Ginzburg-Landau Model: an Amplitude Evolution Equation for Shallow Wake Flows

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Abstract—Linear and weakly nonlinear analysis of shallow wake flows is presented in the present paper. The evolution of the most unstable linear mode is described by the complex Ginzburg-Landau equation (CGLE). The coefficients of the CGLE are calculated numerically from the solution of the corresponding linear stability problem for a one-parametric family of shallow wake flows. It is shown that the coefficients of the CGLE are not so sensitive to the variation of the base flow profile.

Keywords—Ginzburg-Landau equation, shallow wake flow, weakly nonlinear theory.

I. INTRODUCTION

SHALLOW wake flows are quasi-two-dimensional flows behind obstacles (such as islands) in which the horizontal components of the velocity vector are much stronger than the vertical component. A typical measure of shallowness of the flow is the ratio of the transverse length scale of the flow, D, and water depth, H. The flow is assumed to be shallow if the ratio D/H is large enough: $D/H >> 1$. An excellent example of shallow wake flow is shown in [1] where the leaking oil from the tanker Argo Merchant shows a von Karman vortex street flow pattern. Other examples of aeronautical photographs of island wakes in shallow waters are presented in [2] and [3]. Experimentally observed coherent structures in shallow wakes are believed to appear as a result of flow instability [4], [5]. Linear stability of shallow flows is studied experimentally in [6] – [8]. It is shown in [6] that three different flow regimes can be observed in shallow wake flows: steady bubble, unsteady bubble and vortex street. It was found in [3] and [6] that flow patterns behind obstacles depend on shallow wake stability parameter $S = c_f b / H$,

where c_f is the bottom friction coefficient and b is length scale (the diameter of the cylinder in [6]).

Theoretical investigation of linear stability of shallow wake flows is performed in [9] – [12]. Linear stability analyses confirm that the stability characteristics of shallow water flows depend on the magnitude of the stability parameter S. In particular, a flow becomes more stable as the parameter

S increases.

The linear stability theory can be used to determine when a particular flow becomes unstable. The "fate" of the disturbance just above the threshold cannot be predicted by the linear theory. Methods of weakly nonlinear theory are often applied to describe the evolution of the most unstable linear mode when the flow becomes unstable [13] – [14]. Relatively simple amplitude evolution equations such as the complex Ginzburg-Landau equation (CGLE) are used in the literature to analyze spatio-temporal dynamics of complex flows [15], [16]. The popularity of the CGLE is based on the following factors: (1) the model is relatively simple but includes such physical effects as nonlinearity and diffusion, (2) the CGLE is a scalar equation, (3) the CGLE can be derived (in some cases) from the equations of motion, (4) the coefficients of the CGLE can be obtained in closed form (in terms of integrals containing the characteristics of the corresponding linear stability problem), (5) the CGLE can exhibit a rich variety of solutions depending on the values of its coefficients.

In many applications (see, for example, the analysis of the dynamics of the flow behind bluff rings [17] and spatio-temporal dynamics in the wakes of a row of 16 circular cylinders placed close to each other in an incoming flow [18]) the CGLE (or the Landau equation) is used as a phenomenological model equation. In such cases the coefficients of the CGLE are obtained from experimental data.

On the other hand, the CGLE can be derived from the equations of motion. Examples include weakly nonlinear analyses of plane Poiseuille flow [13] and problems related to generation of waves by wind [19], shallow flows behind obstacles such as islands [10], [12] and in the nearshore [14], rapidly decelerated flows in pipes [20] and channels [21].

Despite the fact that the CGLE was successfully applied in practice to model spatio-temporal dynamics of complex flows [17, 18], other sources in the literature suggest that the use of weakly nonlinear theory should be limited. One such an example is the paper [22] where linear and weakly nonlinear theory is applied to the stability analysis of quasi-two-dimensional shear flows such as shallow water flows. It is assumed in [22] that the term representing friction in fluid system is of the form $\vec{f}_R = -\lambda_R \vec{u}$, where λ_R is the coefficient of Rayleigh friction and $\vec{u}$ is the velocity vector. The authors compared their theoretical predictions from the linear stability analysis with experimental data. Reasonable

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agreement was found. On the other hand, it is found in [22] that the Landau constant (the real part of one of the coefficients of the CGLE) is quite sensitive to the shape of the base flow velocity profile. As a result, it is concluded in [22] that it would be impossible to compare directly the theory with experiments since it would be difficult to determine the base flow velocity profile with accuracy up to the third derivative (as it is required by a weakly nonlinear theory). In particular, it is found in [22] that the values of the Landau constant differ by a factor of 3 for two base flow velocity profiles whose linear stability characteristics differ by not more than 20%.

In the present paper linear and weakly nonlinear stability of a one-parametric family of shallow wake flows is investigated. The parameter used in the study represents a slow longitudinal variation of shallow wake flow behind obstacles such as islands. In contrast to [22] where the internal friction is linearly related to the velocity distribution, a nonlinear Chezy formula [23] is used to model bottom friction. The base flow profile used in [9] is adopted in our study. Calculations show that the Landau constant as well as other coefficients of the CGLE are not so sensitive to the shape of the base flow profile. Thus, it is plausible to assume that the CGLE can be used to describe spatio-temporal dynamics of shallow wake flows.

II. LINEAR STABILITY ANALYSIS

Consider the base flow of the form
$$\vec{U} = (U(y), 0) \tag{1}$$
where
$$U(y) = 1 - \frac{2R}{1-R} \frac{1}{\cosh^2(\alpha y)}. \tag{2}$$

The base flow (2) is suggested in [24] after careful analysis of available experimental data for deep water flows behind circular cylinders. The profile (2) is also adopted in the present study. The parameter R is the velocity ratio: $R = (U_m - U_a)/(U_m + U_a)$, where U_m is the wake centerline velocity and U_a is the ambient velocity, and $\alpha = \sinh^{-1}(1)$. It is shown in [10] that under the rigid-lid assumption the linear stability of wake flows in shallow water is described by the following eigenvalue problem:
$$\varphi_1''(U - c + SU) + SU_y\varphi_1' + \left(k^2 - U_{yy} - k^2U - \frac{S}{2}kU\right)\varphi_1 = 0 \tag{3}$$
$$\varphi_1(\pm\infty) = 0, \tag{4}$$
where the perturbed stream function of the flow, $\psi(x, y, t)$, is assumed to be of the form
$$\psi(x, y, t) = \varphi_1(y)\exp[ik(x - ct)] + c.c. \tag{5}$$

Here $\varphi_1(y)$ is the amplitude of the normal perturbation, k is the wavenumber, c is the wave speed of the perturbation, and

$c.c.$ means "complex conjugate". The linear stability of the base flow (2) is determined by the eigenvalues, $c_m = c_{rm} + ic_{im}$, $m = 1, 2, \ldots$ of the eigenvalue problem (3), (4). The flow (2) is linearly stable if $c_{im} < 0$ for all m and linearly unstable if $c_{im} > 0$ for at least one value of m.

The linear stability problem (3), (4) is solved by means of a pseudospectral collocation method based on Chebyshev polynomials. The computational procedure is briefly described below. The interval $-\infty < y < +\infty$ is mapped onto the interval $-1 < r < 1$ by means of the transformation $r = \frac{2}{\pi}\arctan y$. The solution to (3), (4) is sought in the form
$$\varphi_1(r) = \sum_{k=0}^{N} a_k(1 - r^2)T_k(r), \tag{6}$$
where $T_k(r)$ is the Chebyshev polynomial of degree k. The collocation points r_j are
$$r_j = \cos\frac{\pi j}{N}, \quad j = 0, 1, \ldots, N. \tag{7}$$
The derivatives are transformed by the chain rule:
$$\frac{d\varphi_1}{dy} = \frac{2}{\pi}\cos^2\frac{\pi r}{2}\frac{d\varphi_1}{dr},$$
$$\frac{d^2\varphi_1}{dy^2} = \frac{4}{\pi^2}\cos^4\frac{\pi r}{2}\frac{d^2\varphi_1}{dr^2} - \frac{4}{\pi}\sin\frac{\pi r}{2}\cos^3\frac{\pi r}{2}\frac{d\varphi_1}{dr} \tag{8}$$

Substituting (6), (8) into (3), (4) and evaluating the function $\varphi_1(r)$ and its derivatives at the collocation points (7) we obtain the following generalized eigenvalue problem:
$$(B - \lambda C)a = 0 \tag{9}$$
where B and C are complex-valued matrices and $a = (a_1 a_2 \ldots a_N)^T$.

Problem (9) is solved numerically by means of the IMSL routine DGVCCG. The critical values of the stability parameters k, S and c for different values of R are given in Table I (here $S_c = \max_k S$).

TABLE I
CRITICAL VALUES OF THE STABILITY PARAMETER S

R	k	S_c	c
-0.3	0.892	0.11819	0.69814
-0.4	0.909	0.15689	0.65964
-0.5	0.926	0.19548	0.62394
-0.6	0.944	0.23409	0.59083
-0.7	0.962	0.27286	0.55925
-0.8	0.980	0.31189	0.52882

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III. WEAKLY NONLINEAR ANALYSIS

Following [13], in this section the main steps of the derivation of the amplitude evolution equation for the most unstable mode are briefly described. Consider the two-dimensional shallow water equations of the form [10]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{10}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} + \frac{c_f}{2H}u\sqrt{u^2+v^2} = 0, \tag{11}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} + \frac{c_f}{2H}v\sqrt{u^2+v^2} = 0, \tag{12}$$

where u and v are the depth-averaged velocity components in the x and y directions, respectively, H is water depth, p is the pressure.
Suppose that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{13}$$

where $\psi(x,y,t)$ is the stream function of the flow. Eliminating the pressure and using (13) one can rewrite the system (10) – (12) in the form

$$(\Delta\psi)_t + \psi_y(\Delta\psi)_x - \psi_x(\Delta\psi)_y + \frac{c_f}{2h}\Delta\psi\sqrt{\psi_x^2+\psi_y^2} \tag{14}$$
$$+\frac{c_f}{2h\sqrt{\psi_x^2+\psi_y^2}}\left(\psi_y^2\psi_{yy} + 2\psi_x\psi_y\psi_{xy} + \psi_x^2\psi_{xx}\right) = 0$$

Consider a perturbed solution to (14) of the form

$$\psi = \psi_0(y) + \varepsilon\psi_1(x,y,t) + \varepsilon^2\psi_2(x,y,t) + \ldots \tag{15}$$

The parameter ε describes a small deviation of the shallow wake stability parameter S from the critical value S_c:

$$S = S_c(1 - \varepsilon^2) \tag{16}$$

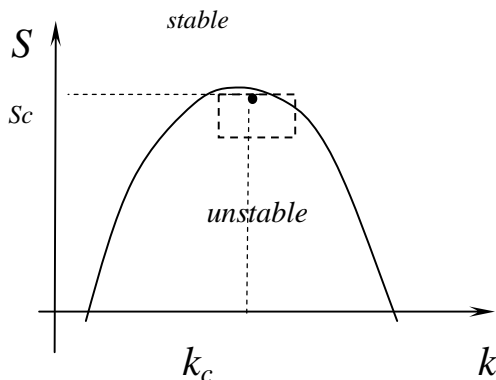Weakly nonlinear theory is applicable in a small neighborhood of the critical point (see Fig. 1):

Fig. 1 Region in the (k,S)-plane (shown as dashed rectangle) where weakly nonlinear theory is applicable

The amplitude evolution equation for the most unstable mode is derived by means of the method of multiple scales. Following [13], the following slow time and longitudinal variables are introduced:

$$\tau = \varepsilon^2 t, \quad \xi = \varepsilon(x - c_g t), \tag{17}$$

where c_g is the group velocity.

The function ψ_1 in (15) is sought in the form

$$\psi_1(x,y,t) = A(\xi,\tau)\varphi_1(y)\exp[ik(x-ct)] + c.c. \tag{18}$$

where A is a slowly varying amplitude.

The linear stability problem (3), (4) is obtained by substituting (15) – (18) into (14), collecting the terms containing ε and using (5). Collecting the terms containing ε^2 the following equation is obtained:

$$L\psi_2 = c_g(\psi_{1xx\xi} + \psi_{1yy\xi}) - 2\psi_{1\xi t} - \psi_{0y}(3\psi_{1xx\xi} + \psi_{1yy\xi})$$
$$-\psi_{1y}(\psi_{1xxx} + \psi_{1yyx}) + \psi_{1x}(\psi_{1xxy} + \psi_{1yyy}) + \psi_{1\xi}\psi_{0yyy}$$
$$-S[(\psi_{1xx} + \psi_{1yy})\psi_{1y} + 2\psi_{1x\xi}\psi_{0y} + \psi_{1yy}\psi_{1y} - 2\psi_{0y}\psi_{0yy}$$
$$+2\psi_{1x}\psi_{1xy}] \tag{19}$$

Here

$$L\psi = \psi_{xxt} + \psi_{yyt} + \psi_{0y}(\psi_{xxx} + \psi_{yyx}) - \psi_{0yyy}\psi_x$$
$$+\frac{c_f}{2h}\left[(\psi_{xx} + 2\psi_{yy})\psi_{0y} + 2\psi_y\psi_{0yy}\right].$$

Similarly, the equation of order ε^3 has the form

$$L\psi_3 = c_g(\psi_{2xx\xi} + 2\psi_{1x\xi\xi} + \psi_{2yy\xi}) - \psi_{1xxt} - \psi_{1yyt} - 2\psi_{2\xi t}$$
$$-\psi_{1\xi\xi t} - 3\psi_{0y}(\psi_{2xx\xi} + \psi_{1x\xi\xi}) - \psi_{1y}(\psi_{2xxx} + 3\psi_{1xx\xi})$$
$$-\psi_{2y}(\psi_{1xxx} + \psi_{1yyx}) - \psi_{1y}(\psi_{2yyx} - \psi_{1\xi yy}) - \psi_{0y}\psi_{2\xi yy}$$
$$+\psi_{2x}\psi_{1xxy} + \psi_{1\xi}\psi_{1xxy} + \psi_{1x}\psi_{2xxy} + 2\psi_{1x}\psi_{1xy\xi} + \psi_{1x}\psi_{2yyy}$$
$$+\psi_{2x}\psi_{1yyy} + \psi_{1\xi}\psi_{1yyy} + \psi_{2\xi}\psi_{0yyy}$$
$$-S[\psi_{2y}(\psi_{1xx} + \psi_{1yy}) + 2\psi_{2yy}\psi_{1y} + 1.5\psi_{1xx}\psi_{1x}^2/\psi_{0y}$$
$$+\psi_{2xx}\psi_{1y} + 2\psi_{1x\xi}\psi_{1y} + 2\psi_{0y}\psi_{2x\xi} + \psi_{1\xi\xi}\psi_{0y} - \psi_{1xx}\psi_{0y}$$
$$-2\psi_{0yy}\psi_{1y} - 2\psi_{0y}\psi_{1yy} + \psi_{1yy}\psi_{2y} - \psi_{1y}\psi_{2yy} + 2\psi_{1x}\psi_{2xy}$$
$$+2\psi_{1x}\psi_{1yy} + 2\psi_{2x}\psi_{1xy} + 2\psi_{1\xi}\psi_{1xy}] \tag{20}$$

The function ψ_2 is sought in the form

$$\psi_2 = AA^*\varphi_2^{(0)}(y) + A_\xi\varphi_2^{(1)}(y)\exp[ik(x-ct)]$$
$$+A^2\varphi_2^{(2)}(y)\exp[2ik(x-ct)] + c.c. \tag{21}$$

The function $\varphi_2^{(0)}(y)$ is the solution of the following boundary value problem

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$$2S[u_{0y}\varphi_{2y}^{(0)} + u_0\varphi_{2yy}^{(0)}] = ik[\varphi_{1y}\varphi_{1yy}^{*} - \varphi_{1y}^{*}\varphi_{1yy}$$
$$+\varphi_1\varphi_{1yyy}^{*} - \varphi_1^{*}\varphi_{1yyy}] - S[k^2\varphi_1\varphi_{1y}^{*} + k^2\varphi_1^{*}\varphi_{1y} \quad (22)$$
$$+2\varphi_{1y}^{*}\varphi_{1yy} + 2\varphi_{1yy}^{*}\varphi_{1y}],$$

$$\varphi_2^{(0)}(\pm\infty) = 0. \quad (23)$$

The function $\varphi_2^{(1)}(y)$ satisfies the equation

$$(iku_0 - ikc)\varphi_{2yy}^{(1)} + (ik^3c - ik^3u_0 - iku_{0yy})\varphi_2^{(1)}$$
$$+S[2u_0\varphi_{2yy}^{(1)} + 2u_{0y}\varphi_{2y}^{(1)} - k^2u_0\varphi_2^{(1)}]$$
$$= (c_g - u_0)\varphi_{1yy} + [-2k^2c + 3k^2u_0 + u_{0yy} - k^2c_g \quad (24)$$
$$-iku_0S]\varphi_1,$$

$$\varphi_2^{(1)}(\pm\infty) = 0. \quad (25)$$

The function $\varphi_2^{(2)}(y)$ is the solution of the boundary value problem

$$2(iku_0 - ikc)\varphi_{2yy}^{(2)} + (8ik^3c - 8ik^3u_0 - 2iku_{0yy})\varphi_2^{(2)}$$
$$+S[2u_0\varphi_{2yy}^{(2)} + 2u_{0y}\varphi_{2y}^{(2)} - 4k^2u_0\varphi_2^{(2)}] \quad (26)$$
$$= ik(\varphi_1\varphi_{1yyy} - \varphi_{1y}\varphi_{1yy}) - S(2\varphi_{1y}\varphi_{1yy} - 3k^2\varphi_1\varphi_{1y}),$$

$$\varphi_2^{(2)}(\pm\infty) = 0. \quad (27)$$

The amplitude evolution equation for A is obtained from the solvability condition for equation (20) and has the form of the complex Ginzburg-Landau equation

$$\frac{\partial A}{\partial \tau} = \sigma A + \delta \frac{\partial^2 A}{\partial \xi^2} - \mu |A|^2 A \quad (28)$$

The coefficients of equation (28) are given by

$$\sigma = \frac{\sigma_1}{\gamma}, \ \delta = \frac{\delta_1}{\gamma}, \ \mu = \frac{\mu}{\gamma}, \quad (29)$$

where

$$\gamma_1 = \int_{-\infty}^{+\infty} \varphi_1^a (\varphi_{1yy} - k^2\varphi_1)dy, \quad (30)$$

$$\sigma_1 = S \int_{-\infty}^{+\infty} \varphi_1^a (2u_0\varphi_{1yy} + 2u_{0y}\varphi_{1y} - k^2u_0\varphi_1)dy, \quad (31)$$

$$\delta_1 = \int_{-\infty}^{+\infty} \varphi_1^a [\varphi_{2yy}^{(1)}(c_g - u_0) + \varphi_2^{(1)}(-k^2c_g - 2k^2c$$
$$+3k^2u_0 + u_{0yy} - 2iku_0S) + \varphi_1(2ikc_g + ikc \quad (32)$$
$$-3iku_0 - US)]dy,$$

$$\mu_1 = \int_{-\infty}^{+\infty} \varphi_1^a \{6ik^3\varphi_2^{(2)}\varphi_{1y}^{*} - 2ik\varphi_{1y}^{*}\varphi_{2yy}^{(2)} + 3ik^3\varphi_1^{*}\varphi_{2y}^{(2)}$$
$$+ik^3\varphi_1(\varphi_{2y}^{(0)} + \varphi^{*(0)}_{2y}) - ik\varphi_{1yy}(\varphi_{2y}^{(0)} + \varphi^{*(0)}_{2y})$$
$$+ik\varphi_{2y}^{(2)}\varphi_{1yy}^{*} - ik\varphi_1^{*}\varphi_{2yyy}^{(2)} + ik\varphi_1(\varphi_{2yyy}^{(0)} + \varphi^{*(0)}_{2yyy}) \quad (33)$$
$$+2ik\varphi_{1yyy}^{*}\varphi_2^{(2)} - 2S[-k^2\varphi_1(\varphi_{2y}^{(0)} + \varphi^{*(0)}_{2y})$$
$$+3k^2\varphi_1^{*}\varphi_{2y}^{(2)} - 1.5k^4\varphi_1^2\varphi_1^{*}/u_0 + 2\varphi_{1yy}(\varphi_{2y}^{(0)} + \varphi^{*(0)}_{2y})$$
$$+2\varphi_{1yy}^{*}\varphi_{2y}^{(2)} + 2\varphi_{1y}(\varphi_{2yy}^{(0)} + \varphi^{*(0)}_{2yy}) + 2\varphi_{2yy}^{(2)}\varphi_{1y}^{*}]\}dy$$

In addition, one needs to calculate the adjoint eigenfunction φ_1^a of the linear stability problem:

$$(iku_0 + 2Su_0)(\varphi_1^a)'' + (2iku_{0y} + 2Su_{0y})(\varphi_1^a)'$$
$$-(ik^3u_0 + u_0k^2S)\varphi_1^a + ikc[(\varphi_1^a)'' - k^2\varphi_1^a] = 0 \quad (34)$$

$$\varphi_1^a(\pm\infty) = 0. \quad (35)$$

The group velocity c_g is given by

$$c_g = \frac{I_1}{I_2}, \quad (36)$$

where

$$I_1 = \int_{-\infty}^{+\infty} [u_0\varphi_{1yy} - \varphi_1(3k^2u_0 + u_{0yy}$$
$$-2k^2c - 2iku_0S)]\varphi_1^a dy$$

$$I_2 = \int_{-\infty}^{+\infty} \varphi_1^a (\varphi_{1yy} - k^2\varphi_1)dy.$$

Solving boundary value problems (22) – (27), calculating φ_1^a and c_g and evaluating integrals in (30) – (33) numerically, the coefficients of the CGLE (28) are obtained for different values of R. The results are summarized in Table II.

TABLE II
COEFFICIENTS OF THE CGLE (28)

R	σ	δ	μ
-0.3	0.063 + 0.004i	0.060 – 0.206i	4.673 + 13.294i
-0.4	0.078 + 0.003i	0.090 – 0.195i	3.796 + 10.938i
-0.5	0.090 + 0.000i	0.115 – 0.184i	3.895 + 10.119i
-0.6	0.100 – 0.003i	0.136 – 0.172i	4.375 + 10.109i
-0.7	0.109 – 0.007i	0.153 – 0.161i	5.149 + 10.590i
-0.8	0.116 – 0.012i	0.167 – 0.152i	6.302 + 11.448i

IV. DISCUSSION

One of the major conclusions drawn from weakly nonlinear analysis applied to quasi-two-dimensional flows in [22] was the effect of strong dependence of the Landau constant μ_r on the form of the base flow profile. Calculations presented in [22] showed that the values of the Landau constant differed by

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a factor of 3 for two base flow velocity profiles whose linear stability characteristics differed by only 20%. As a result, it was concluded in [22] that it would be impossible to apply methods of weakly nonlinear theory in practice since the base flow profile cannot be determined very precisely in experiments. In other words, it was concluded in [22] that the problem of determination of the Landau constant from weakly nonlinear theory is ill-posed so that small variations of the base flow profile lead to large changes in the Landau constant.

The calculations presented in Table I and II in our paper demonstrate that the coefficients of the CGLE are not so sensitive to the variation of the parameter R of the base flow profile (2) as claimed in [22]. In fact, not only the Landau constant is not so sensitive to the changes in the profile (2) but all the coefficients of the CGLE do not vary too much when arge figures and tables may span both columns.

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