

Simulation of a Process Design Model for Anaerobic Digestion of Municipal Solid Wastes

Asok Adak, Debabrata Mazumder and Pratip Bandyopadhyay

Abstract—Anaerobic Digestion has become a promising technology for biological transformation of organic fraction of the municipal solid wastes (MSW). In order to represent the kinetic behavior of such biological process and thereby to design a reactor system, development of a mathematical model is essential. Addressing this issue, a simplistic mathematical model has been developed for anaerobic digestion of MSW in a continuous flow reactor unit under homogeneous steady state condition. Upon simulated hydrolysis, the kinetics of biomass growth and substrate utilization rate are assumed to follow first order reaction kinetics. Simulation of this model has been conducted by studying sensitivity of various process variables. The model was simulated using typical kinetic data of anaerobic digestion MSW and typical MSW characteristics of Kolkata. The hydraulic retention time (HRT) and solid retention time (SRT) time were mainly estimated by varying different model parameters like efficiency of reactor, influent substrate concentration and biomass concentration. Consequently, design table and charts have also been prepared for ready use in the actual plant operation.

Keywords—Anaerobic digestion, municipal solid waste (MSW), process design model, simulation study, hydraulic retention time (HRT), solid retention time (SRT).

I. INTRODUCTION

THE rapid growth of population of India coupled with continual modernization poses significant challenges to the sustainability of the environment. Municipal solid waste (MSW) generation is significantly increasing in Indian urban areas and started creating enormous waste disposal problems in the recent past [1]. The anaerobic digestion is an attractive option for energy generation from the putrescible fraction of MSW as well as for reducing the disposal problem [2].

In anaerobic digestion of MSW, a variety of anaerobic microorganisms perform together to bring about the conversion of organic fraction of MSW. Biological transformation of the organic matter in MSW takes place in three consecutive steps. The first step is enzyme mediated conversions of higher-molecular mass compounds into several

compounds as the source of energy and cell tissue. The second step brings about bacterial conversion of the compounds produced from the first step into identifiable lower-molecular-mass intermediate compounds. The third and also last step is associated with conversion of intermediate compounds into simpler end products especially methane and carbon dioxide [3]. Among the various operational parameters that influence the anaerobic digestion process of MSW, the most important ones are the mass of the organic substrate, moisture content, reaction temperature and the amount of biomass available.

In order to be able to design, predict how the system responds to changes in feed conditions and operate efficiently anaerobic digestion system, appropriate mathematical models need to be developed [4]. Till now, a few process designs models are available for in-vessel anaerobic digestion of MSW. Each design model has its own merits, constraints and focus of output prediction. Singh and Rao [1] developed a first order kinetic model for the gas generation of the anaerobic batch digestion. Nopharatana et al. [5] proposed a kinetic model to simulate the biological reactions occurring in the anaerobic degradation of MSW. In this model three distinguished biomass growth rates for three different phases of anaerobic digestion were considered. However, this model is found to be very complex to apply for the shake of process design. Nwabanne et al. [6] developed a kinetic model of anaerobic digestion of MSW considering Monod's kinetics of biomass growth and substrate utilization. It was applied to estimate the time required for a batch digestion process. Past literature on anaerobic digestion depict that there is no simple mathematical model to describe the anaerobic digestion of MSW under continuous mode. In this present study an effort has been made to develop a simple mathematical model for a continuous flow anaerobic digestion system. The model was simulated to study the influence of different variables like substrate and biomass concentration on the efficiency of the anaerobic digestion and to predict the hydraulic retention time and the solid retention time for any specific case.

II. MATERIALS AND METHODS

A. Background of Process Design Models for Anaerobic Digestion of MSW

Nopharatana et al. [5] developed a kinetic model to simulate the biological reactions occurring in the anaerobic degradation of MSW. The model was a dynamic mass

Asok Adak is with Bengal Engineering and Science University, Shibpur, Howrah -711103, INDIA (phone: 0091-033-26684561; fax: 0091-033-26682916; e-mail: asok@civil.becs.ac.in).

Debabrata Mazumder is with Bengal Engineering and Science University, Shibpur, Howrah -711103, INDIA (phone:0091-033-26684561; fax: 0091-033-26682916; e-mail: debabrata@civil.becs.ac.in).

Pratip Bandyopadhyay is with Bengal Engineering and Science University, Shibpur, Howrah - 711 103, INDIA (phone: 0091-033-2668-4561; e-mail: pratip@civil.becs.ac.in).

balance on a mass of MSW subjected to microbial reactions that converted substrate to products and biomass. The MSW was represented in the model as two components, an insoluble and a soluble fraction. Both the insoluble and soluble fractions are considered to consist of a biodegradable and refractory fraction. The refractory fraction, f , represents a portion of either the soluble or insoluble COD which cannot be degraded anaerobically. The soluble and insoluble biodegradable proportions of the MSW were taken into consideration. Three bacterial groups were considered to mediate the degradation of either soluble or insoluble substrate to CH_4 and CO_2 . The degradation of insoluble substrate was considered to involve an additional enzymatic reaction to catalyze the hydrolysis step, which converted the solid to soluble substrate. The degradation of soluble substrates and organic acids are written as; $r_g = r_{X1}/Y_{X1}$, $r_a = r_{X2}/Y_{X2}$ and $r_f = r_{X3}/Y_{X3}$, where Y_{X1} , Y_{X2} and Y_{X3} are the yield coefficients for the acidogenic bacteria, acetoclastic methane bacteria and hydrogen utilizing methane bacteria, respectively. r_{X1} , r_{X2} and r_{X3} are the growth rates of those respective bacteria. This model is found to be very complex and difficult to apply in the real process design.

A first-order kinetic model developed by Singh and Rao [1] approached anaerobic digestion of complex substrates following a simple basis to compare stable process performance under practical scenario. The substrate concentration was correlated with biogas production (G). The biogas production at any time t was given by.

$$G = G_\infty (1 - e^{-kt}) \dots\dots\dots(1)$$

Where, G_∞ is the ultimate gas production and k is the first order biogas production rate constant. The gas production can also be correlated with substrate concentration as follows:

$$\frac{(G_\infty - G)}{G_\infty} = \frac{B}{B_0} \dots\dots\dots(2)$$

Where, B_0 is the initial substrate concentration and B is the substrate concentration at any time t .

Nwabanne et al. [6] proposed a kinetics model of the anaerobic digestion of MSW which considered both the kinetics of biomass growth and substrate utilization. The rate of reaction was considered to follow the first order reaction in a batch anaerobic digester. The time for batch digestion, t was represented as follows:

$$t = \frac{K_s}{kX} \ln \left(\frac{S_0}{S_e} \right) + \frac{S_0 - S_e}{kX} \dots\dots\dots(3)$$

Where,

- S_0 = Influent substrate concentration, mg/l
- S_e = Effluent substrate concentration, mg/l
- X = Biomass (microorganism) concentration, mg/l
- k = Max rate of substrate utilization, day^{-1}
- K_s = Half-velocity constant, mg/l

This particular model has been studied for batch mode reactor where the substrate concentration gradually decreases without any steady state. The characteristic of continuous mode reactor is completely different because the substrate

concentration approaches to constant under steady state condition. The concentration gradient also becomes uniform at this stage. Thus there is urgent need to develop a mathematical model to analyze the continuous mode reactor for the anaerobic digestion of MSW.

B. Development of Simplistic Process Design Model

A simplistic process design model has been developed for anaerobic digestion of MSW in continuous mode reactor considering kinetics of both biomass growth and substrate utilization. Like the process model of Nwabanne et al. [6] the rate of biochemical reaction is assumed to follow the first order kinetics.

The rate of change of biomass concentration can be expressed as

$$\frac{dX}{dt} = \mu X \dots\dots\dots(4)$$

Where,

X = Biomass concentration (mg/l)

t = Time (day)

μ = Specific growth rate (day^{-1})

μ is again co-related with substrate as per Monod's kinetics.

$$\mu = \mu_{\max} \frac{S}{K_s + S} \dots\dots\dots(5)$$

Where,

μ_{\max} = Maximum specific growth rate (day^{-1})

S = Substrate concentration (mg/l)

K_s = Half velocity constant (mg/l)

Considering the endogenous decay of anaerobic biomass, specific growth rate can be written as

$$\mu' = \mu_{\max} \frac{S}{K_s + S} - K_d \dots\dots\dots(6)$$

Where,

K_d = coefficient of endogenous decay (day^{-1})

The rate of substrate utilization is also correlated with the rate of biomass growth as

$$\frac{dX}{dt} = -Y \frac{dS}{dt} \dots\dots\dots(7)$$

Where,

Y = biomass yield coefficient

The above equation can be expressed as

$$\mu_{\max} \frac{X.S}{K_s + S} = -Y \frac{dS}{dt} \dots\dots\dots(8)$$

$$\text{or, } -\frac{dS}{dt} = \mu_{\max} \frac{X}{Y} \frac{S}{K_s + S} = \frac{kSX}{K_s + S} \dots\dots\dots(9)$$

Where,

$$k = \frac{\mu_{\max}}{Y} = \text{Max rate of substrate utilization } (\text{day}^{-1})$$

In case of continuous mode homogeneous reactor under steady state condition the substrate utilization rate can be

expressed as the rate of substrate removal ($S-S_o$) during the retention period i.e. the hydraulic retention time, HRT (θ). Thus the above equation (Eq. 9) can be modified as

$$\frac{S_o - S}{\theta} = \frac{kSX}{K_s + S} \dots \dots \dots (10)$$

If E be the efficiency (in fraction) of the reactor then the Eq. 10 can be expressed as

$$\theta = \frac{ES_o [K_s + S_o(1-E)]}{kXS_o(1-E)} \dots \dots \dots (11)$$

As soon as the hydraulic retention time is evaluated the volume of the reactor can be determined as follows.

$$V = Q\theta \dots \dots \dots (12)$$

Where,

V = volume of the reactor (m^3)

Q = the flow rate (m^3/day)

The solid retention time, SRT (day) required for the anaerobic digestion system, can be determined as

$$\theta_c = \frac{X}{r_g - K_d X} = \frac{X}{-Y \frac{dS}{dt} - K_d X} \dots \dots \dots (13)$$

Again for the continuous reactor under steady state condition Eq. 13 can be modified as

$$\theta_c = \frac{X}{Y \frac{S_o - S}{\theta} - K_d X} = \frac{X}{Y \frac{ES_o}{\theta} - K_d X} \dots \dots (14)$$

C. Methodology of Simulation

The important process design parameters for the anaerobic digestion of MSW are influent and effluent substrate concentration, biomass concentration, hydraulic retention time (HRT), solid retention time (SRT) and different kinetic constants. Thus for any digestion unit, the variables are the influent and effluent substrate and biomass concentration, hydraulic retention time and solid retention time. All these variables are correlated as per process design equations 11 and 14. In case of a specific MSW, the kinetic constants are fixed. Since HRT and SRT are two control parameters, the simplistic mathematical model developed in the present case was simulated by varying initial substrate concentration, effluent substrate concentration (i.e. varying efficiency of the system) and concentration of biomass to estimate HRT and SRT. The different kinetic constants related to anaerobic digestion have been presented in Table 1 [6].

In the present study, the MSW generated from Kolkata, a Metropolitan city of India was considered. The typical chemical characteristics of said MSW are presented in Table 2 [7]. The biodegradable organics were represented as chemical oxygen demand (COD), in turn theoretical oxygen demand (ThOD) which was calculated stoichiometrically from the carbon content of the MSW. To simulate a soluble state of biodegradable organics upon hydrolysis, a liquid phase of MSW in 1:10 ratio (i.e. 100 g of dry MSW in 1 liter of water) was considered. Since the carbon content of the MSW was 22.36%, the corresponding COD value was calculated to be

$(32/12) \times 22360$ i.e. 59600 mg/l. This value was varied up to $\pm 25\%$ in the simulation process. The food to microorganism ratio (F/M ratio) was varied from 0.8 to 1 and the efficiency was varied in the range of (70-90)%. Consequently, biomass concentration was varied as per F/M ratio depending on substrate COD concentration. The HRT and SRT were calculated by varying the above mentioned parameters and the results are shown graphically in the next section.

TABLE I
 KINETICS PARAMETERS USED FOR THE STUDY

Parameters	Values
k	0.144 day ⁻¹
K_s	21.23 mg/l
k_d	0.038 day ⁻¹
Y	0.367

TABLE II
 TYPICAL CHEMICAL COMPOSITION OF THE MSW AT KOLKATA

Parameters	Value
Moisture (%)	46
pH	8.07
Loss on ignition (%)	38.53
Carbon (%)	22.36
Nitrogen as N (%)	0.76
Phosphorous as P ₂ O ₅ (%)	0.77
Potassium as K ₂ O (%)	0.52
C/N ratio	31.81
Calorific value (KJ/Kg)	5028

III. RESULTS AND DISCUSSION

A. Determination of HRT and SRT for Varying Reactor Efficiency

The HRT and SRT required in a homogeneous continuous flow type reactor under steady state to accomplish the anaerobic digestion of MSW of Kolkata was calculated for different reactor efficiency (E) varying in the range of (70 – 90%). The influent COD was 59600 mg/l and F/M ratio was taken as 0.8, 0.85, 0.90, 0.95 and 1.0 respectively. Thus the biomass concentration was 74500, 70118, 66222, 62737 and 59600 respectively. The variation of HRT for different efficiency has been shown graphically in Fig. 1 and the values are given in Table 3. It is seen that the HRT linearly increases with the increase in efficiency, which is quite expected as per model equation. Again, HRT increases with increase in F/M ratio (i.e. decrease in biomass concentration) representing a series of parallel lines. It is needless to mention that Fig. 1 and Table 3 could be used readily for reactor design.

At the same time, the SRT was also calculated corresponding to the above mentioned data. It is observed that the SRT increases slightly (up to about 14 h) with the increase in reactor efficiency (Fig. 2). The pattern of SRT increase represents a hyperbolic curve. There is no effect of variation of biomass concentration on SRT for varying efficiency at a fixed influent COD concentration.

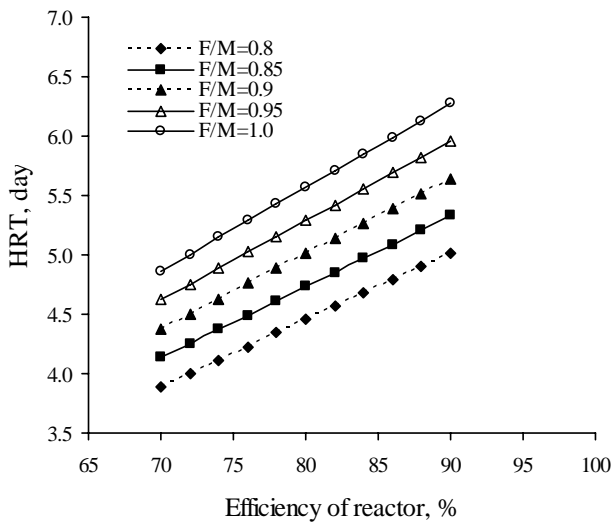


Fig. 1 Variation of HRT with efficiency of anaerobic digestion of MSW

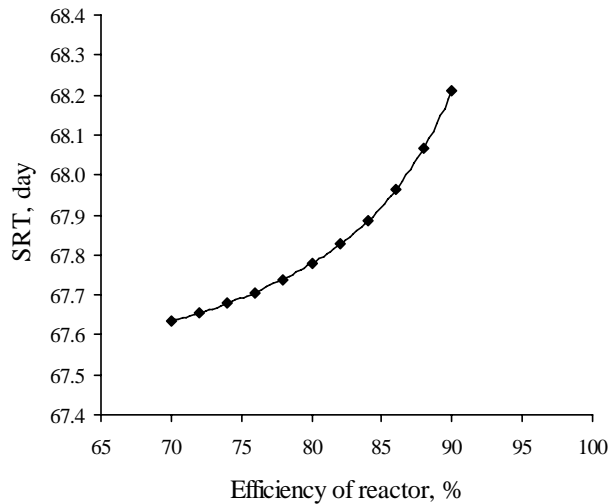


Fig. 2 Variation of SRT with efficiency of anaerobic digestion of MSW

B. Determination of HRT and SRT for Varying Influent Substrate Concentration

The effect of different influent substrate concentration on HRT and SRT value to attain a targeted efficiency in the homogeneous continuous flow type reactor under steady state condition was also studied. The influent concentration was varied in the range of (44700 – 74500) mg/l. The biomass concentration was taken as 74500 and 59600 mg/l corresponding to two F/M ratio of 0.8 and 1.0. The HRT and SRT were calculated for 70, 80 and 90% reactor efficiency in response to F/M ratio of 0.8 and 1.0. The variation of HRT with the variation of influent concentration has been shown in Fig. 3 and the values are provided in Table 4. It is found that the HRT linearly increases with increase in influent substrate concentration for a particular value of reactor efficiency and biomass concentration. HRT also increases with the decrease

in biomass concentration and the increase in reactor efficiency, which is quite reasonable. The variation of SRT with varying influent substrate concentration for different values of reactor efficiency corresponding to two specific F/M ratios has been shown in Fig. 4. It is seen that there is slight decrease in SRT (up to 3.7 h for 70%, up to 5.5 h for 80% and up to 11 h for 90%) with the increase in influent substrate concentration. The profile of SRT demonstrates hyperbolic nature in accordance with the model equation. Similar to the previous case, it is also independent of the biomass concentration.

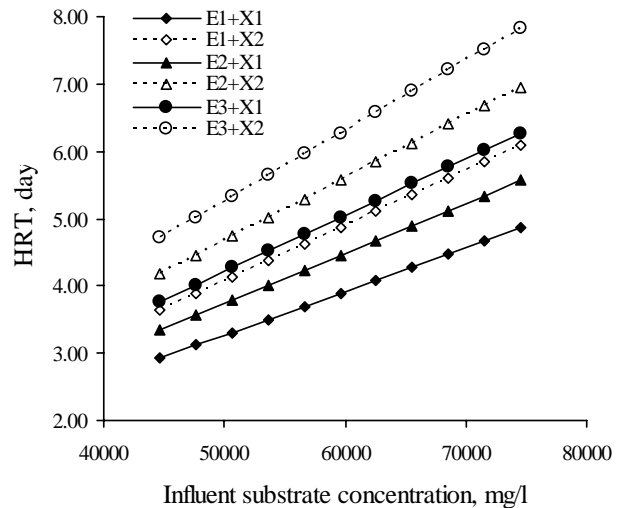


Fig. 3 Variation of HRT of anaerobic reactor for MSW digestion with influent concentration ($E1=70\%$, $E2=80\%$, $E3=90\%$, $X1=74500$ mg/l, $X2=59600$ mg/l)

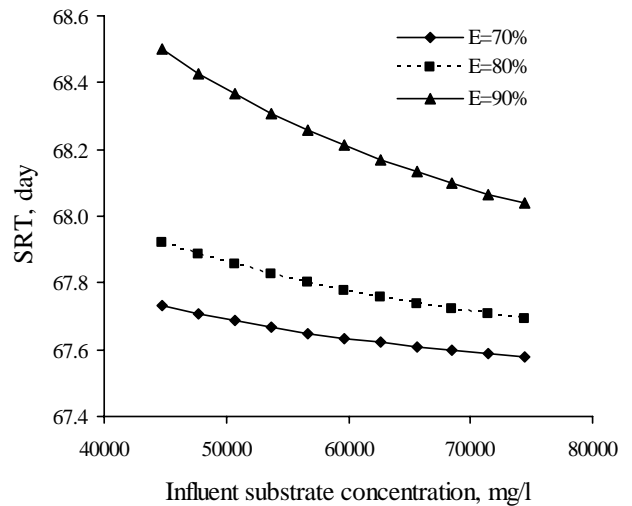


Fig. 4 Variation of SRT of anaerobic reactor for MSW digestion with influent concentration

TABLE III
VARIATION OF HRT AND SRT WITH VARIATION OF EFFICIENCY OF ANAEROBIC DIGESTION OF MSW

Efficiency (%)	Influent COD, S ₀ (mg/l)	Biomass Concentration, X (mg/l)					HRT (day)					SRT (day)
		F/M = 0.8	F/M = 0.85	F/M = 0.9	F/M = 0.95	F/M = 1.0	F/M = 0.8	F/M = 0.85	F/M = 0.9	F/M = 0.95	F/M = 1.0	
70	59600	74500	70118	66222	62737	59600	3.89	4.14	4.38	4.62	4.87	67.63
72	59600	74500	70118	66222	62737	59600	4.01	4.26	4.51	4.76	5.01	67.66
74	59600	74500	70118	66222	62737	59600	4.12	4.37	4.63	4.89	5.15	67.68
76	59600	74500	70118	66222	62737	59600	4.23	4.49	4.76	5.02	5.29	67.71
78	59600	74500	70118	66222	62737	59600	4.34	4.61	4.88	5.15	5.43	67.74
80	59600	74500	70118	66222	62737	59600	4.45	4.73	5.01	5.29	5.57	67.78
82	59600	74500	70118	66222	62737	59600	4.56	4.85	5.14	5.42	5.71	67.83
84	59600	74500	70118	66222	62737	59600	4.68	4.97	5.26	5.55	5.85	67.89
86	59600	74500	70118	66222	62737	59600	4.79	5.09	5.39	5.69	5.99	67.96
88	59600	74500	70118	66222	62737	59600	4.90	5.21	5.52	5.82	6.13	68.07
90	59600	74500	70118	66222	62737	59600	5.02	5.33	5.65	5.96	6.27	68.21

TABLE IV
VARIATION OF HRT AND SRT WITH VARIATION OF INFLUENT SUBSTRATE CONCENTRATION OF ANAEROBIC DIGESTION OF MSW

Influent COD, S ₀ (mg/l)	Reactor Efficiency (%)					Biomass, X (mg/l)					HRT (day)					SRT (day)		
	E1	E2	E3	F/M = 0.8	F/M = 1.0	E1+X1	E1+X2	E2+X1	E2+X2	E3+X1	E3+X2	E1	E2	E2	E2	E2		
44700	70	80	90	74500	59600	2.92	3.65	3.34	4.18	3.77	4.71	67.73	67.92	68.50				
47680	70	80	90	74500	59600	3.12	3.89	3.56	4.45	4.02	5.02	67.71	67.89	68.43				
50660	70	80	90	74500	59600	3.31	4.14	3.79	4.73	4.27	5.33	67.69	67.85	68.36				
53640	70	80	90	74500	59600	3.50	4.38	4.01	5.01	4.52	5.65	67.67	67.83	68.31				
56620	70	80	90	74500	59600	3.70	4.62	4.23	5.29	4.77	5.96	67.65	67.80	68.26				
59600	70	80	90	74500	59600	3.89	4.87	4.45	5.57	5.02	6.27	67.63	67.78	68.21				
62580	70	80	90	74500	59600	4.09	5.11	4.67	5.84	5.27	6.58	67.62	67.76	68.17				
65560	70	80	90	74500	59600	4.28	5.35	4.90	6.12	5.52	6.90	67.61	67.74	68.13				
68540	70	80	90	74500	59600	4.48	5.60	5.12	6.40	5.77	7.21	67.60	67.72	68.10				
71520	70	80	90	74500	59600	4.67	5.84	5.34	6.68	6.02	7.52	67.59	67.71	68.07				
74500	70	80	90	74500	59600	4.87	6.08	5.56	6.95	6.27	7.83	67.58	67.69	68.04				

IV. CONCLUSIONS

In the light of present study it can be concluded that a simplistic process design model for anaerobic digestion of MSW is possible to be developed. The unique model is exclusively based on classical relationships of biological process i.e. substrate utilization rate and biomass growth rate. The developed mathematical model can be employed to predict the process control parameters like the HRT and SRT of a continuous flow type homogeneous reactor under steady state condition. The HRT linearly increases with the increase in efficiency of the reactor, influent substrate concentration and with the decrease in biomass concentration. Moreover, there is linear upward shift of HRT with the enhancement of F/M ratio. The SRT varies slightly, following hyperbolic pattern with the increase in efficiency of the reactor and influent substrate concentration. However, it remains constant with the variation in biomass concentration when HRT is adjusted accordingly. The design charts and tables developed here could be used for designing an MSW digestion system conforming same kinetic constants as used in the present study.

REFERENCES

- [1] O. M. S. Rao and S. P. Singh, "Bioenergy conversion studies of organic fraction of MSW: kinetic studies and gas yield-organic loading relationships for process optimization", *Bioresource Technol.*, vol. 95, pp. 173-185, 2004.
- [2] L. de Baere, "Anaerobic digestion of solid waste: state-of-the-art", *Water Sci. Technol.*, vol. 41, pp. 283-290, 2000.
- [3] G. Tchobanoglous, H. Theisen and S. Vigil, "Integrated Solid Waste Management – Engineering Principle and Management Issues", Mc Graw Hill Inc. USA, 1993.
- [4] G. Lyberatos and I. V. Skiadas, "Modelling of anaerobic digestion – a review", *Global Nest: the Int. J.*, vol. 1, no. 2, pp. 63-76, 1999.
- [5] A. Nopharatana, P. C. Pullammanappallil and W. P. Clarke, "Kinetics and dynamic modelling of batch anaerobic digestion of municipal solid waste in a stirred reactor", *Waste Management*, vol. 27, pp. 595-603, 2007.
- [6] J. T. Nwabanne, O. D. Onukwuli and C.M. Ifeakandu, "Biokinetics of anaerobic digestion of municipal waste", *Int. J. Environ. Res.*, vol. 3, no. 4, pp. 511-516, 2009.
- [7] S. Chattopadhyay, A. Dutta and S. Ray, "Municipal solid waste management in Kolkata, India – A review" *Waste Management*, vol. 29, no. 4, pp. 1449-1458, 2009.