

# Joint Transmitter-Receiver Optimization for Bonded Wireline Communications

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**Abstract**—With the advent of DSL services, high data rates are now available over phone lines, yet higher rates are in demand. In this paper, we optimize the transmit filters that can be used over wireline channels. Results showing the bit error rates when optimized filters are used, and with a decision feedback equalizer (DFE) employed in the receiver, are given. We then show that significantly higher throughput can be achieved by modeling the channel as a multiple input multiple output (MIMO) channel. A receiver that employs a MIMO-DFE that deals jointly with several users is proposed and shown to provide significant improvement over the conventional DFE.

**Keywords**—DFE, MIMO Channels, Receiver Architectures, Transmit Filters.

## I. INTRODUCTION

Optimizing transmit filters for increasing the throughput over linear ISI channels is not a new problem [1]. Moreover, MIMO optimization for multi-user communications has been studied extensively in the area of wireless communications. However, the MIMO optimization over wireline cables using dynamic allocation of transmit power is a fairly new topic of research[2]. The prior research in the area of wireline communications was primarily concerned with single user transmitting over a single white Gaussian-noise channel [3][4]. The single-user 1-D communications problem can also be generalized and studied using the filter banks theory [5]. In [6], a numerical optimization scheme was proposed to optimize an FIR filter bank for communications over 1-D channel that is geared to support multiple users over the same channel. It is easy to envision multiple users sharing the same channel in home or small business environments. With the down-turn in the communications industry, and with the need for even higher data rates, it is still attractive to utilize the copper loop plant to achieve maximum possible throughput. This led to the emergence of industry wide standardization efforts such as the G.BOND project, adopted by the International Telecommunications Union (ITU). G.BOND supports combining multiple DSL channels to increase the aggregate throughput available for a single user. Also, the work done in the dynamic spectrum management project

clearly shows the advantages of jointly optimizing the MIMO channel [7]. In this paper, numerical optimization is performed to maximize the information rate to support single/multiple user(s) operating on single/multiple loops. The authors cannot emphasize enough that the problem formulation as a filter bank [8][9] might look similar to the known multi-dimensional techniques used in wireless communications; however there are many fundamental differences. First, wireline channels are linear-time-invariant. Second, wireline complete synchronization can be guaranteed. Third, the wireline channel can be measured, and previous work has shown efficient ways to estimate the channel characteristics [10].

This paper is organized as follows. In the next section we provide the practical motivation for our work. In section III we present our optimization equations for the transmit filter for the case of a single user utilizing one pair. We present results showing the improvement in bit error rate achieved by such optimization over the conventional square root raised cosine filter. In section IV we move to the multiple users case over more than one pair. We show how to formulate the optimization problem in several different ways that can be used according to the scenario of usage of the telephone pairs as well as the receiver architecture utilized. We first model the problem in the form of a single user suffering from interference. We derive the throughput function to be optimized, and show that our optimization does in fact enhance the bit error rate. We then model the problem as a MIMO system, and derive the MIMO throughput function. We then present results showing the improvement achieved in the throughput. Moreover, we utilize a receiver using a MIMO-DFE[11], and present bit error rate curves to further illustrate our findings. We end with the conclusion

## II. WORK MOTIVATION

One clear application for optimization over MIMO is the xDSL family of applications. With ever growing need for higher bit rates and with many customers causing interference to one another, the current static transmit power spectral density (PSD) allocation proved to be limited. The main reason is that static PSD allocation is designed based on worst-case model and it does not take into consideration the huge variations that happen in the field. The study conducted by the standards body, Committee T1 [7] proved that dynamic allocation of PSD generates significant bit-rate improvement.

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This paper provides a mathematical foundation for this improvement. It also offers a numerical optimization scheme based on that mathematical foundation. One example where this technique would apply is the very-high-speed DSL (VDSL). VDSL implies deploying fiber-optical connection from the central office to the cabinet, then keeping the connection from the cabinet to the home in copper cables. It is widely known that the main impairment for the VDSL is the self-far-end-cross-talk (FEXT). For that application, dynamic optimization of the transit PSD based on the actual FEXT channel will lead to a huge jump in the achievable data rate over a given cable. The technique, however, is not limited to VDSL. It can apply to other variety of xDSL applications. A typical VDSL setup is shown in Figure 1.

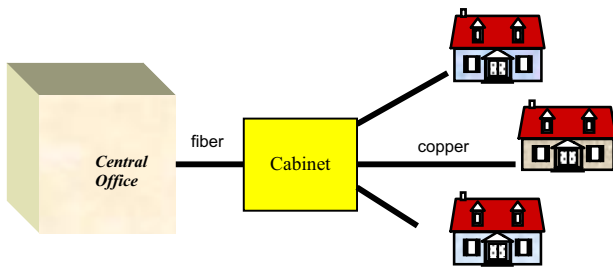


Fig. 1 VDSL Setup

### III. ONE-D OPTIMIZATION

First consider the optimization of a single channel. From a theoretical stand point, one can achieve the maximum throughput by either using infinite length transmit filter or by sending the data in blocks after filtering it by non-stationary filters[3]. Practical schemes have been introduced, however. For example, in the discrete multi-tone (DMT) redundancy is added at the transmitter to map the data symbols on orthogonal tones. The DMT can be viewed in light of filter bank theory as in [5]. In [1], an optimization technique was introduced to maximize the throughput of noisy ISI channels by passing the white input sequence through an FIR linear transmit filter. The MMSE-DFE was the assumed receiver. The  $y$  output of an additive-noise dispersive channel at time step  $k$  is given by

$$y(k) = \sum_{m=0}^{\nu} h(m)x(k-m) + n(k), \quad (1)$$

where  $h(m) = [h(l-1, m) \cdots h(0, m)]^T$  is the  $m^{\text{th}}$  vector of channel coefficients assuming an over-sampling factor of  $l$ , and  $\nu$  is the channel memory. The channel input  $x$  at time step  $k$  is given by

$$x(k) = \sum_{n=0}^{\nu_x} p(n)u(k-n) \quad (2)$$

where  $\{p(i)\}_{i=0}^{\nu_x}$  are the transmit filter coefficients and  $\{u(k)\}$  is an assumed unit energy transmitted sequence. Under the assumption of prior knowledge of the channel and noise characteristics, the channel throughput can be maximized.

This assumption is valid and numerous practical techniques have been published to get such channel and noise information, see for example [10]. As shown in[3], when an MMSE-DFE is used in the receiver, the channel throughput over a block of  $N$  output symbols is given by

$$I = \frac{1}{N+\nu} \log_2 (| I_{N+\nu} + H^h R_{mm}^{-1} H R_{xx} |) \quad (3)$$

where  $I$  is the identity matrix,  $()^h$  denotes the hermitian transpose.  $H$  is the channel matrix whose first block-row equals to the  $(\nu+1)$  length channel impulse response appended by zeros.  $R_{mm}$  and  $R_{xx}$  denote the Toeplitz auto-correlation matrices for the noise and the input sequences, respectively. The channel throughput can be numerically maximized as shown in [1]. In [6] the problem was extended to support the practically used fractionally sampled transmit filter. For this case, the throughput can be written as in (3), except that

$$R_{xx} = P \cdot R_{uu} \cdot P^h \quad (4)$$

where  $P$  is a matrix corresponding to the fractionally sampled transmit filter to be optimized, and  $u$  is the over-sampled input bit sequence. Therefore, assuming uncorrelated input sequence  $R_{uu}$  will be a diagonal matrix, with zeros everywhere except in the positions corresponding to symbols. To attain the resulting information rates for different SNR, equation (3) can be re-defined with substituting  $R_{xx}$  as defined in (4). Simulation showed that the optimal FIR filter consistently achieves better throughput over the raised-cosine as was shown in [6]. Furthermore, Figure 2 shows the improvement in bit error rate achieved by using such optimized transmit filters. In these results, all transmit filters compared were assumed to be of equal length, and the same finite length MMSE-DFE[2] architecture was used in the receiver. Note that although the optimized filter used in generating these results was obtained at a particular SNR, it still showed consistently better bit error rate over the range shown. We have also simulated the raised-cosine filter with several bandwidth expansion factors, and they all showed inferior performance to our optimized fractional filter.

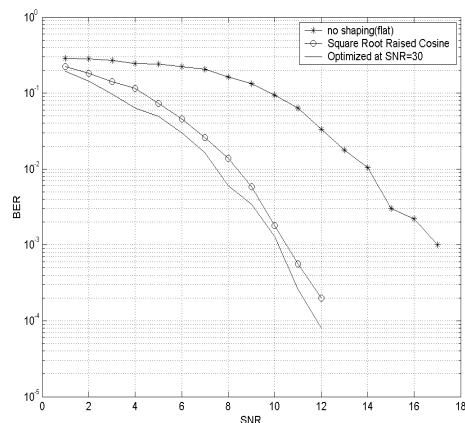


Fig. 2 BER for optimized vs. non-optimized

#### IV. MIMO OPTIMIZATION

##### A. Crosstalk Optimization

The information rate formula can be extended even further to support the crosstalk effect. The main assumption is that the different information sources are independent and as such the crosstalk term will appear as an extra noise at the receive-end. Consider the 2-user 2-D system shown in Fig. 3, the throughput can be written as

$$I_1 = \frac{1}{N + v_1} \log_2(|I_{N+v_1} + H_1^h R_{n_1 n_1}^{-1} H_1 R_{x_1 x_1}|) \quad (5)$$

$$I_2 = \frac{1}{N + v_2} \log_2(|I_{N+v_2} + H_2^h R_{n_2 n_2}^{-1} H_2 R_{x_2 x_2}|) \quad (6)$$

where  $I_1, I_2$  are the throughputs of users 1 and 2- as seen at the  $y_1$  and  $y_2$  outputs- respectively.  $R_{n_1 n_1}$  and  $R_{n_2 n_2}$  will each include the external noise term and the crosstalk term. Under the assumption of independent information sources, one can write

$$R_{n_1 n_1} = R_{mm} + ((P_2 H_{21})^h R_{u_2 u_2} P_2 H_{21}) \quad (7)$$

$$R_{n_2 n_2} = R_{mm} + ((P_1 H_{12})^h R_{u_1 u_1} P_1 H_{12}) \quad (8)$$

where  $R_{mm}$  is the external noise auto-correlation, assuming noise with same characteristics are added to both paths,  $H_{21}$  and  $H_{12}$  are the crosstalk channel matrices for both paths, and the  $P$ 's are the transmit filter matrices. In this formulation, we only consider the FEXT which is the dominant crosstalk impairment for the VDSL.

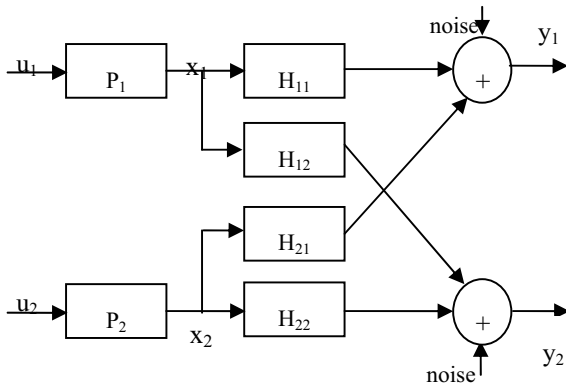


Fig. 3 Simplified 2-input/2-output MIMO system

Now, assume for simplicity that the direct channel is given by  $h_{11}(z) = h_{22}(z) = 0.4070 + 0.8150z^{-1} + 0.4070z^{-2}$  and the FEXT channel is given by  $h_{12}(z) = h_{21}(z) = 0.1 + 0.2z^{-1}$ , i.e. we assume a symmetric MIMO system. We can solve the optimization problem by forcing the two filters  $P_1$  and  $P_2$  to be identical to guarantee fairness among users. Note that in

case of a non-symmetric MIMO system, we can optimize  $I_1 + I_2$  subject to the condition that  $I_1 = I_2$ . Fig. 4 compares between the bit error rate performance obtained when the optimization is performed using equation (5), and that obtained using equation (3), which is when assuming we are dealing with a single user and disregarding the crosstalk channel. In both cases the optimization is performed at an SNR=20, and an MMSE-DFE[12] is used at the receiver. Even in the case of a simple channel such as the one tested here, our optimization outperforms the single user optimization

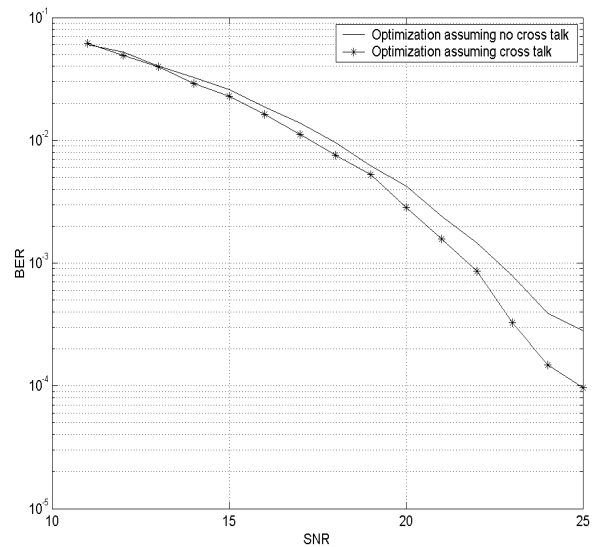


Fig. 4 Optimization considering crosstalk

##### B. Joint Detection

The formulation of the throughput function, discussed in the previous section, assumes that the 2 users will be decoded separately. However, if one assumes joint detection of both users, or that a single user uses both pairs; one can opt for a MIMO MMSE-DFE [11] receiver. In that case, we can extend the derivation of the throughput function derived in [2][3] to MIMO channels. As a simple, though general, extension, assume that a new shaping filter is obtained by convolving the shaping filter with the channel filter and that the channels are all now AWGN channels. One can write

$$y_1(k) = x_{11}(k) + x_{21}(k) + n(k), \quad (9)$$

where  $x_{11}$  is the sequence generated by  $u_1$  being "shaped" by the shaping filter  $\tilde{p}_{11}$ , where  $\tilde{p}_{11}$  is the filter obtained by convolution of  $p_1$  and  $h_{11}$ . It can be easily seen now that we can extend equation (3) to write the throughput at  $y_1$  as:

$$I_1 = \frac{1}{N} \log_2(|I_N + R_{mm}^{-1} [R_{x_{11} x_{11}} + R_{x_{21} x_{21}}]|), \quad (10)$$

where,

$$R_{x_1, x_1} = \tilde{P}_{11} R_{u_1 u_1} \tilde{P}_{11}^H \text{ and } R_{x_2, x_2} = \tilde{P}_{21} R_{u_2 u_2} \tilde{P}_{21}^H .$$

Fig. 5 shows the throughput improvement that can be attained using such formulation for the channels of the last section. Notice that the throughput is even higher than that of a single user system as in such a DFE architecture we use both sequences  $y_1$  and  $y_2$  in decoding each of the two information sequences  $u_1$  and  $u_2$ , and hence the crosstalk term is no longer a liability of our system but rather an “extra” channel we get more bits on. Fig. 6 shows that the bit error rate is also enhanced significantly by using the proposed formulation and receiver architecture.

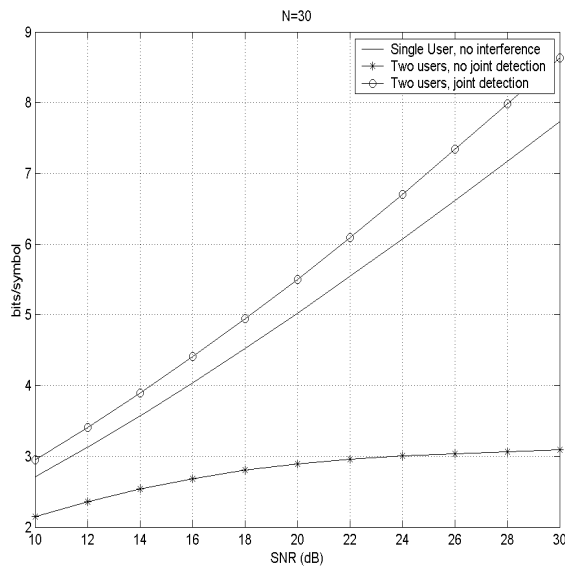


Fig. 5 Throughput improvement using MIMO-DFE

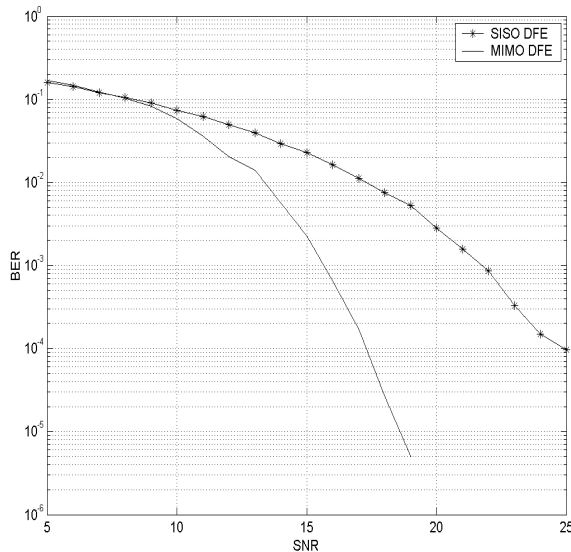


Fig. 6 BER improvement using MIMO-DFE

Again, note that we can also adjust the previous optimization problem to optimize  $I_1 + I_2$ .

## V. CONCLUSION

In this paper, we showed that the bit error rate performance obtained by fractional optimization of transmit filters is better than that using the classical raised cosine filters. Moreover, two new formulations for the optimization of transmit filters in the case of multiple users operating on a communication medium with crosstalk between them were introduced. The first formulation optimizes the transmit filters assuming the other users are interference. The performance obtained by such formulation is better than the performance obtained from optimization neglecting the effect of crosstalk. The second formulation assumes a MIMO-DFE receiver that jointly decodes the users. The throughput, and consequently, the bit error rate performance, attained by this approach is significantly better than those attained by modeling the system as separate users using separate communication mediums.

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