

Double Diffusive Convection in a Partially Porous Cavity under Suction/Injection Effects

Y. Outaleb, K. Bouhade, and O. Rahli

Abstract—Double-diffusive steady convection in a partially porous cavity with partially permeable walls and under the combined buoyancy effects of thermal and mass diffusion was analysed numerically using finite volume method.

The top wall is well insulated and impermeable while the bottom surface is partially well insulated and impermeable and partially submitted to constant temperature T_1 and concentration C_1 . Constant equal temperature T_2 and concentration C_2 are imposed along the vertical surfaces of the enclosure. Mass suction/injection and injection/suction are respectively considered at the bottom of the porous centred partition and at one of the vertical walls.

Heat and mass transfer characteristics as streamlines and average Nusselt numbers and Sherwood numbers were discussed for different values of buoyancy ratio, Rayleigh number, and injection/suction coefficient.

It is especially noted that increasing the injection factor disadvantages the exchanges in the case of the injection while the transfer is augmented in case of suction. On the other hand, a critical value of the buoyancy ratio was highlighted for which heat and mass transfers are minimized.

Keywords—Double diffusive convection, Injection/Extraction, Partially porous cavity

I. INTRODUCTION

DOUBLE-diffusive convection in porous media is mainly motivated by its importance in many natural and engineering applications such as electronic devices, disposal of waste material, chemical transport in packed-bed reactors, heat exchangers, grain-storage installations, drying process, geothermal field, etc. The state of the art has been summarised very well in the literature presented by Nield and Bejan [1], Ingham and Pop [2] and Pop and Ingham [3]. Principal former work relating to such processes is analyzed, primarily, in purely fluid or completely porous configuration. The case of the partially porous enclosures has been much less explored. One can nevertheless quote the works of Mharzi *et al.* [4], Rahli and Bouhade [5], Saeid *et al.* [6], Elsa Baez *et al.* [7], and Bennacer *et al.* [8]. So far, almost all the studies concerning injection and extraction of matter in porous media were limited only to the cases of the vertical or inclined plates. Thus, N. J. Rabadi and E.M. Hamdan [9] analyzed the natural

convection at the level of a tilted surface immersed in a saturated porous medium having variable permeabilities and thermal conductivities, and subjected to a fluid injection. In addition, E. Magyari and B. Keller [10] analytically studied the natural convection along a plate immersed in a porous saturated medium by imposing a temperature profile and a side flow of mass. Lastly, M.A.El Hakiem [11] numerically analyzed the effect of the injection/extraction on the flow of a non-Newtonian fluid along a vertical plate immersed in a porous medium in the case of a mixed convection. Thus, not much work has been reported on non-Darcy mixed convection in a rectangular partially porous enclosure under injection/suction effects which becomes very relevant in the context of electronic devices, drying process, environmental chamber for bacterial culture preservation, etc., have not been considered. Nevertheless, one can quote the work of B.V. Rathish Kumar and S.V.S.S.N.V.G. Krishna Murthy [12] which was focussed on a steady mixed convection inside a vertical square fluid saturated porous enclosure with multiple fluid injections at the bottom wall and multiple suction at the top wall. Thus forced convection is imposed by this combination of suction/injection flow conditions. The free convection is induced by the hot and isothermal left vertical wall. The purpose of the present study is to analyze the behaviour of heat and mass exchange generated by thermosolutal convection in confined media, namely a partially porous cavity. The latter is subjected to boundary conditions of Dirichlet type and with an arrival or departure of fluid on the permeable parts of its walls. The influence of some thermo physical and geometrical parameters, on the transfers and the modes of flow which can appear, is described and analyzed.

II. PHYSICAL MODEL

The study is carried out on a square enclosure compartmentalized in two equal parts, by a porous vertical partition of thickness E (figure 1). The left and right spaces are occupied by the same Newtonien fluid, witch consists of a solvent and an aqueous solution in weak concentration, which saturates the porous partition. The horizontal walls, respectively $y = 0$ and $y = H$, are assumed adiabatic and impermeable, except on the bottom of porous partition where wall is submitted to constant temperature T_1 and concentration C_1 . The same fluid which saturates the porous cavity is injected (or extracted) into the enclosure through this part of the lower wall, with a non null and assumed constant vertical velocity of injection/suction. It is admitted that the exit (or entry) of the fluid injected (or extracted) is done through one of the vertical walls. These last ones are subjected to a T_2 temperature and a C_2 concentration with $T_2 < T_1$ and $C_2 < C_1$.

Y. Outaleb, K. Bouhade and O. Rahli are with LTPMP Laboratory in the Mechanical and Process Engineering Faculty, University of Sciences and Technology Houari Boumediene, BP 32, El Alia, Bab Ezzouar, Algiers, Algeria (e-mail of Y. Outaleb: tamedjout77@yahoo.fr; e-mail of K. Bouhade: kbouhade@usthb.dz ; e-mail of O. Rahli: rahliomar@yahoo.fr).

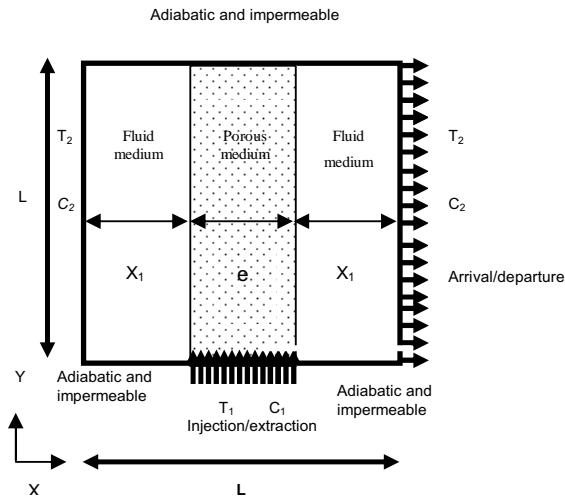


Fig. 1 Schematic of the physical domain

III. MATHEMATICAL FORMULATION

The flow of the fluid through the porous substrate is supposed to be governed by the Darcy-Brinkman model and the various equations, traducing the principles of conservation are formulated by adopting certain simplifying assumptions, namely that the flow is supposed to be laminar and permanent, the fluid assumed incompressible and Newtonian, the viscous dissipation and the compression are neglected in the conservation of energy equation and finally, the Boussinesq approximation is applied and adopted. This last assumption makes it possible to suppose a constant density everywhere, except in the buoyancy term where its expression is a function of the temperature and concentration variations, so that:

$$\rho(T,C) = \rho_{ref} [1 - \beta_T(T-T_{ref}) - \beta_C(C-C_{ref})] \quad (1)$$

Taking into account of these assumptions, and with a suitable choice of reference variables, the controlling equations can be then written in the following non dimensional form:

- Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (2)$$

- Momentum equation:

According to X:

$$\left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = - \frac{\partial P}{\partial X} - GR1.U + GR2. \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (3)$$

According to Y:

$$\left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = - \frac{\partial P}{\partial Y} - GR1.V + GR2. \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + GR3 \quad (4)$$

- Energy equation:

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = GR4. \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (5)$$

- Concentration equation:

$$U \frac{\partial S}{\partial X} + V \frac{\partial S}{\partial Y} = GR5. \left(\frac{\partial^2 S}{\partial X^2} + \frac{\partial^2 S}{\partial Y^2} \right) \quad (6)$$

- Injection velocity (horizontal wall):

$$V_i = -fp.R_K.Ra^{1/2} \quad (7)$$

- Outlet velocity of fluid (one vertical wall):

$$U_s = -fp.R_K.Ra^{1/2} \quad (8)$$

With:

$$GR1 = \varepsilon^2 (Pr/Da) ; GR2 = \varepsilon^2 Pr R_v$$

$$GR3 = \varepsilon^2 Ra Pr(\theta + NS) ; GR4 = R_K ; GR5 = \varepsilon / Le .$$

Where: ε is the porosity, $E=e/L$ the non dimensional thickness of the porous partition, $Pr=v_f/a_f$ the Prandtl number, $Ra=g\beta_T \Delta T K L/v_f a_{eff}$ the Rayleigh number, $Le=a_f/D_f$, the Lewis number, $Da=K/L^2$, the Darcy number, $R_K=k_{eff}/k_f$, the conductivity ratio, $R_v=v_{eff}/v_f$, the viscosity ratio, $N=\beta_C \Delta C/\beta_T \Delta T$, the buoyancy ratio, S , the non dimensional concentration and θ , the non dimensional temperature. fp is a parameter which is equivalent to the factor of injection, introduced by Cheng [13] in his analytical analysis.

$v_f, v_{eff}, a_f, a_{eff}$ are respectively the fluid and the effective viscosities, and the fluid and the effective thermal diffusivities. K is the permeability of the porous matrix and D_f the mass diffusivity.

IV. NUMERICAL RESOLUTION

Governing coupled partial derivative equations together with boundary conditions have been solved using the finite volumes method, the coupling velocity-pressure being treated using algorithm SIMPLER. To respect the dependence between the involved variables, the system of algebraic equations obtained after discretisation, is solved by means of an iterative process said line by line, combining between the algorithm of Thomas and the iterative method of Gauss-Seidel.

The criterion of convergence is based on the relative error on the dependent variables; a value of 10^{-6} was noted sufficient in this case. A good agreement with borderline cases brought back in the literature (Mharzi *et al* [4], Bennacer *et al.* [8]) was especially observed.

V. RESULTS

We present in what follows the influence of the buoyancy ratio, the Rayleigh number and of the factor of injection fp on the average coefficients of heat and solutal exchanges, calculated starting from a heat and mass equilibrium.

A. Effect of buoyancy ratio

Figure 2 and 3 shows that heat and solutal exchange increase when the buoyancy ratio augments in absolute value $|N|$. This increasing of average Nusselt and Sherwood numbers is more significant since one passes from the injection ($fp < 0$) towards the extraction ($fp > 0$). Thus, when the thermal and solutal forces are opposite ($N < 0$), it is noticed that the transfers decrease as the buoyancy ratio decreases in absolute value.

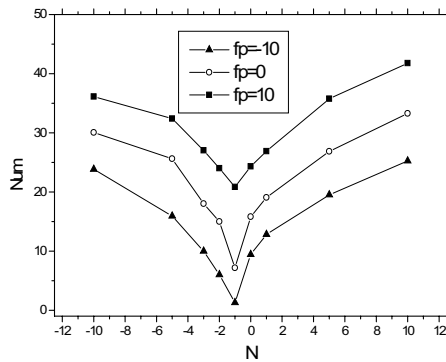


Fig. 2 Average Nusselt number according to the buoyancy ratio for various values of fp. $Ra=10^5$, $E=0.33$, $R_k=R_v=1$, $Pr=0.71$, $\epsilon = 0.5$, $Le=1$, $Da=10^{-3}$

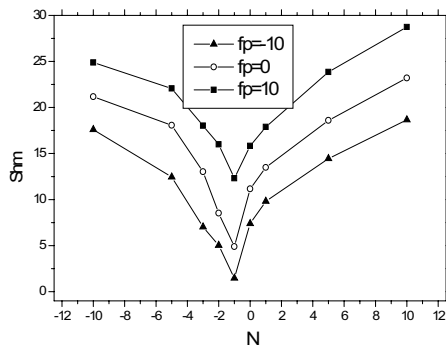


Fig. 3 Average Sherwood number according to the buoyancy ratio for various values of fp. $Ra=10^5$, $E=0.33$, $R_k=R_v=1$, $Pr=0.71$, $\epsilon = 0.5$, $Le=1$, $Da=10^{-3}$

In addition, when the predominance of the solutal force is less pronounced, the effect of the thermal force, which is opposed to the mass one, affects the transfers more and more, until they reach their minimal values. This minimum is at the level of a precise value ($N=-1$) of the buoyancy ratio, which is independent of the value of the factor of injection. Indeed, the curves representing the transfers of heat and mass (respectively Nu_m and Sh_m) present a total inversion of the behaviour at this critical value. This value corresponds to a mutual neutralization of the forces of thermal origin and mass origin. What generates a reduction in the convective movement and

reduces considerably the heat and mass exchanges; this phenomenon is well underlined if there is no matter injection/extraction ($fp=0$). This configuration is well appropriate for the promotion of mass filtration phenomena and heat insulation. On the other hand, when the thermal and solutal forces are co-operating ($N > 0$), thus acting in the same way, they generate an improvement in heat and solutal exchange; what results in higher values of the exchanges coefficients.

B. Effect of Rayleigh number

Figures 4 and 5 illustrate the influence of the Rayleigh number on heat and mass transfers for various factors of injection. It is noted that the increase, in absolute value, of the factor of injection disadvantages the exchanges in the case of the injection ($fp < 0$). This is due to creation by this injection on the base of the cavity, of a thick layer of fluid which is characterized by relatively low gradients of temperature and concentration, which involves thus a reduction of the transfers. On the other hand, in the case of the extraction ($fp > 0$) the mean values of the coefficients of thermal and solutal transfers are more significant as value of fp is high. This amplification of the exchanges can be the consequence of the compression of the boundary layer under the effect of the extraction. This layer becomes thinner and involves thus high heat and solutal gradients.

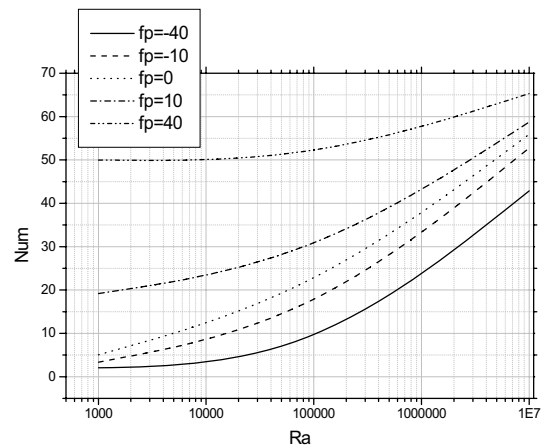


Fig. 4 Average Nusselt number according to Rayleigh number for various values of fp. $N=1$, $E=0.33$, $R_k=R_v=1$, $Pr=0.71$, $\epsilon = 0.5$, $Le=1$, $Da=10^{-3}$

It is also noted that the increase in the Rayleigh number supports the convective movements and consequently the thermosolutal exchange. In addition the reduction of the exchanges with the increase in $|fp|$, during the injection, is clearly accentuated when one considers high values of the Rayleigh number ($Ra=10^{+7}$). Lastly, the influence of Ra is more marked as the factor of injection fp approaches the zero value which represents the absence of any contribution of supplied matter and consequently of any disturbance on the thickness of the boundary layer.

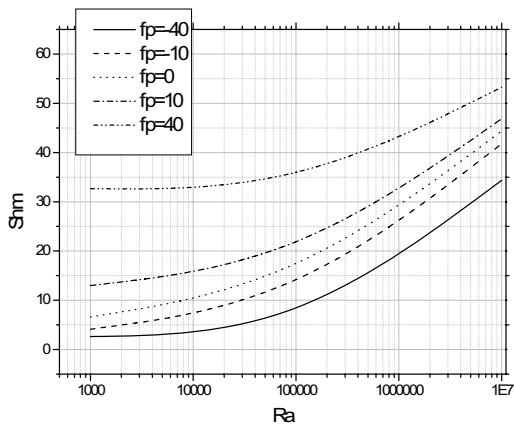


Fig. 5 Average Sherwood number according to Rayleigh number for various values of fp . $N=1$, $E=0.33$, $R_k=R_v=1$, $Pr=0.71$, $\epsilon=0.5$, $Le=1$, $Da=10^{-3}$

C. Streamlines and iso-concentrations

To further analyse the flow and heat and mass fields the corresponding streamlines and iso-concentrations are presented. In figures 6, 7, are plotted the streamlines corresponding respectively to $fp=-40$ (strong injection) and to $fp=+40$ (strong suction). One can notice that for the case of strong injection there is appearance of a unique cell which occupies most of the cavity where rotation is according to the countered time direction generating some difficulties for the displacement of the fluid from the area of injection towards the exit. As the factor fp increases the mode becomes two-cellular (strong suction from the bottom) with formation of two contra-rotating cells. It clearly appears a presence of a flowing zone between the inlet and exit surfaces, which will facilitate the heat and matter displacement. In addition, the observation of iso-concentration (fig. 8 and 9), allows us to note a quasi-stratification on the vertical walls and a relatively high thickness of boundary layer at the level of the zone of injection what decreases mass transfer in the cavity. When one passes from the case of strong injection towards strong suction, it appears a flatness of the mass boundary layer confirming so the higher values of the mass transfer gradients and by there, of the coefficients of transfer.

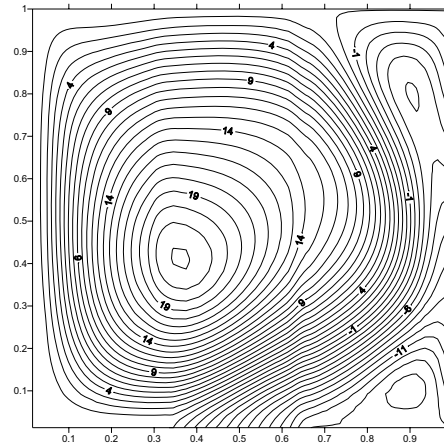


Fig. 6 Current lines for injection from the bottom ($fp=-40$) $Le=1$, $Da=10^{-3}$, $N=R_v=R_k=1$, $Pr=0.71$, $\epsilon=0.5$, $E=0.33$, $Ra=10^{+5}$

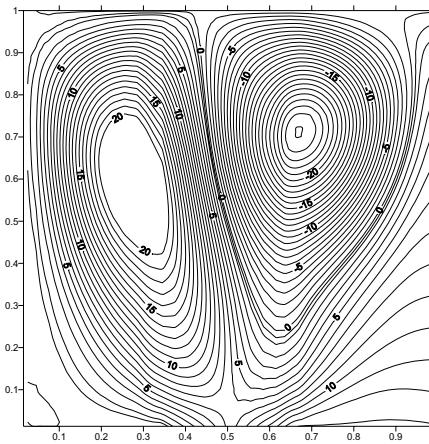


Fig. 7 Current lines for suction from the bottom ($fp=+40$), $Le=1$, $Da=10^{-3}$, $N=R_v=R_k=1$, $Pr=0.71$, $\epsilon=0.5$, $fp=+40$, $E=0.33$, $Ra=10^{+5}$

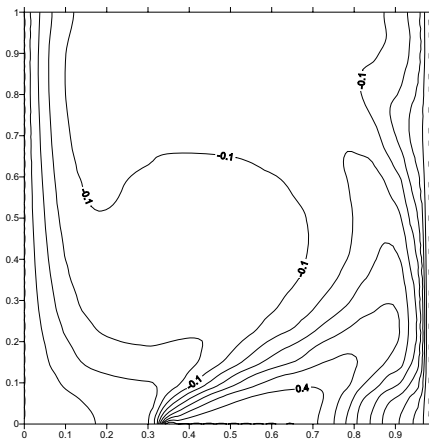


Fig. 8 Iso-concentration lines for injection from the bottom $Le=1$, $Da=10^{-3}$, $N=R_v=R_k=1$, $Pr=0.71$, $\epsilon=0.5$, $fp=-40$, $E=0.33$, $Ra=10^{+5}$

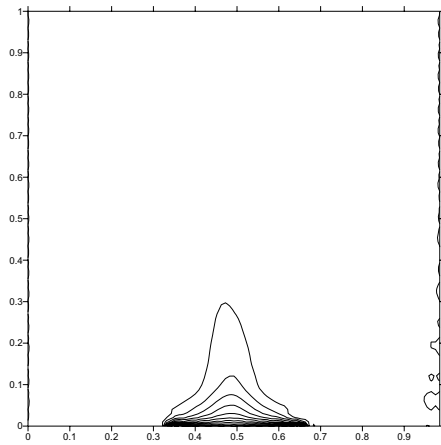


Fig. 9 Current lines for suction from the bottom ($fp=+40$), $Le=1$, $Da=10^{-3}$, $N=R_v=R_k=1$, $Pr=0.71$, $\varepsilon=0.5$, $fp=+40$, $E=0.33$, $Ra=10^{+5}$

VI. CONCLUSION

The analysis of heat and solutal exchanges in a partially porous square enclosure made it possible to show that the rise in the Rayleigh numbers can highly involve the mass and thermal transfers and that whatever the value of the factor of injection. It also appears that it is possible to highlight the existence of a particular (critical) value of the buoyancy ratio for which the transfers are minimized. In addition, the application of a local, parietal, injection/extraction, shows that the phenomenon of extraction on the level of the porous partition is that which improves the transfers, that they are thermal or solutal.

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