World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:7, No:6, 2013

A Design of Fractional-Order PI Controller with Error Compensation

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***Abstract*—**Fractional-order controller was proven to perform better than the integer-order controller. However, the absence of a pole at origin produced marginal error in fractional-order control system. This study demonstrated the enhancement of the fractional-order PI over the integer-order PI in a steam temperature control. The fractional-order controller was cascaded with an error compensator comprised of a very small zero and a pole at origin to produce a zero steady-state error for the closed-loop system. Some modification on the error compensator was suggested for different order fractional integrator that can improve the overall phase margin.

***Keywords*—**Fractional-order PI, Ziegler-Nichols tuning, Oustaloup's Recursive Approximation, steam temperature control.

I. Introduction

PID controller is still dominating the feedback control applications until today. The simple control strategy based on the accumulation over some operation on the error signal has made it easy to understand and robust enough to solve many industrial problems. That is why the research on optimizing the PID controller is still going on until today. The advancement of the three terms controller in the form of fractional-order PID (FO-PID) control has becoming more popular since the last 10 years. This new technique is proven to provide more flexibility and ability to enhance modeling and control of systems' dynamics [1].

Integer-order approximation for fractional-order system had been investigated since 1960s in other research area such as chemistry and mechanical systems [2]. Some approximation techniques are based on continued fraction expansion (CFE), curve fitting or identification methods and power series expansion (PSE). Oustaloup's Recursive Approximation (ORA) is among the most popular approximation technique. The technique used recursive poles and zeros distribution within specified frequency range to assimilate the frequency response of the fractional-order transfer function.

Applications of fractional-order models in control theory had been considered only twenty-years after that. The idea of fractional-order controller was first proposed by A. Oustaloup through *Commande Robuste d'Ordre Non Entier* (CRONE) controller in 1991.

Later on, I. Podlubny had initiated the fractional order PID in the form of $PI^{\lambda}D^{\mu}$ in 1999 [3] involving an integrator of order λ and differentiator of order μ of less than 1. The generalization of the PID with fractional power of λ and μ was demonstrated by many to give better performance compared to the integer PID. However, up till now there is no systematic way to set the value for λ and μ [1].

Recently, more studies had been concentrated on the method for FO-PID tuning [4], [5]. Generally, the design specifications were looking for an infinite gain margin and constant phase margin around the cross-over frequency to gain robust control towards gain variations [6]. The solutions were then obtained by solving a linear numerical optimization problems as had been reported in [7], [8]. Another tuning approach was by utilizing the Ziegler-Nichols tuning rules based on information of its frequency and step response. The rules were successfully applied by [9] and [10] in their studies.

This paper investigates the application of fractional PI (FO-PI) controllers to control steam temperature of a distillation process. The steam was applied for essential oil extraction which needs to be regulated around 85ºC to preserve the quality of yield. Ziegler-Nichols tuning rule was applied for the PI controller's gain and the fractional order was adjusted for the FO-PI based on frequency response specification and steady-state error requirement.

The main issue that will be discussed in this paper is on the steady-state error compensation technique that is necessary when implementing the fractional-order controller. The approximation of fractional terms for integrator using ORA technique missed a pole at the origin as opposed to the integer-order PI. Its absence in the FO-PI controller has become a weakness despite of its advantages. Two error compensator schemes had been introduced by Feliu et al. [7] and Axtell [9] and was popularly applied ever since. This paper discussed on the application of both schemes and proposed some modification for improvement.

This paper is organized as follows: Section II outlines the Oustaloup's approximation for fractal operators used to implement the FO-PI. Section III described briefly on the system and its modeling. Section IV discussed on the Ziegler-Nichols tuning rules based on the process reaction curve method. Section V discussed on the configuration of the FO-PI based on the Ziegler-Nichols parameter. Section VI presents the comparison between the PID and FOPID based on simulation study over the identified model. Finally, conclusions were drawn for the issue being discussed.

This work was supported in part by the UiTM Research Intensive Faculty (RIF) fund (660/2012).

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II. Oustaloup's Recursive Approximation Method

The implementation of fractal controller involved the technique of approximating the integer order systems to represent the fractional order system. Some of the techniques are continued fraction expansion (CFE), curve fitting or identification methods and power series expansion (PSE). These techniques had been discussed and demonstrated in [2]. In identification methods, the approximation was analyzed in frequency domain to obtain a rational function whose frequency response fits the frequency response of the irrational function. This method was derived by Oustaloup himself and known as Oustaloup's recursive approximation (ORA). This method is based on the approximation of a function in the form:

$$H(s) = s^m, m \in \Re \tag{1}$$

This function can be approximated by series of rational function synthesized as follows:

$$\hat{\mathbf{H}}(s) = k \prod_{n=1}^{N} \frac{1 + s/\omega_{z,n}}{1 + s/\omega_{p,n}} \tag{2}$$

The approximation of H(s) only valid in the boundary of low and high cut-off frequency [ω_l; ω_h]. N represents the number of poles and zeros which should be chosen beforehand. High value of N permitted good approximation but increased the computational complexity. On the other hand, low value results in simpler approximation but could cause appearance of ripple in gain and phase behavior. Low and high frequency band limitations could avoid the use of infinite numbers of rational transfer function besides limiting the high frequency gain of the derivative effect [11]. The poles and zeros are calculated using the following recursive equations:

$$\begin{aligned} \omega_{z,1} &= \omega_l \sqrt{\eta} \\ \left.\begin{aligned} \omega_{p,n} &= \omega_{z,n}\alpha \\ \omega_{z,n+1} &= \omega_{p,n}\eta \end{aligned}\right\} &, n = 1 \ldots N \end{aligned} \tag{3}$$

where

$$\alpha = \left(\frac{\omega_h}{\omega_l}\right)^{\mu/N} \text{ and } \eta = \left(\frac{\omega_h}{\omega_l}\right)^{1-\mu/N}$$

III. Fractional-Order PI Control

The transfer function of FOPID is given by

$$C(s) = K_p\left(1 + \frac{1}{T_i s^{\lambda}} + T_d s^{\mu}\right) \tag{4}$$

where K_p, T_i, and T_d are controller gain while λ and μ are the integral and differential power in real number. Fractional PID is generalization of the integer PID such that

- If λ =1 and μ=1, we obtain a classical PID.
- If λ=1 and μ=0, we obtain a PI controller.
- If λ=0 and μ=1, we obtain a PD controller.
- If λ=0 and μ=0, we obtain a P controller

Hence, if λ and μ were set to arbitrary value between 0 and 1, the controller can be configured to behave within these four possibilities [5], [9], [12].

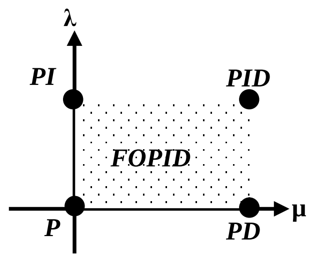

Fig. 1 Fractional PID control space

This is the main advantage of the FO-PID. Other than that, FO-PID was acknowledged by many researchers to provide better control especially to a class of fractal system. Furthermore, FO-PID is less sensitive to changes in process parameters and the controller parameters itself. There were five parameters can be tuned instead of three in the conventional version and thus more design specifications can be achieved from the λ and μ adjustment [4].

The frequency response for differentiator and integrator using ORA was shown in Figs. 2 and 3 respectively. The magnitude and phase of each function related to fractional power m is given by,

$$\left.\begin{aligned} 20\lg\left|\hat{s}^m\right|_{s=j\omega}\right| &= 20m\lg(\omega)dB \\ \angle\hat{s}^m\Big|_{s=j\omega} &= \frac{\pi m}{2} \end{aligned}\right\}\omega_l \leq \omega \leq \omega_h \tag{5}$$

where m represents the magnitude of λ and μ and will be used throughout this paper. The gain and phase can be adjusted between ±20 dB/dec and ±90º. This characteristic enable for more accurate design of the PID controller.

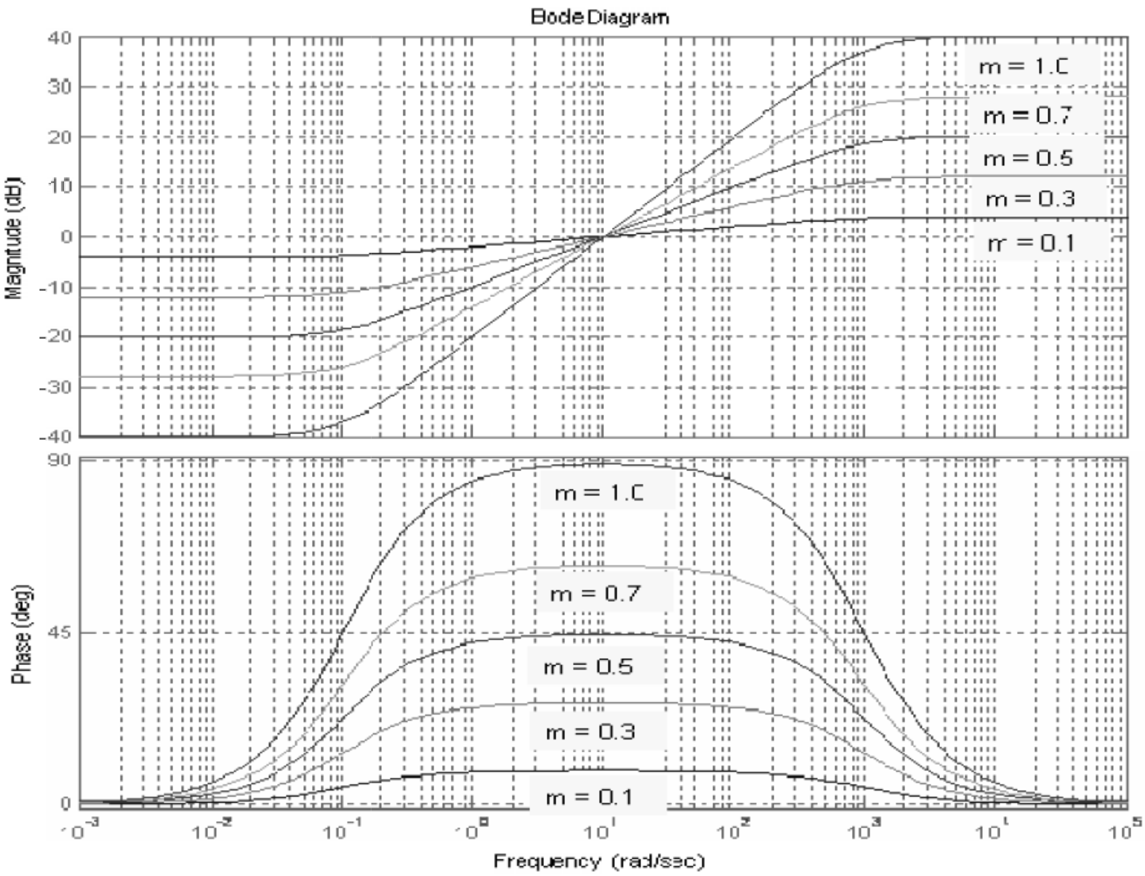

Fig. 2 Bode diagram of ORA on s^m

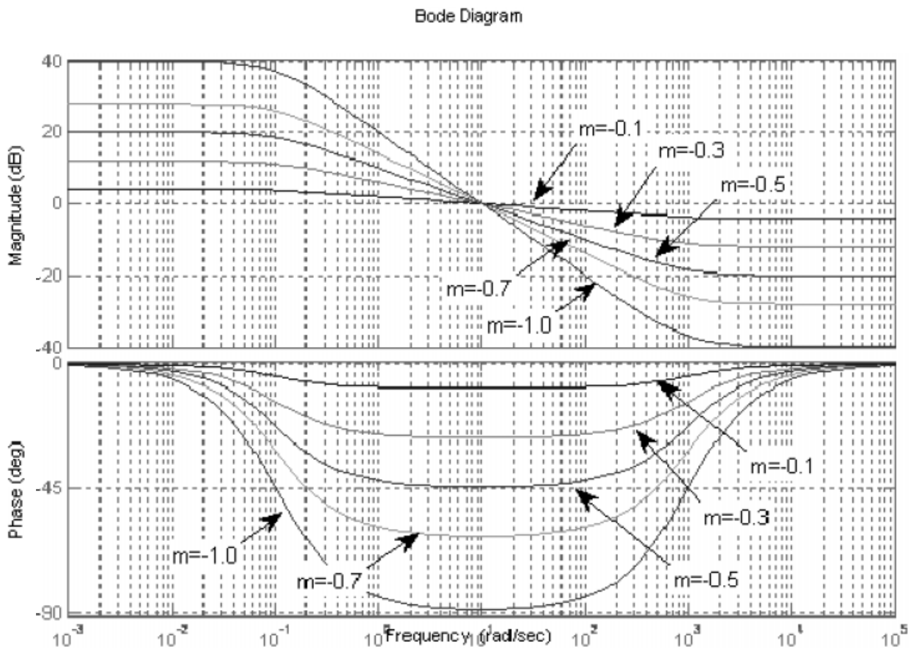

Fig. 3 Bode diagram of ORA on s^{-m}

However, applying ORA to an integrator never generates a pole at the origin and hence the controller will not be able to track the set point without steady-state error [13]. The approach currently used to resolve this matter was by introducing a pure integrator and split the fractional integrator into two parts. This method was introduced by Axtell [9] and described in (6):

$$\frac{1}{s^{\lambda}} = s^{1-\lambda} \times \frac{1}{s} \quad (6)$$

The ORA was then applied to the fractional function of $s^{1-\lambda}$. This approach was used by S.B. Ramiro [9] but conversely, M. Farshad [14] has proved that this approach produced inaccurate results. Applying a pure integrator will modify the overall frequency response and thus, the output will not be as expected and may cause instability. Alternatively, the steady-state error can be improved by increasing the system's type by introducing the following term as proposed by Feliu-Batlle et al. [7]:

$$G_e(s) = \frac{s+n}{s} \quad (7)$$

where n being a small value so that high frequency specifications were maintained and the system gain was not altered drastically. This approach was applied in this research for steady-state error compensation but with some modifications. The effect of each steady-state error compensation technique discussed above was described through Bode plots of the integrator terms and the composite PI controller. Fig. 4 represents the effect from error compensator described by (6) while Fig. 5 represents (7). Both conditions were simulated for m=-0.5.

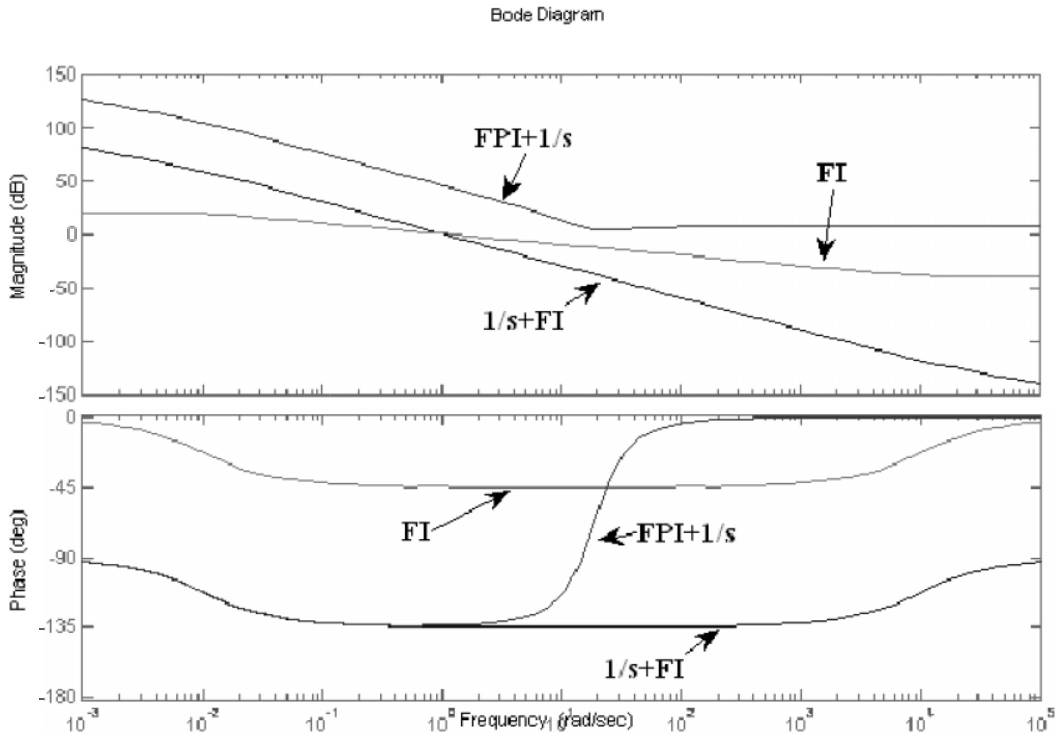

Fig. 4 Bode plot of FO-PI with error compensation in (6) when m=-0.5

When the pure integrator was cascaded to the fractional integrator (FI), both magnitude and phase characteristics were totally altered for the whole frequency range. The phase was shifted down and reduced the phase margin. Consequently the overshoot will increase. The phase was no more maintained around the crossover frequency and hence, gain changes will not be tolerated. On the contrary, using the second method just increased the system's type and maintains all other behaviors around specified frequency range. The overall magnitude specifications can be achieved by a simple gain adjustment.

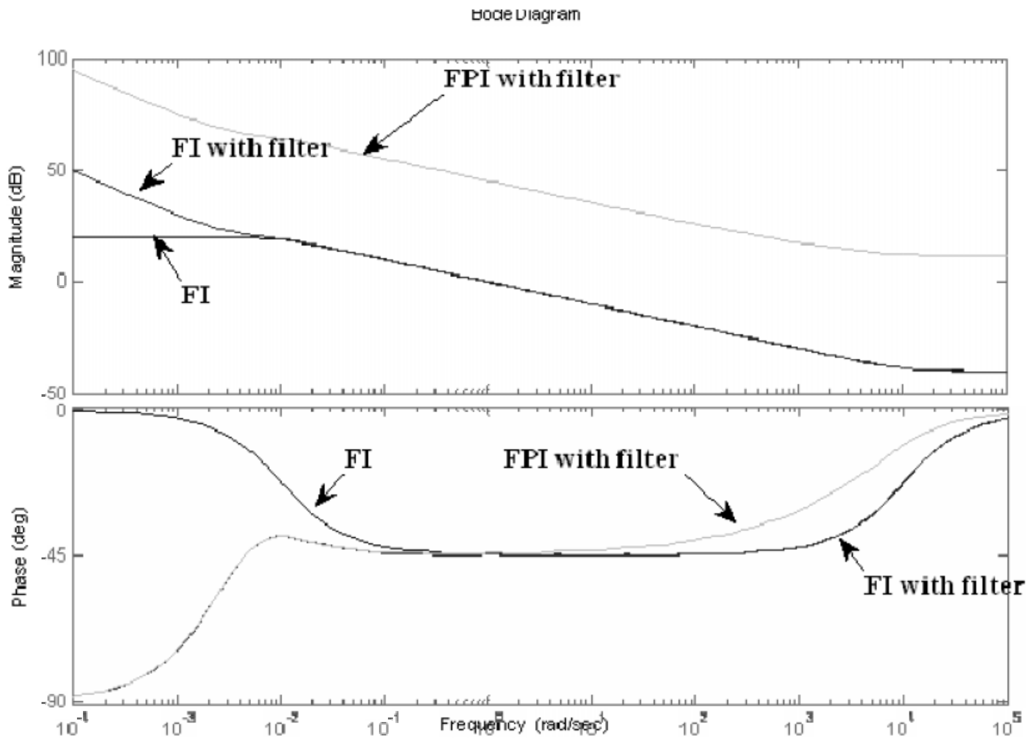

Fig. 5 Bode plot of FO-PI with error compensation in (7) when m= -0.5

IV. PID Tuning Rules

This study applied Ziegler-Nichols PI tuning based on process reaction curve. This rule only accurately applied to a process with an s-shaped step response. The s-shaped step response can be estimated by a first-order plus dead-time (FOPDT) system in the following form:

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \tag{8}$$

where K is the process gain, θ is apparent dead-time and τ is the time constant or the pole of the process. All the parameters can be acquired easily from the step response test around the operating point. A step response of the process to be controlled was given in Fig. 6. The approximated FOPDT model was given in (9).

$$G'(s) = \frac{4.6e^{-25s}}{280s + 1}; 80°C < T < 100°C \tag{9}$$

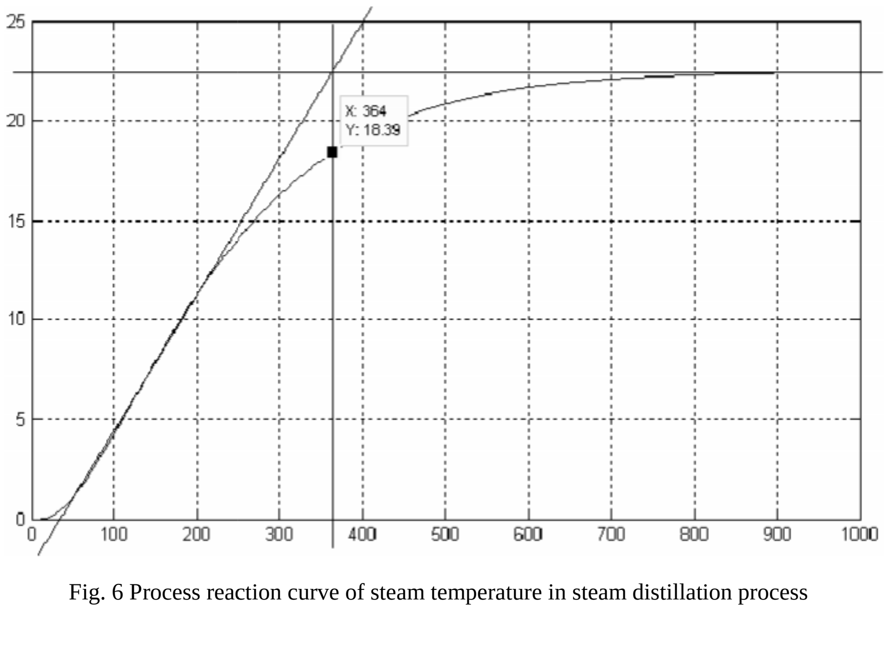

Fig. 6 Process reaction curve of steam temperature in steam distillation process

The controller parameters are then calculated according to the rules listed in Table I for K=4.6, θ=25 sec, and τ=280 sec. For the standard PID structure, the following PI controller was obtained;

$$C(s) = 2.19\left(1 + \frac{0.027}{s}\right) \tag{10}$$

TABLE I
ZIEGLER-NICHOLS PID TUNING RULES FROM FOPDT

	K_c	T_i	T_d
P	$\left(\frac{1}{K}\right)\left(\frac{\tau}{\theta}\right)$	-	-
PI	$\left(\frac{0.9}{K}\right)\left(\frac{\tau}{\theta}\right)$	3.3θ	-
PID	$\left(\frac{1.2}{K}\right)\left(\frac{\tau}{\theta}\right)$	2.0θ	0.5 θ

V. SIMULATION RESULTS

The results were based on simulation study of the FOPDT model given in (9). PI parameters were obtained using the Ziegler-Nichols rules and were compared with the FO-PI having the same controller settings. The integrator was approximated using ORA with N =4, ω_L=0.01rad/s and ω_H=10000rad/s. The approximate transfer function was multiplied with gain, k so that the Bode magnitude crossed 0 dB (unity gain) at 1rad/s as experimented in [15].

A. FO-PI without Error Compensation

The first experiment was implemented without any error compensator for the FO-PI. The objective was to get the idea on the effect of FO-PI and to gauge its limitation. The controllers were compared for temperature regulation at 85ºC. The result for IO-PI was given in Table II. The response has high overshoot but zero steady-state error. The settling time reported in this paper includes the input offset of 10s which should be subtracted from the reported value.

TABLE II
IO-PI CLOSED-LOOP PERFORMANCES

Rise time (s)	Settling time (s)	OS (%)	Steady-state error (ºC)
20	314	74.53	0

The FO-PI was implemented with the same controller settings. The integral term was varied for integer order, m = -0.1, -0.5, and -0.9. For each case, the gain k was adjusted accordingly as mentioned in previously. The output response together with the output from IO-PI was shown in Fig. 7 and the performance criterions were given in Table III.

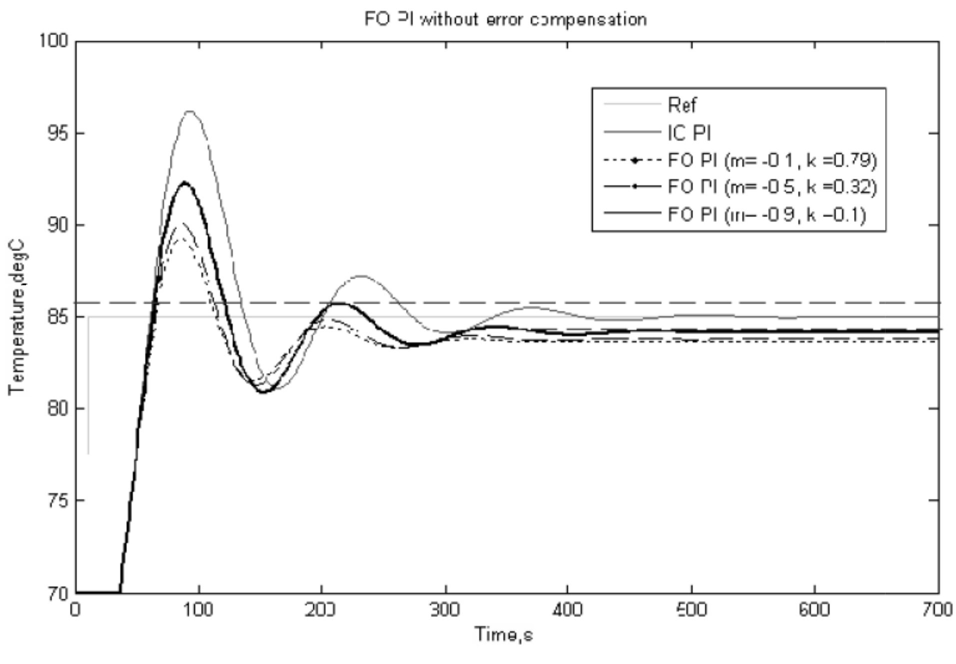

Fig. 7 FO-PI closed-loop responses without error compensation

In terms of overshoot, the FO-PI version was better than the IO-PI. The overshoot was reduced with the fractal power. But, none of the FO-PI output can regulate the temperature at the set point and not even settled within the ±5% (±0.75ºC) boundary. This was due to the absence of pole at the origin as discussed previously.

TABLE III
FO-PI (WITHOUT ERROR COMPENSATION) CLOSED-LOOP PERFORMANCES

m	k	OS (%)	Steady-state error (ºC)
-0.1	0.79	28.27	1.33
-0.5	0.32	33.93	1.21
-0.9	0.1	48.50	0.87

B. FO-PI with Fixed Error Compensation

The next stage of evaluation accommodated an error compensator described by (7). The compensator was design for m=-0.5 where n will remain fixed at 0.03rad/s. The fractional power was varied for m= -0.1, -0.5 and -0.9. The incompetency of the compensator can be observed in Fig. 8. The output performance was given in Table IV.

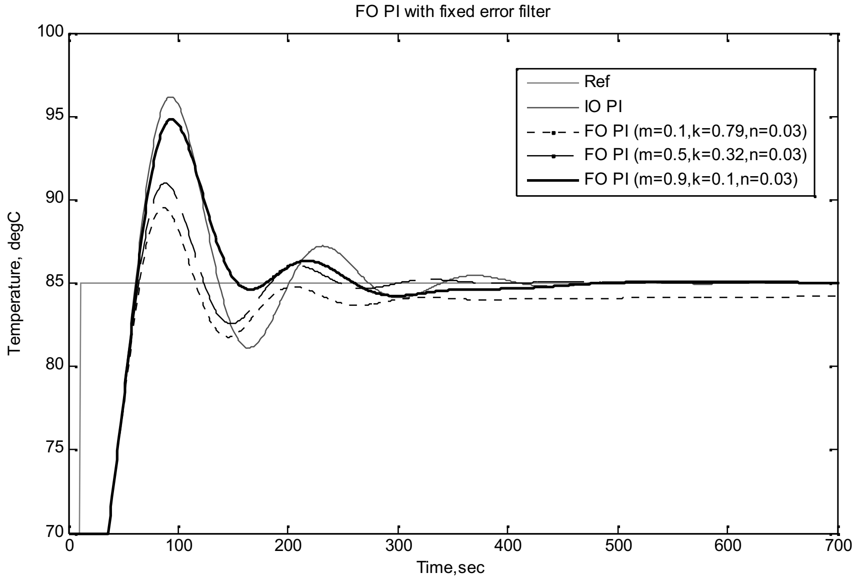

Fig. 8 FO-PI closed-loop responses with fixed error compensation

The overall transient was a bit different from case A. Obviously, steady-state error was improved and eliminated for m=-0.5 and -0.9. The error when m=-0.1 was reduced. However, overshoot in the output was worse for every order of m but the settling time was improved compared to IO-PI.

TABLE IV
FO-PI (WITH FIXED ERROR COMPENSATION) CLOSED-LOOP PERFORMANCES

m	k	n	Settling time (s)	OS (%)	Steady-state error (ºC)
0.1	0.79	0.03	-	30.07	0.84
0.5	0.32	0.03	224	40	0
0.9	0.1	0.03	294	65.47	0

C. FO-PI with Error Compensation of Variable n

From the results obtained in section A and B, it can be concluded that the zero of the error compensator had some impact on the overall system’s response and should be adjusted to get better response for different order of m. So, this study proposed an adjustable n which is the zero of the error compensator transfer function. The frequency response of the compensator is shown in Fig. 9.

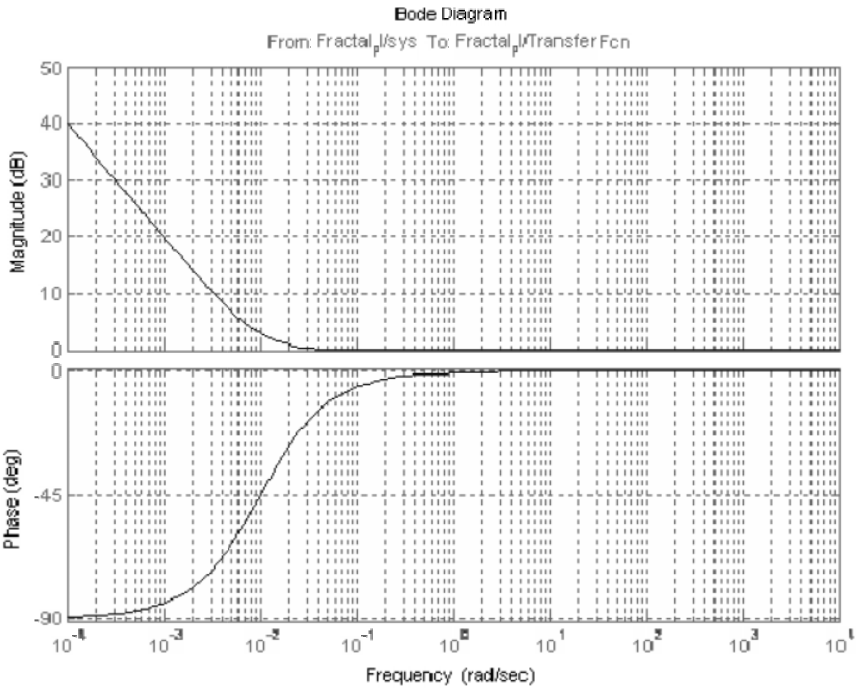

Fig. 9 Open-loop Bode for the error compensator

The movement of zero had significant impact on the phase margin especially for m=-0.5 and -0.9 while for m=-0.1, the presence of the filter was very dominant due to small magnitude and phase of the fractional integrator. For m=-0.9, great improvement in %OS was observed when n= 0.03 compared to n=0.003. The overall results were presented in Fig. 10 and closed-loop performance was given in Table V.

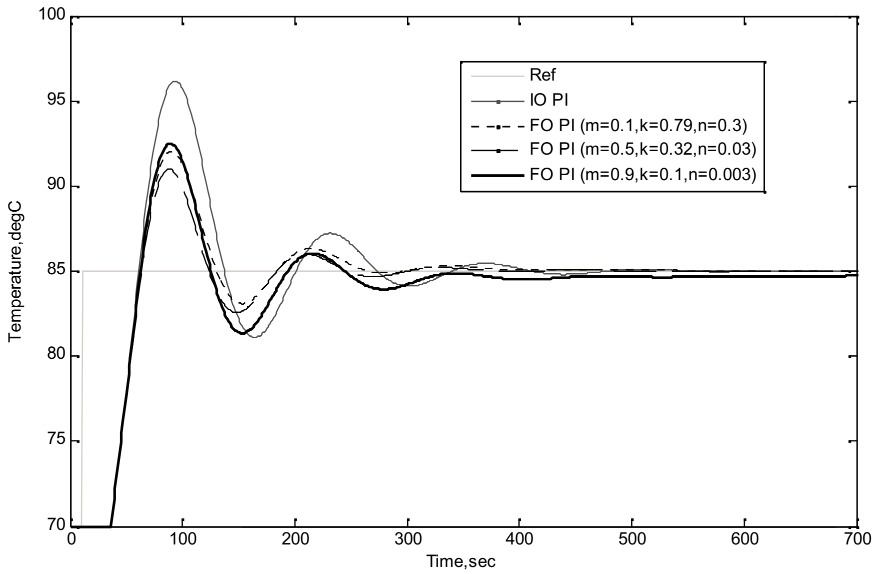

Fig. 10 FO-PI closed-loop responses with variable error compensation

TABLE V
FO-PI (WITH VARIABLE ERROR COMPENSATION) CLOSED-LOOP PERFORMANCES

m	k	n	Settling time (s)	OS (%)
0.1	0.79	0.3	239	46.93
0.5	0.32	0.03	225	40.00
0.9	0.1	0.003	301	50.07

VI. CONCLUSIONS

This study demonstrated the improvement of fractional-order PI over the integer-order PI. The inherent steady-state error was eliminated using an error compensator comprised of a very small zero and a pole at origin. This study also show that the compensator was not generalized for every order of the fractional integrator but should be tuned for better phase margin specification which is the indicator for the output overshoot. Likewise, the error compensator dominating the closed-loop response for cases where fractional-order is less than 0.5. Further investigation should be done in this area to find a better solution for steady-state error compensation in fractional-order controller.

ACKNOWLEDGMENT

The authors would like to thank the Faculty of Electrical Engineering, UiTM Malaysia for research facilities provided during the research. We also would like to express our gratitude for all members of PiCO Research group for their comments and supports.

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