Simulating Discrete Time Model Reference Adaptive Control System with Great Initial Error

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Abstract—This article is based on the technique which is called Discrete Parameter Tracking (DPT). First introduced by A. A. Azab [8] which is applicable for less order reference model. The order of the reference model is (n-l) and n is the number of the adjustable parameters in the physical plant.

The technique utilizes a modified gradient method [9] where the knowledge of the exact order of the nonadaptive system is not required, so, as to eliminate the identification problem. The applicability of the mentioned technique (DPT) was examined through the solution of several problems.

This article introduces the solution of a third order system with three adjustable parameters, controlled according to second order reference model. The adjustable parameters have great initial error which represent condition.

Computer simulations for the solution and analysis are provided to demonstrate the simplicity and feasibility of the technique.

Keywords—Adaptive Control System, Discrete Parameter Tracking, Discrete Time Model.

I. INTRODUCTION

THE model reference adaptive control (MRAC) technique has been a popular approach to the control of the systems operating in the presence of the parameter and environmental variations. In such a scheme, the dynamic characteristics of the plant are specified in a reference model and the input signal or the controllable parameters of the plant are adjusted, continuously or discretely, so that its response will duplicate that of the model as closely as possible. The identification of the plant dynamic performance is not necessary and hence a fast adaptation can be achieved.

A large number of practical control problems can be formulated as discrete time model reference adaptive system. Examples are: Process Control [1], Power Systems Control [2], Signal Processing [3], etc. The available techniques for the solution for such problems are of the following type: local parametric optimization theory [4], Lyapunov function techniques [5], [6] and hyperstability and positively concepts [7]. Most of these techniques avoid the identification problem, but stresses that the order of the reference model must be the same as that of the nonadaptive system.

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In this paper a modified gradient tracking technique is proposed to solve the discrete model reference adaptive system when the exact order of the nonadaptive system is not known. The method requires that the order of the reference model be equal to (n-1) where n is the number of the controllable parameters of the discrete plant.

The mentioned Discrete Parameter Tracking (DPT) technique is utilized to solve a third order nonadaptive system having three adjustable parameters controlled according to a second order reference model. The adjustable parameters suffer a great initial error which represents a severe practical operating condition for many of the mentioned available adaptive techniques.

The organization of the rest of the paper is as follows: in section 2 the principle adaptation is presented followed by the mathematical function of the proposed technique illustrated through the mentioned third order discrete time linear variant physical system, representing our nonadaptive portion and a second order discrete input-output relationship representing the reference model. The concept is illustrated in section 3 through the solution of the mentioned problem and conclusion appears in section 4.

II. PRINCIPLE OF ADAPTATION

Consider the following single-input/single-output discrete model reference adaptive system shown in Fig. (1) and represented by:

$$g_{3}c(k-3) + \alpha_{2}c(k-2) + \alpha_{1}c(k-1) + \alpha_{0}c(k) = r(k)$$
 (1)
and

$$g_3 = constant. \tag{2}$$

$$\begin{array}{l} \alpha_{2}(k) = g_{2}(k) + h_{2}(k), \\ \alpha_{1}(k) = g_{1}(k) + h_{1}(k) \\ \alpha_{2}(k) = g_{2}(k) + h_{2}(k) \end{array}$$
(3)

 $\alpha_0(k) = g_0(k) + h_0(k)$.] Where r(k) is the input sequence, c(k) is the output of the adjustable system, k is the sample number, $\alpha_0(k)$, $\alpha_1(k)$ and $\alpha_2(k)$ are the adjustable system parameters, $h_0(k)$, $h_1(k)$ and $h_2(k)$ are the controllable adjustable system parameters, $g_0(k)$, $g_1(k)$, and $g_2(k)$ are the parts of system parameters which change due to acting disturbances.

(b) The desired input-output relationship (Reference Model).

$$A_{2}y(k-2) + A_{1}y(k-1) + A_{0}y(k) = r(k)$$
(4)

Where y(k) is the desired output of the reference mode, and A_2 , A_1 and A_0 are constants reference model parameters.



Fig. 1 Block diagram of the proposed system.

The parameters, $g_2(k)$, $g_1(k)$ and $g_0(k)$ are assumed to vary over extreme ranges within the operating environments of the nonadaptive system. Assuming also that there exists a value of $h_2(k)$, $h_1(k)$ and $h_0(k)$ for which the system will behave like the chosen model. The input information to the adaptation mechanism is the generalized error e(k) and its output will be discretely adjusting values of $h_2(k)$, $h_1(k)$ and $h_0(k)$. The objective is to formulate the adaptation mechanism equations. If no limitations are placed on the values which may be assumed by $h_2(k)$, $h_1(k)$ and $h_0(k)$, then regardless of what values $g_2(k)$, $g_1(k)$ and $g_0(k)$ take on, the output of both the physical plant and the reference model will be approximately identical whenever some chosen function of the error will be minimized. The error function used in this work is:

$$f(e) = \frac{1}{2} [q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)]^2$$
(6)

Where q_0 , q_1 and q_2 are constant factors.

The objective of the adaptation mechanism is the minimization of the error function which is a quadratic form of the error and depends indirectly on the differences:

$$\delta_2(k) = \alpha_2(k) - A_2 \tag{7}$$

$$\delta_1(k) = \alpha_1(k) - A_1 \tag{8}$$

$$\partial_0(k) = \alpha_0(k) - A_0 \tag{9}$$

Assuming that $g_2(k)$, $g_1(k)$ and $g_0(k)$ vary slowly as compared to the basic time constants of the nonadaptive adjustable system and the reference model, and the adaptive mechanism will be designed to adjust the parameters $h_2(k)$, $h_1(k)$ and $h_0(k)$ at a rate which is much greater than the rate of variation of $g_2(k)$, $g_1(k)$ and $g_0(k)$, one can use the gradient optimization technique which leads to the following basic adaptation rule:

$$\Delta \alpha_2 = -k_2 grad[f(e)] = -k_2 \frac{\partial f(e)}{\partial \alpha_2}$$
(10)

$$\Delta \alpha_1 = -k_1 grad[f(e)] = -k_1 \frac{\partial f(e)}{\partial \alpha_1}$$
(11)

$$\Delta \alpha_0 = -k_0 grad[f(e)] = -k_0 \frac{\partial f(e)}{\partial \alpha_0}$$
(12)

Where k_2 , k_1 and k_0 are arbitrary positive constants.

If the development where to continue to be based upon equations (10), (11) and (12). The resulting design would require explicit knowledge of $\alpha_2(k)$, $\alpha_1(k)$ and $\alpha_0(k)$ and consequently $g_2(k)$, $g_1(k)$ and $g_0(k)$.

Now suppose that $\alpha_2(k)$, $\alpha_1(k)$ and $\alpha_0(k)$ are treated as constants and A_2 , A_1 and A_0 are to be adjusted so as to cause $\delta_2(k)$, $\delta_1(k)$ and $\delta_0(k)$ to approach the same value, then the variations in A_2 , A_1 and A_0 become:

$$\Delta A_2 = -k_2 grad[f(e)] = -k_2 \frac{\partial f(e)}{\partial A_2}$$
(13)

$$\Delta A_{1} = -k_{1}grad[f(e)] = -k_{1}\frac{\partial f(e)}{\partial A_{1}}$$
(14)

$$\Delta A_0 = -k_0 grad[f(e)] = -k_0 \frac{\partial f(e)}{\partial A_0}$$
(15)

The objective is not to change A_2 , A_1 and A_0 . However to change $\alpha_2(k)$, $\alpha_1(k)$ and $\alpha_0(k)$. Since the same change in δ_2 , δ_1 and δ_0 , can be obtained by subtracting ΔA_2 , ΔA_1 and ΔA_0 from α_2 , α_1 and α_0 rather than adding them to A_2 , A_1 and A_0 . Now the error function f(e) is driven to minimum by determining the appropriate increments for A_2 , A_1 and A_0 and applying the negative of these increments to α_2 , α_1 and α_0 .

The resulting equations for the variation in α_2 , α_1 and α_0 are:

$$\Delta \alpha_2 = k_2 \frac{\partial f(\ell)}{\partial A_2} \tag{16}$$

$$\Delta \alpha_1 = k_1 \frac{\partial f(e)}{\partial A_1} \tag{17}$$

$$\Delta \alpha_0 = k_0 \frac{\partial f(e)}{\partial A_0} \tag{18}$$

Since it is assumed that the adaptation mechanism will be designed so as to adjust the parameters h_2 , h_1 and h_0 in a rate much greater than the changes in g_2 , g_1 and g_0 (i.e. $\Delta h_i \gg \Delta g_i$). Equations (16), (17) and (18) can be approximated as:

$$\Delta h_2 = h_2(k) - h_2(k-1) = k_2 \frac{\partial f(e)}{\partial A_2}$$
(19)

$$\Delta h_1 = h_1(k) - h_1(k-1) = k_1 \frac{\partial f(e)}{\partial A_1}$$
⁽²⁰⁾

$$\Delta h_0 = h_0(k) - h_0(k-1) = k_0 \frac{\partial f(e)}{\partial A_0}$$
(21)

When equation (6) was substituted into (19), (20) and (21) the partial derivative was carried out, equations (19), (20) and (21) yield to:

$$\Delta h_2 = -k_2 \Big[q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) \Big] \times \\ \left[q_0 \frac{\partial y(k)}{\partial A_2} + q_1 \frac{\partial y(k-1)}{\partial A_2} + q_2 \frac{\partial y(k-2)}{\partial A_2} \right]$$

$$\Delta h_2 = -k \Big[q_2 e(k) + q_2 e(k-1) + q_2 e(k-2) \Big] \times$$
(22)

$$\Delta h_{1} = -k_{1} \Big[q_{0} e(k) + q_{1} e(k-1) + q_{2} e(k-2) \Big] \times \\ \left[q_{0} \frac{\partial y(k)}{\partial A_{1}} + q_{1} \frac{\partial y(k-1)}{\partial A_{1}} + q_{2} \frac{\partial y(k-2)}{\partial A_{1}} \right]$$
(23)

$$\Delta h_0 = -k_0 [q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)] \times \left[a_1 \frac{\partial y(k)}{\partial y(k-1)} + a_2 \frac{\partial y(k-2)}{\partial y(k-2)} \right]$$
(24)

$$\left[q_0 \frac{\gamma(\gamma)}{\partial A_0} + q_1 \frac{\gamma(\gamma)}{\partial A_0} + q_2 \frac{\gamma(\gamma)}{\partial A_0}\right]$$

The terms $\frac{\partial y(k-1)}{\partial A_i}$ and $\frac{\partial y(k-2)}{\partial A_i}$ in the above equations can be represented by:

$$\frac{\partial y(k-2)}{\partial A_i} = \frac{\partial}{\partial A_i} D^2[y(k)]$$
(25)

$$\frac{\partial y(k-1)}{\partial A_i} = \frac{\partial}{\partial A_i} D[y(k)]$$
(26)

Where D represents the delay operation. Again assuming slow variation, the order of the two linear operators in the righthand side of equation (25) and (26) can be interchanged, yielding to:

$$\frac{\partial y(k-2)}{\partial A_i} = D^2 \frac{\partial y(k)}{\partial A_i}$$
(27)

$$\frac{\partial y(k-1)}{\partial A_i} = D \frac{\partial y(k)}{\partial A_i}$$
(28)

Now introducing the notation:

$$u_i(k) = \frac{\partial}{\partial A_i} y(k) \tag{29}$$

The adaptive adjusting mechanism equations could be written as:

$$h_{2}(k) = h_{2}(k-1) -k_{2}[q_{0}e(k) + q_{1}e(k-1) + q_{2}e(k-2)] \times [q_{0}u_{2}(k) + q_{1}u_{2}(k-1) + q_{2}u_{2}(k-2)]$$
(30)

$$h_{1}(k) = h_{1}(k-1) -k_{1}[q_{0}e(k) + q_{1}e(k-1) + q_{2}e(k-2)] \times [a_{2}u_{1}(k) + a_{2}u_{1}(k-1) + a_{2}u_{1}(k-2)]$$
(31)

$$h_{0}(k) = h_{0}(k-1) -k_{0}[q_{0}e(k) + q_{1}e(k-1) + q_{2}e(k-2)] \times [q_{0}u_{0}(k) + q_{1}u_{0}(k-1) + q_{2}u_{0}(k-2)]$$
(32)

The only unknown quantities in the above equations are $u_2(k)$, $u_1(k)$ and $u_0(k)$.

Consider the difference equation of the reference model, and taking the partial derivative of both sides with respect to the parameter A_0 , and interchanging the two linear operators as equation (27) and (28), employing the notation introduced in equation (29), we get:

$$A_2 u_0(k-2) + A_1 u_0(k-1) + A_0 u_0(k) = -y(k)$$
(33)

Similarly, but differentiating with respect to A_1 and A_2 , yield to:

$$A_{2}u_{1}(k-2) + A_{1}u_{1}(k-1) + A_{0}u_{2}(k) = -y(k-1)$$
(34)
$$A_{2}u_{2}(k-2) + A_{1}u_{2}(k-1) + A_{0}u_{2}(k) = -y(k-2)$$
(35)

Equations (33), (34) and (35) represent three difference equations with available forcing functions and their solutions provide the values of $u_0(k)$, $u_1(k)$ and $u_2(k)$ for equations (30), (31) and (32).

The complete set of difference equations which describe the adaptive system operation can be written as:

$$g_{3}c(k-3) + \alpha_{2}c(k-2) + \alpha_{1}c(k-1) + \alpha_{0}c(k) = r(k)$$

$$A_{2}y(k-2) + A_{1}y(k-1) + A_{0}y(k) = r(k)$$

$$A_{2}u_{0}(k-2) + A_{1}u_{0}(k-1) + A_{0}u_{0}(k) = -y(k)$$

$$A_{2}u_{1}(k-2) + A_{1}u_{1}(k-1) + A_{0}u_{1}(k) = -y(k-1)$$

$$A_{2}u_{2}(k-2) + A_{1}u_{2}(k-1) + A_{0}u_{2}(k) = -y(k-2)$$

$$\begin{aligned} h_2(k) &= h_2(k-1) \\ &- k_2 \big[q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) \big] \\ &\times \big[q_0 u_2(k) + q_1 u_2(k-1) + q_2 u_2(k-2) \big] \\ h_1(k) &= h_1(k-1) \\ &- k_1 \big[q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) \big] \\ &\times \big[q_0 u_1(k) + q_1 u_1(k-1) + q_2 u_1(k-2) \big] \\ h_0(k) &= h_0(k-1) \end{aligned}$$

$$\begin{array}{c} n_{0}(k) & n_{0}(k-1) \\ & -k_{0} \left[q_{0} e(k) + q_{1} e(k-1) + q_{2} e(k-2) \right] \\ & \times \left[q_{0} u_{0}(k) + q_{1} u_{0}(k-1) + q_{2} u_{0}(k-2) \right] \end{array}$$
(36)

III. SOLUTION OF EXAMPLE

Using the proposed technique, computer simulation was carried out to adapt the following example:

(a) The adjustable system.

$$g_3c(k-3) + (g_2 + h_2)c(k-2) + (g_1 + h_1)c(k-1) + (g_0 + h_0)c(k) = r(k)$$

Where the parameters g_0 , g_1 and g_2 are assumed to vary and the numerical values of the system parameters were chosen initially to be:

$$(g_0 + h_0) = 7.96$$
 $(g_1 + h_1) = -11.82$

 $(g_2 + h_2) = 5.86 \qquad \qquad g_3 = -1$

and h_i are assumed initially to be equal to zero.

(b) The reference model.

$$2.25y(k-2) - 7.5y(k-1) + 6.25y(k) = r(k)$$

(c) The input:

The input signal r(k) was chosen to be sampled square pulses of magnitude equal to ± 2.0 and duration of 100 sec.

Simulation Results:

Simulation were carried out with the system parameters g_2 , g_1 and g_0 varying inearly according to the following equations:

$$g_{2}(k) = -5.86 - 0.01k$$

$$g_{1}(k) = -11.82 - 0.01k$$

$$g_{0}(k) = 7.96 - 0.01k$$

$$0 \le k \le 100$$

$$= 0.0 \qquad k > 100$$
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According to the adaptation technique described in section 2, the set of difference equations describing the dynamics of the adaptive mechanism are:

$$\begin{aligned} h_{2}(k) &= h_{2}(k-1) \\ &- k_{2} \Big[q_{0} e(k) + q_{1} e(k-1) + q_{2} e(k-2) \Big] \\ &\times \Big[q_{0} u_{2}(k) + q_{1} u_{2}(k-1) + q_{2} u_{2}(k-2) \Big] \\ h_{1}(k) &= h_{1}(k-1) \\ &- k_{1} \Big[q_{0} e(k) + q_{1} e(k-1) + q_{2} e(k-2) \Big] \\ &\times \Big[q_{0} u_{1}(k) + q_{1} u_{1}(k-1) + q_{2} u_{1}(k-2) \Big] \\ h_{0}(k) &= h_{0}(k-1) \\ &- k_{0} \Big[q_{0} e(k) + q_{1} e(k-1) + q_{2} e(k-2) \Big] \\ &\times \Big[q_{0} u_{0}(k) + q_{1} u_{0}(k-1) + q_{2} u_{0}(k-2) \Big] \\ 2.25 u_{0}(k-2) - 7.5 u_{0}(k-1) + 6.25 u_{0}(k) = - y(k). \\ 2.25 u_{1}(k-2) - 7.5 u_{1}(k-1) + 6.25 u_{1}(k) = - y(k-1). \\ 2.25 u_{2}(k-2) - 7.5 u_{2}(k-1) + 6.25 u_{2}(k) = - y(k-2). \end{aligned}$$

The constant parameters of the adaptive mechanism were obtained by trial and error technique, (any set of parameters that secure the stability of the adaptive process), intently we choose them near optimum to represent inaccuracy which can be seen as part of initial error, to be:

$$k_0 = 30;$$
 $k_1 = 30;$ $k_2 = 30;$
 $q_0 = 3;$ $q_1 = -3$ $q_2 = 1$

The results of simulation are shown in Fig.s 2 through 6 which show the proper function of adaptive process even with the inaccurate choice of the adjusting mechanism parameters.

Now we pass to the main object of this paper, which is the great initial error, so, we start by changing the initial value of g_0 to be equal to 6, the results of this case are given in Figs. 7 through 11. Then $g_0(0) = 7$ and the results are shown in Figs. 12 through 16. Finally $g_0(0) = 9$ with results shown in Figs. 17 through 21.



Fig. 2 h_i for optimum parameters.



Fig. 3 Model Y & System C for optimum parameters.



Fig. 4 Error between Model & System for optimum parameters.



Fig. 5 h_0 versus h_1 for optimum parameters.

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Fig. 8 Model *Y* & System *C* for $g_0 = 6$.







Fig. 13 Model *Y* & System *C* for $g_0 = 7$.

Y(k),C(k),

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Fig. 14 Error between Model & System for $g_0 = 7$.



Fig. 17 h_i for $g_0 = 9$.

00

600





Fig. 19 Error between Model and System for $g_0 = 9$.





From the last results Figs. 2 through 21, it is clear the proper functioning of the introduced technique.

We did the same for the initial value of g_1 to be equal to -11, -12 and -13, the results are given in Figs. 22 through 26, 27 through 31 and 32 through 36 respectively.

Then, again we did the same for the initial value of g_2 to be equal 5, 6 and 7 and the results for this case are given in Figs. 37 through 41, 42 through 46 and 47 through 51 respectively.



Fig. 24 Error between Model & System for $g_1 = -11$.



Fig. 28 Model Y and System C for $g_1 = -12$.

Y(k),C(k),

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Fig. 29 Error between Model & System for $g_1 = -12$.







Fig. 33 Model *Y* and System *C* for $g_I = -13$.



Fig. 34 Error between Model & System for $g_1 = -13$.





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Fig. 40 h_0 versus h_1 for $g_2 = 5$.



Fig. 44 Error between Model & System for $g_2 = 6$.

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Fig. 49 Error between Model & System for $g_2 = 7$.



Comments:

- 1) It is of much importance to know that the desired output (reference model output) could be obtained from the nonadaptive portion. i.e. there is a set of parameters that can make the two outputs approximately identical.
- 2) The aim of the introduced example was only to prove that the adaptive adjusting mechanism designed according to the introduced Discrete Parameter Tracking (DPT) technique can sustain the problem of great initial error which is practically common problem.
- 3) In the above example, the behavior of the adaptive mechanism was studied for other initial values of g_0 , g_1 and g_2 , the results proved to be working satisfactory. (The results are not included).

IV. CONCLUSION

The introduced Discrete Parameter Tracking (DPT) technique which is based on the concept of modified gradient technique in contrary to other known techniques which stress that the order of reference model must be the same as that of the nonadaptive system, this technique enables us to solve the discrete time model reference adaptive system when the exact order of the nonadaptive system is not known. The method requires that the order of reference model is chosen in relation to the number of controlable parameters, and that it exist a set of the controllable parameters for which the output of the nonadaptive portion will duplicate the reference model output.

Computer simulation shows that if the parameters of the adaptive adjusting mechanism are properly chosen to be optimum or even near the optimum, the technique may sustain severe practical problems such as great initial error.

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