Abstract—Inter-symbol interference if not taken care off may cause severe error at the receiver and the detection of signal becomes difficult. An adaptive equalizer employing Recursive Least Squares algorithm can be a good compensation for the ISI problem. In this paper performance of communication link in presence of Least Mean Square and Recursive Least Squares equalizer algorithm is analyzed. A Model of communication system having Quadrature amplitude modulation and Rician fading channel is implemented using MATLAB communication block set. Bit error rate and number of errors is evaluated for RLS and LMS equalizer algorithm, due to change in Signal to Noise Ratio (SNR) and fading component gain in Rician fading Channel.

Keywords—Least mean square (LMS), Recursive least squares (RLS), Adaptive equalization, Bit error rate (BER), Rician fading channel, Quadrature Amplitude Modulation (QAM), Signal to noise ratio (SNR).

I. INTRODUCTION

In any wireless communication link, the channel induced distortion results in Inter-Symbol Interference (ISI), which, if left uncompensated, causes higher error rates. The solution to the ISI problem is to design a receiver that employs a means for compensating or reducing the ISI in the received signal. An adaptive equalizer is the best compensator for the ISI problem [1]. Adaptive equalization developed by Lucky, have an algorithm dependent on peak distortion criteria which led to zero forcing equalizer as in [2]. To facilitate data transmission over the channel, some form of modulation is used, so that the spectral component of the transmitted signal resides inside the pass band of the channel. In general, ISI which is caused mainly by the dispersion in the channel and thermal noise generated at the receiver input is the key area of concern.

A Quadrature Amplitude Modulation (QAM) technique is used for the simulation model, in QAM the information bits are encoded in both the amplitude and phase of the transmitted signal. Thus QAM has two degree of freedom, which makes it more spectrally efficient than technique like M-ary Frequency Shift Keying (MFSK), M-ary Phase Shift Keying (MPSK) and others. It can encode the maximum number of bits per symbol for a given average energy [3, 4]. Some common square constellation such as 4-QAM and 16-QAM are shown in figure 1.

Fig. 1 Square constellation of 4-QAM (triangle) and 16-QAM (circle)

The transmitted signal may be represented as [3, 4]

\[ S_i(t) = R_i e^{j \theta_i} \left[ g(t)e^{-j 2 \pi f_t} \right] = A_i \cos(\theta_i) g(t)(\cos(2\pi f_s t) - A_i \sin(\theta_i) g(t)(\sin(2\pi f_s t) \right), 0 \leq t \leq T_s \]

where pulse shape \( g(t) \), must maintain the orthogonal property i.e.

\[ \int_0^T g^2(t) \cos^2(2\pi f_s t) dt = 1 \]

and \[ \int_0^T g^2(t) \cos(2\pi f_s t) \sin(2\pi f_s t) dt = 0 \]

The energy in \( S_i(t) \) is

\[ E_{s_i} = \int_0^T s_i^2(t) dt = A_i^2 \]

The distance between any pair of symbols in the signal constellation is

\[ d_{ij} = \| S_i - S_j \| = \sqrt{(S_{i1} - S_{j1})^2 + (S_{i2} - S_{j2})^2} \]

for square signal constellations, where \( S_i \) and \( S_j \) take values on \((2i-1-L,L)\) with \( i = 1, 2, \ldots, L \), the minimum distance between

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signal point reduces to $d_{\text{min}}=2d$ constellation points are used to send $2f$ bits/symbol or $l$ bits per dimension, where $l=0.5\log_{2}M$. In square constellation it takes approximately 6dB more power to send an additional 1 bit/dimension or 2bits/symbol while maintaining the same minimum distance between constellation points [2-4]. It is hard to find a gray coded mapping for QAM where all adjacent symbols differ by a single bit. To recover the signal at the receiver is an important issue in a wireless communication system, where channel behavior is not constant and to be observed closely. In wireless communication system a small scale fading can be generally incorporated by Rayleigh or Rician probability density function (PDF). Assuming Rayleigh or Rician fading Channel means, that the fading amplitude are Rayleigh or Rician distributed random variable, whose value affect the signal amplitude (finally power) of the received signal. A Rayleigh fading has multiple reflective paths, which are large in numbers and there is no dominant line of sight (LOS) propagation path. Fading is Rician distributed if a dominant LOS is present [4-6]. The fading amplitude $r_i$ at $i^{th}$ instant can be represented as in equation 6,

$$r_i = \sqrt{(x_i + \beta)^2 + y_i^2}$$

where $\beta$ is the amplitude of the specular component and $x_i, y_i$ are samples of zero mean stationary Gaussian random process each with variance $\sigma^2$. Factor $K$ in Rician distribution is the ratio of specular to defuse energy, given by equation 7,

$$K = \frac{\beta}{2\sigma^2}$$

Rician K factor may vary from $K=0$ to $\infty$ (minimum to maximum limit).

In general PDF of Rician is represented as in equation (8)

$$f_{\text{Ric}} (r) = \frac{r}{\sigma^2} \exp \left( - \frac{r^2 + \beta^2}{2\sigma^2} \right) I_0 \left( \frac{r\beta}{\sigma^2} \right), r \geq 0$$

where $I_0[.]$ is the zero order modified Bessel function of the first kind. If there is no dominant propagation path $K=0$ and $I_0[1]=1$ yield the worst case Rayleigh PDF given by equation (9)

$$f_{\text{Ray}} (r) = \frac{r}{\sigma^2} \exp \left( - \frac{r^2}{2\sigma^2} \right), r \geq 0$$

A typical plot of PDF of Rician and Rayleigh is shown in figure 2 and 3 respectively.

The cumulative distribution function (CDF) takes the shape denoted by equation (10)

$$C_{\text{Ric}} (r) = 1 - \exp \left( - \gamma \sum_{m=1}^{\infty} \frac{\beta}{2\sigma^2} I_m \left( \frac{r\beta}{\sigma^2} \right) \right)$$

where $\gamma=(k+1)^2/2\sigma^2$. As can be seen from equation 10, it is very difficult to evaluate the PDF, due to the summation of an infinite number of terms [4, 6-8]. However in practical terms it is sufficient to increase $m$ to a value, where the last term contribution is less than 0.1%.

A typical Rician fading envelope for input sample period: 1.0000e-004, maximum Doppler shift: 200, path delays: 0, Average path gain in dB: 0, normalize path gains: 1, path gains: 1.3432+ 0.5966i, Channel filter delay: 0, Reset before filtering: 1, Number of samples processed: 1000, is shown in figure 2. A typical Rayleigh fading envelope for equalizers are used at the receiver to alleviate the ISI problems caused by delay spread. Mitigation of ISI is required when the modulation symbol time $T_s$ is on the order of the channels RMS delay spread $\sigma_{\text{rms}}$ [2, 5-9]. Higher data rate applications are more sensitive to delay spread and generally require high performance equalizer or other ISI mitigation techniques. As wireless channel varies over time, the equalizer must learn the frequency or impulse responses of the channel referred as training and then update its estimate of the frequency response as the channel changes referred as tracking. The process of equalizer training and tracking is often referred to as adaptive equalization, since the equalizer adapts to the changing channels.

Adaptive equalizers require algorithms for updating the filter tap coefficient during training and tracking. Algorithms generally incorporate tradeoff between complexity, convergence rate and numerical stability [1, 3, 4, 10]. Different types of equalizer, structure and algorithms used are shown in table I.
An adaptive equalizer is customarily placed in the receiver with the channel output as the source of excitation applied to the equalizer, different parameters are adjusted by means of Least mean square (LMS) or Recursive least squares (RLS) algorithm to provide an estimate of each symbol transmitted [1, 3, 9]. A tutorial treatment of adaptive equalization including LMS and RLS algorithm that were developed during the period 1965–1975 is effectively explained in [2]. A comprehensive treatment of LMS and RLS algorithm present in [1-3] is being employed for modeling the structure and evaluating the results.

The LMS algorithm have capability to adaptively adjust the tap coefficients of a linear equalizer or a Decision feedback equalizer (DFE) is basically a stochastic steepest descent algorithm in which the true gradient vector is approximated by an estimate obtain directly from the data [1, 10]. The major advantage of this algorithm lies in its computational simplicity. However, the price paid for the simplicity is slow convergence. In order to obtain faster convergence, it is necessary to device more complex algorithm involving additional parameters. In particular, if the matrix is $N\times N$ and has eigen value $\lambda_0, \lambda_1, \ldots, \lambda_N$, we may use an algorithm that contains $N$ parameters one for each of the eigen value. In deriving faster convergence algorithm, the choice can be least squares approach or recursive least squares approach [10, 11]. In this approach we deal directly with the received data in minimizing the quadratic performance index, where as previously we minimized the expected value of the squared error. The challenge faced by user of adaptive filtering is first to understand the capabilities and limitations of various adaptive filter algorithms and secondly, to use this understanding in the selection of the appropriate algorithm for the application [2, 10, 11]. Adaptive filter, employing different algorithm are used for various application such as system identification, equalization, predictive coding, spectrum analysis, noise cancellation, seismology, electrocardiography etc.

The paper has been divided into V sections as follows, Section I deals with brief introduction. Structure of model is described in section II. LMS and RLS algorithm are described in section III. Section IV includes different assumptions made prior to simulation and result analysis after simulation. Concluding remarks are in section V. References are included at the end.

II. STRUCTURE OF THE MODEL

A model of communication system consisting of random integer generator, QAM modulator, Rician fading channel, gain control, equalizer and QAM demodulator is implemented using MATLAB block set as shown in figure 4. Simulation is being carried out, by varying signal to noise ratio and fading component gain of Rician fading channel for the algorithm RLS and LMS and the output is observed in the form of bit error rate (BER), number of errors and the number of bits processed [5].

![Fig. 4 Structure of the model](image)

**Random integer generator:** The Random Integer Generator block generates uniformly distributed random integers in the range $[0, M-1]$, where $M$ is the $M$-ary number. The $M$-ary number can be either a scalar or a vector. If it is a scalar, then all output random variables are independent and identically distributed. If the $M$-ary number is a vector, then its length must equal the length of the initial seed. If the initial seed parameter is a constant, then the returning noise is repeatable. The block generates scalar (1x1 2-D array), vector (1-D array), or matrix (2-D array) output, depending on the dimensionality of the constant value parameter and the setting of the interpret vector parameters as 1-D parameter. The output of the block has the same dimensions and elements as the constant value parameter.

**QAM modulator:** The Rectangular QAM Modulator modulates using $M$-ary Quadrature amplitude modulation with a constellation on a rectangular lattice. The parameter $M$ in $M$-ary must have the form $2^K$ for some positive integer $K$. The output is a baseband
representation of the modulated signal. Rician fading channel; The Rician Fading Channel block implements a baseband simulation of a Rician fading propagation channel. The input can be either a scalar or a frame-based column vector. Fading causes the signal to spread and become diffuse. The K factor parameter, which is part of the statistical description of the Rician distribution, represents the ratio between direct-path (un-spread) power and diffuse power. The ratio is expressed linearly, not in decibels. While the gain parameter controls the overall gain through the channel, the K factor parameter controls the gain's partition into direct and diffuse components [3, 9, 12-14].

III. DESCRIPTION OF THE ALGORITHM

In the communication system model implemented, two types of algorithm are used for the simulation purpose; they are Least Mean Square Algorithm and Recursive Least Squares Algorithm.

A. Least Mean Squares Algorithm

LMS filter is built around a transversal (i.e. tapped delay line) structure. Two practical features, simple to design, yet highly effective in performance have made it highly popular in various application. LMS filter employ, small step size statistical theory, which provides a fairly accurate description of the transient behavior. It also includes H* theory which provides the mathematical basis for the deterministic robustness of the LMS filters [1-3].

The LMS algorithm is a linear adaptive filtering algorithm, which in general, consists of two basics procedure a filtering process, which involve, computing the output of a linear filter in response to the input signal and generating an estimation error by comparing this output with a desired response and an adaptive process, which involves the automatics adjustment of the parameter of the filter in accordance with the estimation error. The combination of these two processes working together constitutes a feedback loop, as illustrated in figure 5. LMS algorithm is built around a transversal filter, which is responsible for performing the filtering process. A weight control mechanism responsible for performing the adaptive control process on the tape weight of the transversal filter [2, 3, 12-14]. LMS algorithm is summarized in appendix.

B. Recursive Least Squares Algorithm

The RLS filter overcomes some practical limitations of the LMS filter by providing faster rate of convergence and good performance.

In the RLS algorithm the method of least squares is extended to develop a recursive algorithm for the design of adaptive transversal filter as shown in figure 6. Given the least squares estimate of the tape weight vector of the filter at iteration (n-1), we compute the updated estimate of the vector at iteration n upon the arrival of new data. An important feature of this filter is that its rate of convergence is typically an order of magnitude faster than LMS filter, due to the fact that the RLS filter whitens the input data by using the inverse correlation matrix of the data, assumed to be zero mean [1, 2, 14-16]. The Improvement is achieved at the expense of an increase in computational complexity of the RLS filter. RLS algorithm is summarized in appendix.

IV. SIMULATION AND RESULT ANALYSIS

Simulation is carried out in two parts. First part dealing with RLS equalizer algorithm and second part dealing with LMS equalizer algorithm.

Assumption made for first and second part are as follows; in random integer block M-ary number = 4; in QAM block M = 4, min. distance = 2, phase offset (radian) = 0 and sample per symbol = 1; in Rician fading channel block, Specular component gain (dB) = [-10 -60], Fading component gain = variable, maximum Doppler shift (Hz) = 0.0001 per symbol period, SNR (dB) = Variable; in equalizer block adaptive algorithm = RLS or LMS, number of weights = 6, reference tap = 4.

Simulated output values in terms of BER and number of errors by varying values of SNR (dB) in Rician fading
channels are plotted in figure 7. Similarly by varying the fading component gain in Rician fading channel, the obtained BER and number of errors are plotted in figure 8.

that there is large variation of number of errors for a gain variation of 10dB to 20dB for both the algorithm. When the gain is varied from 20dB to 80dB there is small variation in number of errors (in the range of 10 to 15) for RLS algorithm, on the other hand for LMS algorithm the number of errors is large (in the range of 60 to 70).

It may be noted from figure 8, that BER is significantly low between 0.01059 to 0.001286 (a difference of 0.009304) and between 0.026 to 0.006431, (a difference of 0.019569) for RLS and LMS algorithm respectively, i.e. RLS adapted system dominates on LMS adopted system for variation of fading component gain from 0-1 to 0-7. When fading component gain is ranging from 0-7 to 0-9 a significant improvement in BER is observed in RLS algorithm in between 0.001641 to 0.001286, which is better than LMS, where the BER values are in between 0.005749 to 0.006431. Similarly a better performance in terms of Number of errors are observed from 807 to 98 for RLS and 1981 to 490 for RLS and LMS adapted system respectively by varying the value of fading component gain from 0-1 to 0-9. All above results indicated that RLS algorithm adapted communication system is better.

V. CONCLUSION

Inter-symbol interference caused due to channel induce distortion can be effectively overcome and a BER of low value 0.001286 can be obtained by adapting RLS algorithm at the receiver. An adaptive equalizer employing RLS equalizer is a better option over LMS equalizer, if performance in terms of BER and number of errors in a communication system having Rician fading channel is concerned. In contrast, RLS algorithm are model dependent also tracking behavior may be inferior, unless care is taken to minimize the mismatch between the mathematical model on which they are based and the underlying physical process responsible for generating the input data. Stochastic gradient algorithm such as the LMS algorithm are model independent and exhibit good tracking behavior.

APPENDIX

LMS algorithm may be summarized as follows, based upon wide-sense stationary stochastic signal [1-3].

Parameters: \( M = \) number of tapes; \( \mu = \) step size

\[
0 < M < \frac{2}{S_{\text{max}}} \tag{11}
\]

where \( S_{\text{max}} \) is the maximum value of the power spectral density of the tape input \( u(n) \) and filter length \( M \) is moderate to large.

Limitation: If prior knowledge of the tape weight vector \( \hat{\Omega}(n) \) is not available, set

\[
\hat{\Omega}(n) = 0 \tag{12}
\]

Data:
- Given \( u(n) = [u(n), u(n-1), \ldots, u(n-M+1)]^T \)
- \( d(n) = \) desired response at time \( n \)
To be computed ̂\(n+1)\) = estimate of tape weight vector at time \(n+1\)

**Computation:** For \(n = 0, 1, 2, 3, \ldots\) compute

Estimation of error

\[ e(n) = d(n) - y(n) \]  

(13)

\(y(n)\) filter output

Tape weight adaptation

\[ \hat{\omega}(n+1) = \hat{\omega}(n) + \mu u(n) e(n) \]  

(14)

RLS algorithm may be summarized as follows [1-2],

**Parameter:** Initial weight vector

\[ \hat{\omega}(0) = 0 \]  

(15)

Customary practice is to set \(\hat{\omega}_0 = 0\)

\[ P(0) = \sigma_0^2 I \]  

(16)

\(P\) is inverse correlation matrix and \(\sigma\) is regularization parameter; positive constant for high SNR and negative constant for low SNR

**Computation:** For each instant of time \(n=1, 2, 3\ldots\) compute

\[ \pi(n) = P(n-1)u(n) \]  

(17)

an intermediate quantity for computing \(k(n)\)

\[ k(n) = \frac{\pi(n)}{A + u(n)H(n)\pi(n)} \]  

(18)

time varying gain vector

\[ \zeta(n) = d(n) - H(n)\hat{\omega}(n-1)u(n) \]  

(19)

priori estimation error

\[ \hat{\omega}(n) = \hat{\omega}(n-1) + k(n)\zeta(n) \]  

(20)

tape weight vector and

\[ p(n) = \frac{1}{2} p(n-1)\hat{\omega}(n) u(n) \]  

(21)

\{M by M inverse correlation matrix\}

**Acknowledgment**

Author is thankful to the technical support provided by Electronics and Communication Engineering Department, MNIT, Jaipur, Rajasthan (India).

**References**


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