

Assessment of the Influence of External Earth Terrain at Construction of the Physic-mathematical Models or Finding the Dynamics of Pollutants' Distribution in Urban Atmosphere

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Abstract—There is a complex situation on the transport environment in the cities of the world. For the analysis and prevention of environmental problems an accurate calculation hazardous substances concentrations at each point of the investigated area is required. In the turbulent atmosphere of the city the well-known methods of mathematical statistics for these tasks cannot be applied with a satisfactory level of accuracy. Therefore, to solve this class of problems apparatus of mathematical physics is more appropriate. In such models, because of the difficulty as a rule the influence of uneven land surface on streams of air masses in the turbulent atmosphere of the city are not taken into account. In this paper the influence of the surface roughness, which can be quite large, is mathematically shown. The analysis of this problem under certain conditions identified the possibility of areas appearing in the atmosphere with pressure tending to infinity, i.e. so-called "wall effect".

Keywords—Air pollution, concentration of harmful substances, physical-mathematical model, urban area.

I. INTRODUCTION

THE state-of-art problematic relevant to the field of vehicular traffic flow ecology is observed in the article that is common and quite typical to the bulk of cities worldwide. Motor transport is ranked as a first among the series of anthropogenic air pollution sources of the large cities. The constantly growing number of vehicles brings about an increasingly large level of contamination by such a noise and electromagnetic pollution produced by motor vehicle emissions, pollution of urban territory by all kinds of solid waste, distortion of water and tectonic balance with the environment, some landscape distortions, etc. This, in its turn, results in some irresolvable economic consequences [1-3].

Therefore, atmospheric air pollution is one of the most acute ecological problems of transport ecology [4]. At that, certain concentrations of harmful substances and the possibility of predicting their concentration values make it possible to take corresponding decisions aimed at reducing the impact of those substances upon people and urban environment.

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This would enable one to minimize the impact of transport system upon people and reduce large indirect damages (observed at present) [3-5].

Therefore, to resolve the problem successfully, some data describing abundance of harmful substances in urban atmosphere is needed. For that purpose there exist a number of mathematical models based on various methods of assessment and forecast of ecological parameters, and describing the dynamics of harmful substances propagation.

To evaluate the results of implemented solutions, the apparatus of mathematical statistics is traditionally applied. The usage of the apparatus of mathematical statistics and other mathematical toolsets where development of a model is based on the results of single shots and integral environmental measurements (in particular, those of urban atmospheric environment) aimed at determination of the dynamics of exhaust gases concentration in the urban atmosphere – is connected with the problem of low accuracy of measurements. According to engineering practice, a computation error may number in the hundreds of a percent [6]. Mathematical statistics normally uses interpolation methods of modeling and extrapolation models to estimate ecological parameters. It is shown in [6-8] that neither of the abovementioned models is capable of presenting an acceptable quality level in turbulent environment of a city. However, those models may be used, for instance, to investigate laminar flows in nature where concentration changes are not so fast.

It has been shown in [6] that methods of mathematical statistics are not capable of solving transport ecology problems in the optimal way. That is a main reason why the apparatus of mathematical physics is used to investigate ecological problems as a major research tool applied to study the problem of urban atmospheric pollution produced by motor vehicles, – enabling one to achieve a resulting high accuracy based on initial data. Due to existing complexities in models' development (formalization, calculation, etc.), number of such models is relatively law [6-8]. Models of that kind enable one to investigate the ecology of urban quarters in a more profound and detailed way, revealing "sore" issues; after a decision has been made, those models also make it possible to validate mathematically the expected positive changes resulting from those decisions, and to test the expected results in practice. Such mathematical models aimed at finding concentrations of harmful substances in urban environment creating, taking into account the nature of the investigated phenomenon.

In models based on mathematical physics apparatus, because of the difficulty as a rule the influence of uneven land surface on streams of air masses in the turbulent atmosphere of the city are not taken into account. In this paper the influence of the surface roughness, which can be quite large, is mathematically shown.

II. STATEMENT OF THE PROBLEM

As is known [9-12], the Earth's relief has an impact on the character of air or liquid flow around a body. Ignoring the Earth's relief at construction of mathematical models of movement of admixtures in air and water environment prevents from differentiating the arising air or water flow disturbance. In many scientific articles [6-8, 13-15, 16-17,18 and etc], dedicated to studies of dynamics of admixtures in air /liquid, the corresponding mathematical models are constructed in canonical domains (1D is either finite or infinite bar; 2D is either a rectangle or a strip; 3D is either bounded or unbounded parallelepiped or a sphere etc.). Consequently, the presence of asperities of the area of the ground under the study, specifically, a studied section of a modern urban street with vehicular movement, has been ignored in these models.

Here the following task is being considered. Let there be two air masses $S1$ and $S2$, divided by a surface ∂S . It is supposed that $S1$ and $S2$ are moving along the earth plane, i.e., along a horizontal plane at different velocities g_1 and g_2 accordingly. Asperities of the ground cannot but entail disturbances of both air and water flow and, therefore, our objective in this Section is to differentiate an unperturbed state, which the air masses $S1$ and $S2$ could have in the absence of asperities, and a turbulent state, which takes place in reality. For this objective we will, first, speculate as regards an unperturbed state of the air masses $S1$ and $S2$:

- characteristic qualities, such as temperature, pressure, density etc., are changed only vertically, i.e., these characteristic qualities depend only upon the height, where the ground surface is taken as a reference mark;
- velocities g_1 and g_2 of air masses $S1$ and $S2$ are constants, at that: $g_1 \neq g_2$;
- the temperature T_1 of the flow $S1$ changed in a linear fashion [10,11], namely:

$$T_1(x_3) \stackrel{\text{def}}{=} T_{Land} + \alpha_1 x_3,$$

where T_{Land} is a temperature on the ground surface, $\alpha_1 < 0$ is a temperature gradient, x_3 - is a height variable from the ground surface;

- the temperature T_2 of the flow $S2$ changed in a linear fashion [10,11], namely:

$$T_2(x_3) \stackrel{\text{def}}{=} T_h + \alpha_2 \cdot x_3,$$

where T_h is a temperature of the surface ∂S of division of

air masses $S1$ and $S2$, $\alpha_2 < 0$ is a temperature gradient, h is height of the surface ∂S of division from the ground, in other words, h is the height of an unperturbed surface of division, x_3 is the height variable from the ground surface;

- the motion of the air masses $S1$ and $S2$ takes place equally in all vertical layers parallel to the flow $S1$ (Fig. 1).

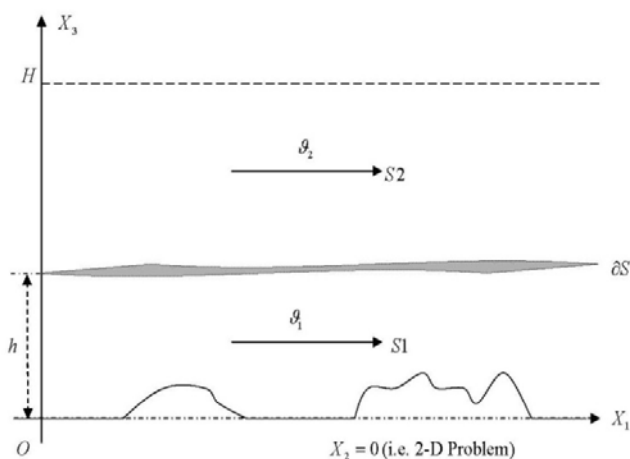


Fig.1 Motion of air masses $S1$ and $S2$ inclusive of urban relief

In order to construct an adequate mathematical model of the above-described problem at the made speculations, let's denote by $p_1(x_3)$ and $p_2(x_3)$ the pressure of the air masses $S1$ and $S2$, accordingly; by $\rho_1(x_3)$ and $\rho_2(x_3)$ the density of the air masses $S1$ and $S2$, accordingly; by $g_1(x_3)$ and $g_2(x_3)$ the velocity of the air masses $S1$ and $S2$, accordingly. As shown above, the temperature of the flows $S1$ and $S2$ is denoted by $T_1(x_3)$ and $T_2(x_3)$, accordingly.

The values p_i , ρ_i , g_i and T_i ($i=1,2$) are the characteristic qualities of unperturbed flows of air masses $S1$ and $S2$. Our objective is to determine (i.e., differentiate) analogous characteristic qualities of turbulent flows, where disturbance is entailed by the presence of asperities on the ground surface (to be more exact, on the surface of the studied section of urban street with vehicular movement). Therefore alongside with the symbols p_i , ρ_i and g_i ($i=1,2$) we will introduce also such symbols as \tilde{p}_i , $\tilde{\rho}_i$, \tilde{g}_i and w_i ($i=1,2$), where \tilde{p}_i is the disturbance in pressure of the flow i S_i ($i=1,2$); $\tilde{\rho}_i$ is disturbance of thickness of i -th flow S_i ($i=1,2$); \tilde{g}_i is disturbance of "horizontal" speed of i -th flow S_i ($i=1,2$); w_i is the disturbance in the "vertical" velocity of i -th flow S_i ($i=1,2$). Then the correct (real, true) values of these characteristic qualities obviously will be equal to $p_i + \tilde{p}_i$, $\rho_i + \tilde{\rho}_i$, $g_i + \tilde{g}_i$ and w_i ($i=1,2$), and with respect to these true characteristic qualities we can write

corresponding motion equations[10-11,19-20]:

$$\frac{1}{\rho_i + \tilde{\rho}_i} \frac{\partial(p_i + \tilde{p}_i)}{\partial x_1} + (\vartheta_i + \tilde{\vartheta}_i) \frac{\partial(\vartheta_i + \tilde{\vartheta}_i)}{\partial x_1} + w_i \frac{\partial(\vartheta_i + \tilde{\vartheta}_i)}{\partial x_3} = 0; \quad i = 1, 2, \quad (1)$$

$$(\vartheta_i + \tilde{\vartheta}_i) \frac{\partial w_i}{\partial x_1} + w_i \frac{\partial w_i}{\partial x_3} + \frac{1}{\rho_i + \tilde{\rho}_i} \frac{\partial(p_i + \tilde{p}_i)}{\partial x_3} + g = 0; \quad i = \overline{1, 2}, \quad (2)$$

$$\frac{\partial\{(\rho_i + \tilde{\rho}_i)(\vartheta_i + \tilde{\vartheta}_i)\}}{\partial x_1} + \frac{\partial\{w_i(\rho_i + \tilde{\rho}_i)\}}{\partial x_3} = 0, \quad i = \overline{1, 2}, \quad (3)$$

where

$$0 < x_1 < L; \quad 0 < x_3 < h, \quad i = 1;$$

$$0 < x_1 < L; \quad h < x_3 < H, \quad i = 2;$$

$$p_i = p_i(x_3), \quad \rho_i = \rho_i(x_3), \quad \vartheta_i = \vartheta_i(x_3), \quad \tilde{p}_i = \tilde{p}_i(x_1, x_3),$$

$$\tilde{\rho}_i = \tilde{\rho}_i(x_1, x_3), \quad \tilde{\vartheta}_i = \tilde{\vartheta}_i(x_1, x_3)$$

$$\text{and } w_i = w_i(x_1, x_3) \quad (i = 1, 2).$$

To the (1)-(3) systems it is necessary to add the heat balance equation. Since we assumed that the arising disturbances are entailed solely by a relief of the studied section of a city (of course, there are also other impacts that generate disturbances of the cited characteristic qualities of air flow[9,19,21]), then heat inflow should be equal to zero. Consequently, to the equations (1)-(3) it is necessary to add a condition of lack of heat inflow, namely,

$$(\vartheta_i + \tilde{\vartheta}_i) \frac{\partial}{\partial x_1} \left(\frac{p_i + \tilde{p}_i}{(\rho_i + \tilde{\rho}_i)^\beta} \right) + w_i \frac{\partial}{\partial x_3} \left(\frac{p_i + \tilde{p}_i}{(\rho_i + \tilde{\rho}_i)^\beta} \right) = 0, \quad (4)$$

where $\beta = 1.41$ [9, 21].

III. FINDING THE SOLUTION OF THE EQUATIONS' SYSTEM

So, the true characteristic qualities of turbulent airflows are determined (of course, in the presence of corresponding boundary conditions and conjugation conditions on the surface ∂S of division of the air masses $S1$ and $S2$) by a system of equations (1)-(4). Ditto as in the study [20], in this section we will be restricted to the study of a question of finding of asymptotic expression of solutions of the system (1)-(4) on below assumptions:

- the flow disturbances are taken as small that their degrees above the first order might be ignored;
- smallness of irregularity of the surface of the urban street section under consideration is taken (against the maximum spatial dimensions of the area under consideration).

These two assumptions allow dividing the system (1)-(4) into two parts, namely, the first system contains equation of unperturbed motion, but the second part of the system contains equation of turbulent motion. The idea of division of the system (1)-(4) into the above – said parts is based on the idea of equating in the system of equations (1)-(4) of finite summands and first – order infinitesimals that allows reducing the initial system (1)-(4) to below two equation systems:

- a linear equation system for unperturbed air mass;
- a linear equation system for disturbed air mass.

Later we have the equation of disturbed flow:

$$\begin{cases} \vartheta_i \frac{\partial \xi_i}{\partial x_1} + \frac{\partial \tilde{p}_i}{\partial x_3} + g \tilde{\rho}_i = 0, \\ \vartheta_i \frac{\partial \eta_i}{\partial x_1} + \frac{\partial \tilde{p}_i}{\partial x_1} = 0, \\ \vartheta_i \frac{\partial \tilde{\rho}_i}{\partial x_1} + \frac{\partial \eta_i}{\partial x_1} + \frac{\partial \xi_i}{\partial x_3} = 0, \\ \vartheta_i \frac{\partial \tilde{p}_i}{\partial x_1} - \beta RT_i \vartheta_i \frac{\partial \tilde{\rho}_i}{\partial x_1} + \{(\beta - 1)g - \beta R \alpha_i\} \xi_i = 0. \end{cases} \quad (5)$$

So, the problem resides in finding of asymptotic solution of the system (5). To this effect we will first study one auxiliary problem, which resides in finding of solution of the system (5) provided that the form of the studied section of urban street is defined by equation

$$x_3 = X_3(x_1) = \cos(\gamma x_1). \quad (6)$$

In this situation, it is obvious that it has to be found a solution of the system (5) in the form

$$\begin{cases} \eta_i(x_1, x_3) = \eta_i^{(1)}(x_3) \cos(\gamma x_1), \\ \tilde{P}_i(x_1, x_3) = \tilde{p}_i^{(1)}(x_3) \cos(\gamma x_1), \\ \tilde{\rho}_i(x_1, x_3) = \tilde{\rho}_i^{(1)}(x_3) \cos(\gamma x_1), \\ \xi_i(x_1, x_3) = \xi_i^{(1)}(x_3) \sin(\gamma x_1), \\ X_{3,\partial S}(x_1) = C \cos(\gamma x_1), \end{cases} \quad (7)$$

where $C \equiv const.$, $X_{3,\partial S}(x_1)$ is the disturbance of the surface of the division ∂S .

Until we claim satisfaction of the system (5) by the functions from (7), we will try to lay down conditions on the ground surface and on the surface of the division ∂S . Since the ground surface normal component of particle velocity, adjacent to the surface, is obviously equal to zero.

Further, written the required conditions form necessary conjugation conditions on the ground surface and on the surface of ∂S division of two mediums $S1$ and $S2$. Now we can claim that the functions represented in (7) satisfy the equation of the system (5) of the turbulent airflow and to given conditions. Alternately plugging the functions from (7) into corresponding equation of the system (5) and considering the conjugation appropriate conditions, we will derive a system of equations as follows:

$$\gamma \mathcal{G}_i(x_3) \xi_i^{(1)}(x_3) + \left\{ \tilde{\rho}_i^{(1)}(x_3) \right\}' + g \tilde{\rho}_i^{(1)}(x_3) = 0, \quad (8)$$

$$\mathcal{G}_i(x_3) \eta_i^{(1)}(x_3) + \tilde{\rho}_i^{(1)}(x_3) = 0, \quad (9)$$

$$\gamma \mathcal{G}_i(x_3) \tilde{\rho}_i^{(1)}(x_3) + \gamma \eta_i^{(1)}(x_3) - \left\{ \xi_i^{(1)}(x_3) \right\}' = 0, \quad (10)$$

$$\gamma \mathcal{G}_i(x_3) \tilde{\rho}_i^{(1)}(x_3) - \gamma \beta R T_i(x_3) \mathcal{G}_i(x_3) \tilde{\rho}_i^{(1)}(x_3) + \left\{ \beta R \alpha_i - (\beta - 1) g \right\} \xi_i^{(1)}(x_3) = 0, \quad (11)$$

$$\xi_1^{(1)}(0) + \mathcal{G}_1(0) \rho_{Land} C \gamma = 0, \quad (12)$$

$$\tilde{\rho}_1^{(1)}(h) + \tilde{\rho}_2^{(1)}(h) = C g (\rho_1(h) - \rho_2(h)), \quad (13)$$

$$C \gamma (\mathcal{G}_1(h) - \mathcal{G}_2(h)) \rho_1(h) \rho_2(h) = \rho_2(h) \xi_1^{(1)}(h) - \rho_1(h) \xi_2^{(1)}(h), \quad (14)$$

$$\rho_2(h) \mathcal{G}_2(h) \xi_1^{(1)}(h) = \rho_1(h) \mathcal{G}_1(h) \xi_2^{(1)}(h). \quad (15)$$

Now, having (8)-(15), we will do as follows:

– from (9) we will derive

$$\tilde{\rho}_i^{(1)}(x_3) = -\mathcal{G}_i(x_3) \eta_i^{(1)}(x_3); \quad (16)$$

– from (11) and (16) we will derive

$$\gamma \mathcal{G}_i^2(x_3) \eta_i^{(1)}(x_3) - \gamma \beta R T_i(x_3) \mathcal{G}_i(x_3) \tilde{\rho}_i^{(1)}(x_3) - \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \xi_i^{(1)}(x_3) = 0; \quad (17)$$

– from (10) we will derive

$$\tilde{\rho}_i^{(1)}(x_3) = \frac{\left\{ \xi_i^{(1)}(x_3) \right\}' - \gamma \eta_i^{(1)}(x_3)}{\gamma \mathcal{G}_i(x_3)}; \quad (18)$$

– from (17) and (18) we will derive

$$\eta_i^{(1)}(x_3) = \frac{\beta R T_i(x_3) \left\{ \xi_i^{(1)}(x_3) \right\}' + \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \xi_i^{(1)}(x_3)}{\gamma \left\{ \beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right\}}; \quad (19)$$

– from (16) and (19) we will derive

$$\tilde{\rho}_i^{(1)}(x_3) = -\frac{\beta R T_i(x_3) \left\{ \xi_i^{(1)}(x_3) \right\}' + \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \xi_i^{(1)}(x_3)}{\frac{\gamma}{\mathcal{G}_i(x_3)} \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right)}; \quad (20)$$

– from (18) and (19) we will derive

$$\tilde{\rho}_i^{(1)}(x_3) = -\frac{\mathcal{G}_i^2(x_3) \left\{ \xi_i^{(1)}(x_3) \right\}' + \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \xi_i^{(1)}(x_3)}{\gamma \mathcal{G}_i(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right)}. \quad (21)$$

So, the desirable functions $\eta_i^{(1)}(x_3)$, $\tilde{\rho}_i^{(1)}(x_3)$ and $\tilde{\rho}_i^{(1)}(x_3)$ ($i=1,2$), as may be inferred from equations (19)-(21), are expressed through the functions $\xi_i^{(1)}(x_3)$ and $\left\{ \xi_i^{(1)}(x_3) \right\}'$. Considering these expressions in a single remained "untouched" equation (8), we will derive

$$\begin{aligned} & \gamma^2 \mathcal{G}_i^2(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right)^2 \xi_i^{(1)}(x_3) \\ & - g \mathcal{G}_i^2(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \left\{ \xi_i^{(1)}(x_3) \right\}' \\ & - g \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \xi_i^{(1)}(x_3) - \\ & - \beta R T_i(x_3) \mathcal{G}_i(x_3) \mathcal{G}_i'(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \left\{ \xi_i^{(1)}(x_3) \right\}' \\ & - \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \mathcal{G}_i(x_3) \mathcal{G}_i'(x_3) \\ & \times \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \xi_i^{(1)}(x_3) \\ & - \beta R T_i'(x_3) \mathcal{G}_i^2(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \left\{ \xi_i^{(1)}(x_3) \right\}' \\ & - \beta R T_i(x_3) \mathcal{G}_i^2(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \left\{ \xi_i^{(1)}(x_3) \right\}'' \\ & - \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \mathcal{G}_i^2(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \left\{ \xi_i^{(1)}(x_3) \right\}' \\ & + \beta^2 R^2 T_i(x_3) T_i'(x_3) \mathcal{G}_i^2(x_3) \left\{ \xi_i^{(1)}(x_3) \right\}' \\ & - 2 \beta R T_i(x_3) \mathcal{G}_i^3(x_3) \mathcal{G}_i'(x_3) \left\{ \xi_i^{(1)}(x_3) \right\}' \\ & + \beta R \left\{ (\beta - 1) g - \beta R \alpha_i \right\} T_i'(x_3) \mathcal{G}_i^2(x_3) \xi_i^{(1)}(x_3) \\ & - 2 \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \mathcal{G}_i^3(x_3) \mathcal{G}_i'(x_3) \xi_i^{(1)}(x_3) = 0. \end{aligned}$$

In the last ordinary differential equation concerning the function $\xi_i^{(1)}(x_3)$ we will take account of the equality $T_i'(x_3) = -\alpha_i$ ($i=1,2$), and thereafter group together the corresponding coefficients at $\xi_i^{(1)}(x_3)$, $\left\{ \xi_i^{(1)}(x_3) \right\}'$ and $\left\{ \xi_i^{(1)}(x_3) \right\}''$. Then we will derive the following ordinary linear differential equation of the second order concerning the desirable function $\xi_i^{(1)}(x_3)$:

$$\begin{aligned} & \beta R T_i(x_3) \mathcal{G}_i^2(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \left\{ \xi_i^{(1)}(x_3) \right\}'' \\ & + \left\{ g \beta^2 R T_i(x_3) \mathcal{G}_i^2(x_3) + 2 \beta R \alpha_i \mathcal{G}_i^4(x_3) \right\} \\ & - \left\{ -g \beta \mathcal{G}_i^4(x_3) - \alpha_i \beta^2 R^2 T_i(x_3) \mathcal{G}_i^2(x_3) \right\} \\ & \times \left\{ \xi_i^{(1)}(x_3) \right\}' + \left\{ g \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \right. \\ & - \beta^2 \mathcal{G}_i^2(x_3) \left(\beta R T_i(x_3) - \mathcal{G}_i^2(x_3) \right) \\ & \left. + \beta R \alpha_i \left\{ (\beta - 1) g - \beta R \alpha_i \right\} \mathcal{G}_i^2(x_3) \right\} \xi_i^{(1)}(x_3) = 0. \quad (22) \end{aligned}$$

We will study the derived equation (22). In the first instance let's single out that the coefficient at the top-order derivative $\{\xi_i^{(1)}(x_3)\}''$ will equal zero at two points, namely, where

$$\beta RT_i(x_3) \mathcal{G}_i^2(x_3) = 0.$$

and at the points, where $\beta RT_i(x_3) - \mathcal{G}_i^2(x_3) = 0$.

In other words, the equation (22) is the equation with two regular singular points.

Considering that $\mathcal{G}_i(x_3) \neq 0 \quad \forall x_3 \quad (i=1,2)$, then a point at which $T_i(x_3) = 0$ corresponds to the first singular point. Since $T_{Land} \neq 0$ (i.e. the value is not equal to zero in all points of earth's surface) in the expression $T_1(x_3) \stackrel{def}{=} T_{Land} + \alpha_1 x_3$, then we will get that for the second medium $S2$ this singular point locates at its upper boundary and, consequently, it cannot reside inside of the first medium $S1$.

The second singular point, for which

$$\mathcal{G}_i^2(x_3) = \beta RT_i(x_3) \quad (i=1,2), \quad (23)$$

might reside either inside the first or the second layer. Therefore at this singular point we may require the roundedness of the functions $\xi_i^{(1)}(x_3)$, $\eta_i^{(1)}(x_3)$, $\tilde{p}_i^{(1)}(x_3)$ and $\tilde{\rho}_i^{(1)}(x_3)$. In order to study the second singular point, which according to the formula (23) might be interpreted as a point at the flow velocity $\mathcal{G}_i(x_3)$ equal to sound velocity $\beta \cdot R \cdot T_i(x_3)$, we will introduce a new independent variable φ according to the formula

$$\varphi \stackrel{def}{=} \frac{\beta RT_i(x_3) - \mathcal{G}_i^2(x_3)}{\mathcal{G}_i^2(x_3)}. \quad (24)$$

The introduced by convention (24) variable φ , essentially simplifies the procedure of statement of conditions contributing to the finiteness of the functions $\xi_i^{(1)}(x_3)$, $\eta_i^{(1)}(x_3)$, $\tilde{p}_i^{(1)}(x_3)$ and $\tilde{\rho}_i^{(1)}(x_3)$.

Considering the new independent variable φ , determined by the equality (24), the equation (22) takes the following more simplified form:

$$\begin{aligned} \varphi(\varphi+1) \left\{ \xi_i^{(1)}(\varphi) \right\}_\varphi'' + \frac{\alpha_i R(\varphi-1) - g\varphi}{\alpha_i R} \left\{ \xi_i^{(1)}(\varphi) \right\}_\varphi' \\ + \frac{\left(\begin{array}{l} \beta(g - \alpha_i R)(\beta R \alpha_i + g\varphi) \\ -g(\beta R \alpha_i + g\varphi) - \gamma^2 \mathcal{G}_i^4(\varphi) \varphi^2 \end{array} \right)}{\beta^2 R^2 \alpha_i^2} \xi_i^{(1)}(\varphi) = 0. \end{aligned} \quad (25)$$

Now then, the characteristic qualities of the equation (25)

are $\lambda_1 = 0$ and $\lambda_2 = 2$. Directly one can easily make sure that the solution of the equation (25), which corresponds to the characteristic quality $\lambda_1 = 0$, has a logarithmic singularity. If

we pass on to the earlier notations of variable x_3 (24), then according to the formula (20) we will derive that at the point $\varphi = 0$ the pressure $\tilde{p}_i^{(1)}(x_3)$ becomes infinite. Consequently, it is necessary to take only such solution of the equation (25), which corresponds to the characteristic quality $\lambda_2 = 2$. A solution corresponding to this characteristic quantity is effective in the second layer $S2$ and it reduces the function $\xi_i^{(1)}(x_3)$ to zero, which means reduction of the "vertical" velocity $w_2(x_1, x_3)$ to zero. This may be interpreted as the presence of a certain surface in a form of a solid boundary on which there is a singular point $\varphi = 0$, i.e. whereon there is

$$\varphi = \frac{\beta RT_2(x_3) - \mathcal{G}_2^2(x_3)}{\mathcal{G}_2^2(x_3)} = 0,$$

that means

$$\mathcal{G}_2^2(x_3) = \beta RT_2(x_3). \quad (26)$$

$$\mathcal{G}_2^2(x_3) = \sin^2(x_3) \quad \beta \cdot R \cdot T_2(x_3) = 1,41 \cdot 8,31 \cdot (0,01) \cdot x_3$$

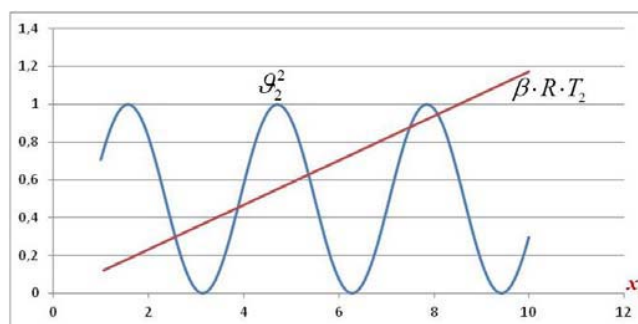


Fig. 2 An example of the occurrence of a "solid boundary"(at the intersection of two lines)

In other words, the singular point $\varphi \in S2$, at which there is the equality (26), defines a certain solid boundary (an example is shown on Fig.2), i.e. the surface described by the equality (26).

This effect was discovered in the '70s for the aquatic environment. In this article for the first time such effect is described for air environment.

IV. SOLUTION OF FULL PROBLEM

After of the unconventional solution of this system of equations as a result of which can be found the solution of the whole problem which may be written in the form of Fourier integral in the following form:

$$\begin{aligned} \xi_i^{(General)}(x_1, x_3) \\ = \frac{1}{\pi} \int_0^{+\infty} d\sigma_1 \int_{-\infty}^{+\infty} \xi_i^{(1)}(\sigma_1, x_3) X_3(\sigma_2) \sin(\gamma(x_1 - \sigma_2)) d\sigma_2, \end{aligned} \quad (27)$$

$$\begin{aligned} \eta_i^{(General)}(x_1, x_3) \\ = \frac{1}{\pi} \int_0^{+\infty} d\sigma_1 \int_{-\infty}^{+\infty} \eta_i^{(1)}(\sigma_1, x_3) X_3(\sigma_2) \cos(\gamma(x_1 - \sigma_2)) d\sigma_2, \end{aligned} \quad (28)$$

$$\begin{aligned} \tilde{\rho}_i^{(General)}(x_1, x_3) \\ = \frac{1}{\pi} \int_0^{+\infty} d\sigma_1 \int_{-\infty}^{+\infty} \tilde{\rho}_i^{(1)}(\sigma_1, x_3) X_3(\sigma_2) \cos(\gamma(x_1 - \sigma_2)) d\sigma_2, \end{aligned} \quad (29)$$

$$\begin{aligned} p_i^{(General)}(x_1, x_3) \\ = \frac{1}{\pi} \int_0^{+\infty} d\sigma_1 \int_{-\infty}^{+\infty} p_i^{(1)}(\sigma_1, x_3) X_3(\sigma_2) \cos(\gamma(x_1 - \sigma_2)) d\sigma_2, \end{aligned} \quad (30)$$

$$\begin{aligned} X_{3,\partial S}(x_1) \\ = \frac{1}{\pi} \int_0^{+\infty} d\sigma_1 \int_{-\infty}^{+\infty} X_{3,\partial S}(\sigma_1) Z(\sigma_2) \cos(\gamma(x_1 - \sigma_2)) d\sigma_2, \end{aligned} \quad (31)$$

where \tilde{p}_i , $\tilde{\rho}_i$, $\xi_i^{(1)}$, $\eta_i^{(1)}$ and $X_{3,\partial S}$ are the solutions of the same problem, on condition (6).

Remark. When the solutions \tilde{p}_i , $\tilde{\rho}_i$, $\xi_i^{(1)}$, $\eta_i^{(1)}$ have regular exceptional points (exactly this case was studied above) and the poles (i.e., irregular exceptional points) on the real coordinate axis, then in the formulas (27)-(31), by which the desirable solutions are determined $\xi_i^{(General)}(x_1, x_3)$, $\eta_i^{(General)}(x_1, x_3)$, $\tilde{\rho}_i^{(General)}(x_1, x_3)$, $p_i^{(General)}(x_1, x_3)$, $X_{3,\partial S}(x_1)$ of the general problem, the corresponding integrals are understood in the matter of Cauchy value [22].

V. CONCLUSION

1. It is mathematically proved that the presence of roughness (irregularity) on the earth's surface influences the way of flowing around the object by the air flow or liquid. Therefore, ignoring actual terrain of the Earth in the construction of mathematical models for the description of impurities motion both in atmospheric and aqueous environments does not allow identifying and evaluating the resulted disturbances of water or air flow. For evaluating characteristic values disturbances of air masses (such as temperature, pressure, density) caused by the presence of by the circumterrestrial layer of the investigated layered "parallelepiped" area of urban environment, the complete system of equations of perturbed air flow is derived and the question of determination of asymptotic solution of the complete system constructed is investigated on the basis of the motion equations and the equation of the heat inflow balance.

2. In the Cartesian coordinate system the hydro-thermodynamic model of the atmospheric processes in mesoscale is build, which consists of:

- the equation of atmospheric dynamics (it is the consequence of the momentum conservation law);
- the potential temperature transfer equation (it is the consequence of the first law of thermodynamics);
- the mass fraction of the water vapor transfer equation, i.e. the equation of specific atmospheric humidity (it is the consequence of the continuity equation).

3. Since the study area is the layered "parallelepiped" area, the corresponding initial-boundary conditions of the mixed type and the consistency conditions are constructed in the article, which are connected with the formed hydro-thermodynamic equations.

4. The performed investigation of the influence of the urban terrain on air flow of the atmosphere proved that by means of using definite, non-unidirectional flows of air masses, the high- pressure occurs among them; this pressure turns the vertical speed into zero. This effect can be interpreted as presence of some surface in a form of a solid "wall".

ACKNOWLEDGMENT

Present article was executed within the framework of the following two International Projects (Grants): (1) European Social Fund Project No. 2009/0159/1DP/1.1.2.1.2./09/IPIA/VIAA/006 (for the first author resounding support in Realization of the Doctoral Program "Telematics and Logistics" of the Transport and Telecommunication Institute, Riga, Latvia) (for the first author); (2) The European Social Fund (ESF) Project No. 1DP/1.1.1.2.0/09/APIA/VIAA/142 "Elaboration of effective analytical and numerical methods for solving of direct and inverse mathematical physics problems in material, economics and environment sciences" (for the second author).

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