2} \\ \mathbf{z}_{m-1} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{43}$$

where

$$W_{ij} = \begin{cases} \frac{2I_{2n}}{t_f} - F & \text{if } i = j = 0 \\ \frac{F}{(2i+3)} & \text{if } i = 0, 1, \dots, m-2 \quad \text{and} \quad j = i+1 \\ \frac{-F}{(2i-1)} & \text{if } i = 1, 2, \dots, m-1 \quad \text{and} \quad j = i-1 \\ \frac{2I_{2n}}{t_f} & \text{if } i = j = 1, 2, \dots, m-1 \\ \bigcirc_{2n} & \text{otherwise} \end{cases} \tag{44}$$

and

$$\mathbf{v}_0 = \frac{2\mathbf{z}(0)}{t_f} \tag{45}$$

are $n \times m$ matrices, and $\phi(t)$ is given in Eq. (7). Substituting Eqs. (53) and (54) into Eq. (31), we have

$$\begin{aligned} J &= \frac{1}{2}\int_0^{t_f} \phi^T(t)\left[Y^TQY + U^TRU\right]\phi(t)dt \\ &= \frac{1}{2}\int_0^{t_f} \phi^T(t)\left[M+N\right]\phi(t)dt \end{aligned} \tag{57}$$

where $M = Y^TQY$ and $N = U^TRU$ are $m \times m$ symmetric matrices and can be computed easily using Eqs. (55), (56) and (32). Now substituting $\phi(t)$, *M* and *N* in Eq. (57) and utilizing the orthogonality property of SLPs given in Eq. (1), we have

$$J = \frac{x_f t_f}{2}\sum_{j=0}^{m-1}\left[qM(j,j) + rN(j,j)\right] \tag{58}$$

where

$$M(j,j) = \sum_{i=0}^{n-1}\frac{y_{ij}^2}{(2i+1)} \tag{59}$$

and

$$N(j,j) = \sum_{i=0}^{n-1}\frac{u_{ij}^2}{(2i+1)} \tag{60}$$

for $j = 0, 1, 2, \dots, m-1$.

IV. An Illustrative Example [1], [3]

Let $x_f = 4,\ t_f = 1,\ f(x) = 1 + x,\ q = r = 1$. We compute the control law $-u(x,t)$ and output $y(x,t)$ at $x = 0, 1, 2, 3, 4$ by considering $m = n = 5$ and $\triangle t$(step size)= 0.01 in the proposed method. From Figures 1-5, one can see that results match exactly with the ones in [3]. Moreover, the results obtained via spatially discretized model [1] are also shown in Figures 1-5 for comparison purpose. One may opine that this comparison is meaningless as it should be done with exact $u(x,t)$ and $y(x,t)$. In the present situation, exact $u(x,t)$ and $y(x,t)$ are unknown unfortunately.

Nevertheless, we say that the proposed SLP approach is more accurate than the finite difference method [1]. This is primely because, expansion of $y(x,t)$ in terms of SLP is continuous while it is discrete in finite difference method. The discrete nature of finite difference method leads to computational errors which increase as we move towards the boundaries ($x = 0$ and $x = 4$) of the specimen. This is apparent in Figures 1-5, i.e. the values of $-u(x,t)$ and $y(x,t)$ obtained via SLP method and the method in [1] are almost matching for $x = 2$ in Figure 3 and it is not so for $x = 0, 1, 3, 4$ in Figures 1, 2, 4, 5. The values of *J* are shown in Table 1.

Moreover, computational time and memory space requirements are comparatively low in the case of SLP approach, see Table 2. This clearly demonstrates the superiority of proposed SLP approach over the finite difference method. All the computations are done with MATLAB 7 and Pentium 4 CPU 3.00 GHz, 1 GB RAM system.

TABLE I
J VALUES

Method	J value
Sage and White	15.38371873364494
Proposed SLP	15.00019510801434

V. Conclusion

Optimal control law of a linear distributed parameter system is computed using SLP. The proposed approach is simple, straightforward and recursive, and therefore is computationally

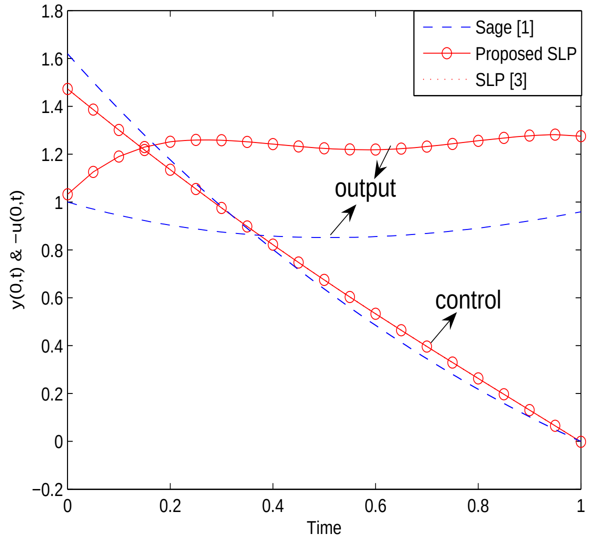

Fig. 1. Optimal control and output for x=0

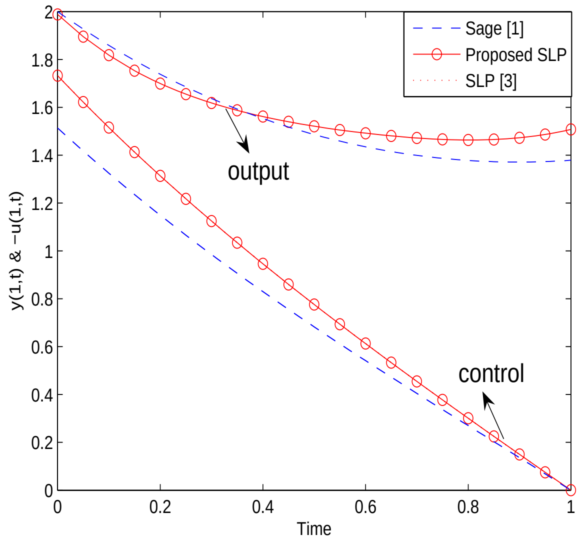

Fig. 2. Optimal control and output for x=1

TABLE II
COMPUTATIONAL TIME AND WORK-SPACE MEMORY

Method	Time in sec.	Memory in kb.
Sage and White	1.250	42.2
Proposed SLP	0.359	11.8

attractive. Moreover, the computational methodology followed to develop the algorithms in Section 3 is new.

ACKNOWLEDGMENT

The author would like to thank Dr. B. M. Mohan, Professor, Department of Electrical Engineering, IIT-Kharagpur, India, for his help and inspiration in this work. Also the author would like to thank Prof. (Dr.) Manojranjan Nayak, the President of the Siksha ‘O’ Anusandhan University, Bhubaneswar, Odisha, (the sponsoring institute) for his kind encouragement and support to pursue the Ph.D work at IIT Kharagpur.

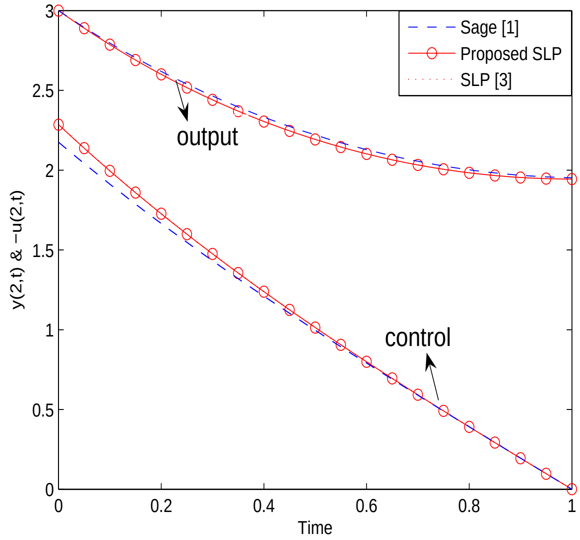

Fig. 3. Optimal control and output for x=2

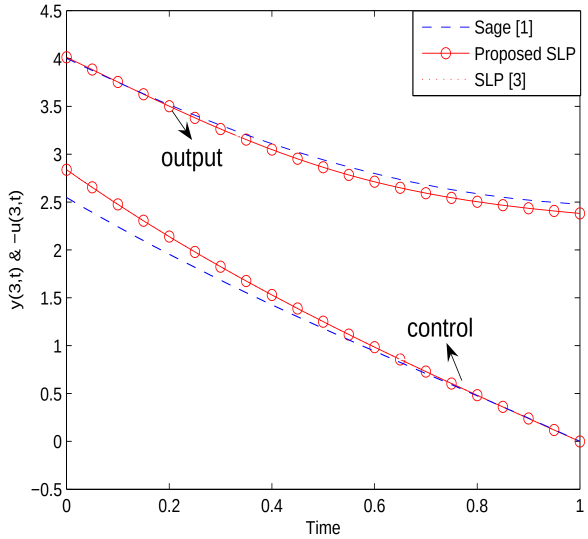

Fig. 4. Optimal control and output for x=3

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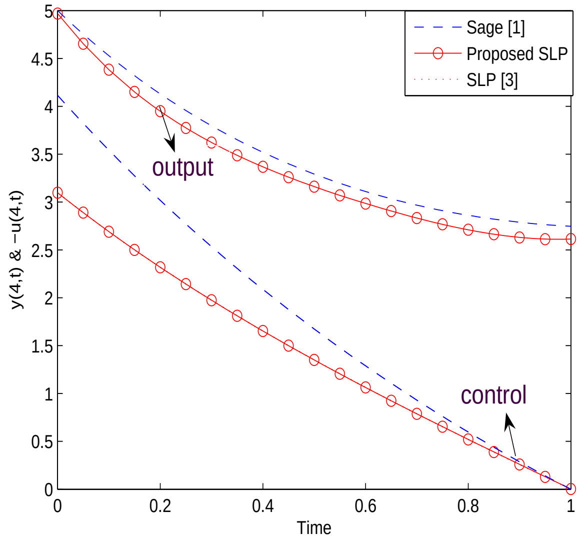

Fig. 5. Optimal control and output for x=4

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Sanjeeb Kumar Kar received his B.Tech. in Electrical Engineering from College of Engineering and Technology, OUAT, Bhubaneswar in 1993, M. Tech in Control System from Indian Institute of Technology, Kharagpur in 2004. Since 1998 he has been with the Department of Electrical Engineering at Institute of Technical Education and Research, Siksha 'O' Anusandhan University, Bhubaneswar, where he is an assistant professor. Prior to that he serves one year as lecturer in CET, Bhubaneswar and three years in industries. At present he is perusing his Ph. D. at IIT Kharagpur (in study leave).