World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:1, No:12, 2007

Vibration Base Identification of Impact Force Using Genetic Algorithm

R. Hashemi and M.H.Kargarnovin

Abstract— This paper presents the identification of the impact force acting on a simply supported beam. The force identification is an inverse problem in which the measured response of the structure is used to determine the applied force. The identification problem is formulated as an optimization problem and the genetic algorithm is utilized to solve the optimization problem. The objective function is calculated on the difference between analytical and measured responses and the decision variables are the location and magnitude of the applied force. The results from simulation show the effectiveness of the approach and its robustness vs. the measurement noise and sensor location.

Keywords— Genetic Algorithm, Inverse problem, Optimization, Vibration.

I. INTRODUCTION

AN important type of inverse problems in structural mechanics is force determination from measured response of the structure to applied force. Indeed in many practical applications it is difficult and some times impossible to directly measure the dynamic force that are acting on a vibrating structure, hence it could be beneficial to compute the time history and location of applied loads indirectly, using structural response measurement together with a dynamic model of structure. In the field of engineering structures a number of force identification techniques have been already developed by various investigators.

Stevens has given an excellent survey of the literature on the force identification [1]. Also Chan et al. [2,3] and Zhu and Law [4] have presented a theoretical background of various moving force identification methods. An inverse weighted pseudo algorithm was presented by Parloo et al. [5,6] to estimate the forces that are acting on a structure starting from the measured response spectra. In this method force identification requires the inversion of the complete frequency response of the complete frequency response functions (FRF) matrices. Wang et al. [7,8] and Thite and Thompson [9] developed a prediction algorithm for unknown impact and harmonic forces. These models could estimate the force amplitude and its location simultaneously, but it is time-consuming and sensitive to location of measurement. Zhu and

Lu [10] presented a time domain method to identify both concentrated and distributed loads on beam and plate structures.

Flores et al. [11] presented an optimization-based inverse procedure for the determination of external loads applied to a given mechanical structure, by using information concerning the dynamic behavior of the system and its corresponding finite element model. The influence of the stress-stiffening effect on the dynamic characteristics of structural systems was used to establish a relation between the dynamic responses and the applied external forces. The identification procedure was illustrated by means of numerical simulations and experimental tests, in which a heuristic technique known as Life Cycle model was used.

Obata and Miyamori [12] investigated the dynamic response characteristics of pedestrian bridges and to develop a human walking force model to assist in the development and design of pedestrian bridges. Human walking force parameters were identified by a genetic algorithm (GA) from experimental forced vibration data. The results of the dynamic response obtained by the GA were in agreement with the experimental results. Therefore, they concluded that the GA system is useful in the identification of pedestrian walking forces, and from the experimental and calculated results, it was considered that the walking force model identified by the GA is substantially accurate.

In this paper, the identification of impact force acting on a simply supported beam is addressed. It is assumed that the acceleration response of the structure to impact force is known, and the location and magnitude of the unknown force are sought for. The analytical acceleration response of the beam is derived in time and frequency domains. The problem is then formulated as an optimization problem, in which the objective function is considered as the location and magnitude of the impact force. The genetic algorithm is then adopted to solve this optimization problem. Simulation studies are conducted to assess the proposed approach and the effect of measurement noise and sensor location is also investigated.

II. THEORETICAL BACKGROUND

Consider a simply supported beam with a span length L, constant flexural stiffness EI, constant mass per unit length ρ and viscous damping C. The effect of shear deformation and rotary inertia are not taken into account (Euler-Bernoulli beam). As shown in Fig. 1, if the ideal impact force is applied at $x = x_j$, the equation of motion can be expressed as [1]:

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$$o\frac{\partial^2 v(x,t)}{\partial t^2}+C\frac{\partial v(x,t)}{\partial t}+EI\frac{\partial^4 v(x,t)}{\partial x^4}=P_j\delta(t)\,\delta(x-x_j) \quad (1)$$

where $v(x,t)$ is the beam deflection at point x and time t and $\delta(t)$ is the Dirac delta function. Based on the modal superposition, if the nth mode shape function of the beam is $\phi_n(x)=\sin(n\pi x/L)$, the solution of equation (1) can be expressed as:

$$v(x,t)=\sum_{n=1}^{\infty}\sin(n\pi x/L)\,q_n(t) \quad (2)$$

where n is the mode number and $q_n(t)$ is the nth modal displacement. After substituting equation (2) into equation (1) and integrating the resultant equation with respect to x, between 0 and L, then using the boundary conditions and the properties of Dirac delta function, the equation of motion in terms of the modal displacement $q_n(t)$ can be expressed as:

$$\frac{\partial^2 q_n(t)}{\partial t^2}+2\xi_n\omega_n\frac{\partial q_n(t)}{\partial t}+\omega_n^2 q_n(t)=P_n(t) \quad (3)$$

where $\omega_n=\frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho}}$, $\xi_n=\frac{C}{2\rho\omega_n}$ and $P_n(t)=P_j\,\delta(t)\phi_n(x_j)$ are the nth modal frequency, the modal damping and the modal force, respectively. If the impact force is known, equation (3) can be solved for zero initial conditions to yield modal displacement as:

$$q_n(t)=\frac{1}{\omega_{d_k}}\int_0^t P_n(\tau)e^{-\xi_n\omega_n(t-\tau)}\sin\omega_{d_K}(t-\tau)\,d\tau,\ n=1,2,... \quad (4)$$

where $\omega_{d_K}=\omega_n\sqrt{1-\xi_n^2}$. The solution of equation (4) for $q_n(t)$ can be derived as:

$$q_n(t)=\frac{Pj\,\phi_n(x_j)}{\omega_{d_K}}e^{-\xi_n\omega_n t}\sin(\omega_{d_K}t) \quad (5)$$

By substituting equation (5) in equation (2), the dynamic deflection of the beam at point $x=x_i$, subjected to the impact force acting at $x=x_j$, can be obtained as:

$$v(x_i,t)=\sum_{n=1}^{\infty}\frac{\phi_n(x_i)\phi_n(x_j)P_j}{\omega_{d_n}}e^{-\xi_n\omega_n t}\sin(\omega_{d_n}t) \quad (6)$$

By twice differentiating the above equation with respect to time, the acceleration response of the beam can be derived as:

$$a(x_i,t)=a_i(t)=\sum_{n=1}^{\infty}\frac{\phi_n(x_i)\phi_n(x_j)P_j}{\omega_{d_n}}e^{-\xi_n\omega_n t}\times$$
$$\big((2\xi_n^2\omega_n^2-\omega_n^2)\sin(\omega_{d_n}t)-2\xi_n\omega_n\omega_{d_n}\cos(\omega_{d_n}t)\big) \quad (7)$$

Performing the Fourier transform on equation (7), the acceleration of beam can be obtained in frequency domain as:

$$A(x_i,\omega)=A_i(t)=$$
$$\sum_{n=1}^{\infty}\frac{\phi_n(x_i)\phi_n(x_j)P_j}{(\omega_n^2-\omega^2)+i(2\xi_n\omega_n\omega)}\times(-\omega_n^2-i2\xi_n\omega_n\omega) \quad (8)$$

III. FORMULATION OF FORCE IDENTIFICATION AS AN OPTIMIZATION PROBLEM

The problem of force identification can be regarded as an inverse problem, and inverse problem solving methods should be more efficient to such a problem.

The basic idea of inverse problem solving is to determine the system inputs if the system outputs are known. On the other hand, in the direct problem the aim is just to seek or predict the system outputs from the given inputs. The successful resolution of direct problems consists of building a simulated system able to accurately predict the experimental outputs from the same experimental inputs. Such simulation is successful if the error between experimental outputs and numerical results is acceptably small.
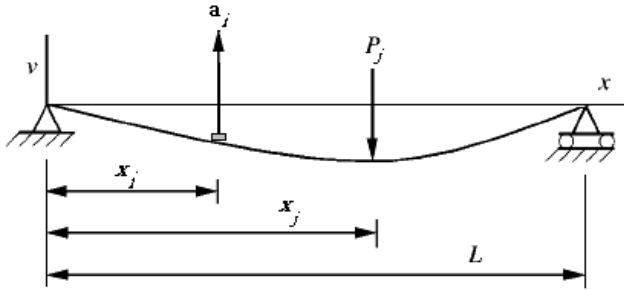
The general idea of the inverse problem solving can be depicted as following. First step is doing numerical simulation for the considered system. Then, use an optimization method to minimize the difference between the results of the numerical simulation and the actual outputs of the process.

In the case of force identification problem, the direct problem is to obtain the dynamic response of the structure given the exact location and magnitude of the force. However, in the inverse problem, it is assumed that the dynamic response of the structure to the force is known and the location and magnitude of the force are sought for.

Consider the proportionally damped simply supported beam subject to an unknown impact force at $x=x_j$ as shown in Figure 1. The system modal parameters (natural frequencies and mode shapes) are assumed to be known from modal analysis. The beam acceleration response at $x=x_i$ to the impact force can be measured by the accelerometer and denoted as $\hat{a}_i(t)$. This response is consistent with the theoretically estimated acceleration response for a specific force location and magnitude, as shown in equation (7) and denoted as $a_i(t)$. So, the problem is to update the parameters of the theoretical model such that the difference between the theoretical and measured responses is minimized. This parameter updating problem can be formulated as an optimization problem as follows:

$$\underset{\theta\in\Theta}{\text{Minimize}}\,J_t(\theta)=\sum_{r=1}^{N_t}\big|a_i(t_r)-\hat{a}_i(t_r)\big| \quad (9)$$

wherein θ is a solution to the problem, which is a vector containing the decision variables, Θ is the solution space defining the lower and upper bounds of the decision variables, and N_t is the considered number of time points.

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P_j : Unknown impact force, $\mathbf{a}_i$: Measured acceleration

Figure 1 Impact force on a simple supported beam.

The optimization problem can also be formulated in frequency domain as:

$$\underset{\theta \in \Theta}{\text{Minimize}} J_\omega(\theta) = \sum_{r=1}^{N_\omega} \left| A_i(\omega_r) - \hat{A}_i(\omega_r) \right| \qquad (10)$$

in which, $A_i(\omega)$ and $\hat{A}_i(\omega)$ are the theoretical and experimental frequency response of the beam, respectively, and N_ω is the considered number of frequency points.

An important characteristic of this optimization problem is that the objective function has a large number of local optimums; therefore, gradient-based optimization methods may not converge to a global solution. In this case, gradient-free optimization algorithms, such as Genetic Algorithm, can be used.

A. Implementation of genetic algorithm

The Genetic Algorithm (GA) primarily formulated by Holland [13], is a probabilistic global search and optimization method that mimics the metaphor of natural biological evolution. GA operates on a population of individuals (potential solutions), each of which is an encoded string (chromosome), containing the decision variables (genes).

The structure of a GA is composed by an iterative procedure through the following five main steps:

- Creating an initial population (G_0).
- Evaluation of the performance of each individual or chromosome (c_k) of the population, by means of a fitness function to be maximized.
- Selection of individuals for reproduction of a new population.
- Application of genetic operators: Crossover and Mutation.
- Iteration of steps 2 to 4 until a termination criterion is fulfilled.

B. Problem encoding and initialization

The definition and the number of the decision variables are critical for the optimization process. All variables are encoded into a chromosome. Traditionally, GA uses binary strings as chromosome representation. In our problem, the candidate solution is the location and magnitude of the impact force. The location of the force is determined by the associated element number (j), which is a discrete variable and the magnitude of

the force is represented by P. These variables are coded in a chromosome using a binary coding scheme as:

$$\theta = (j, P) \qquad (11)$$

To start the algorithm, an initial population of individuals (chromosomes) is defined. We configure the GA, so that it creates a fixed number of initial individuals at random from the whole solution space. An important parameter in initialization is the population size. In general, the population size affects both the ultimate performance and the efficiency of GA.

C. Fitness function

To apply GA, a fitness function is required in order to evaluate the status of each solution and improve the solution. The fitness value is associated with each individual, expressing the performance of the related solution with respect to a fixed objective function to be minimized. The fitness function is defined as:

$$F(x) = \frac{b_1}{b_2 + J(x)} \qquad (12)$$

where F is the fitness value and b_1 and b_2 are some constants which are used to prevent division by zero and to make the fitness value to be in a wide range. If we plot the fitness values with respect to all possible force locations and magnitudes, a surface will be obtained. For instance, Figure 2 shows the fitness function for a sample case, in which a 50 N force is acting on the 20th element of the simply supported beam of Figure 1, and the accelerometer is placed on the 10th element. As it is clear from this figure, the fitness function has a lot of local maxima, implying the need to use a gradient-free optimization algorithm.
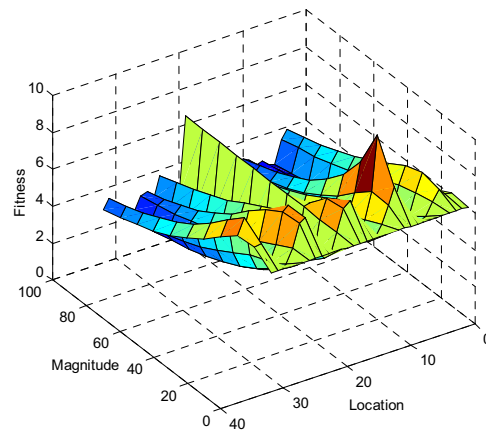

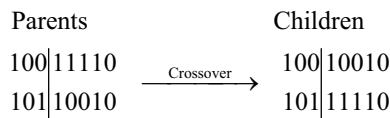*Figure 2 Fitness function for a sample case*

D. Genetic operators

Following the evaluation of the fitness of all chromosomes in the population, the genetic operators are applied to produce a new population. During this process a number of genetic operators are used. The most important genetic operators are selection, crossover, and mutation which are briefly described here.

Selection is the mechanism for selecting the individuals with high fitness over low fitted ones to produce the new

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individuals for the next population. The selection function adopted here is the roulette wheel method in which the probability to choose a certain individual is proportional to its fitness, as follows [13]:

$$\text{Prob}\big[c_i \text{ is selected}\big] = \frac{F(c_i)}{\sum F(c_k)} \qquad (13)$$

Crossover is the method of merging the genetic information of two individuals (parents) to produce the new individuals (children). In the simplest case, this process is realized by cutting two chromosomes at a randomly chosen position, with a probability of p_c and swapping the two tales, as is visualized below [14]:

Parents   Children

$$100|11110 \xrightarrow{\text{Crossover}} 100|10010$$
$$101|10010 \qquad\qquad 101|11110$$

The parameter p_c is called the crossover rate and controls the rate at which solutions are subjected to crossover. As p_c increases, however, solution can be disputed faster than selection can exploit them. Typical values of p_c are in the range of 0.5–1.0.

Mutation is a probabilistic random deformation of the genetic information for an individual. The positive effect of mutation is the preservation of genetic diversity. This process can be handled by altering each bit randomly with a small probability, p_m, as depicted below [14]:

For parameter p_m, which is called mutation rate, large values will transform the GA into a purely random search algorithm. However, too small values will cause the premature convergence of GA to suboptimal solutions. Typically, p_m is chosen in the range of 0.005–0.1[14].

mutated bit
↓
$$10011110 \xrightarrow{\text{Mutation}} 10011010$$
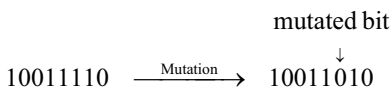
The process of reproduction continues until a termination criterion is fulfilled. In this work, the termination criterion is the maximum number of generations; that is, the reproduction continues until the number of generations reaches to a specified maximum limit. Figure 3 shows the flowchart of the optimization process via GA.

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IV. SIMULATION AND RESULTS

To evaluate the performance of the proposed GA-based force identification method, we consider the simply supported steel beam, as shown in Figure 1. The material properties and dimensions of the beam are summarized in Table 1.

The step size and range of variations for each decision variable are illustrated in Table 2. The beam is equally divided into 64 elements and the magnitude of the force is assumed not to exceed 127 N. Therefore, the solution space contains 64×128 points and the chromosome is to be formed by a 15-bit binary string representing θ with 7-bit resolution for j and 8-bit for P.
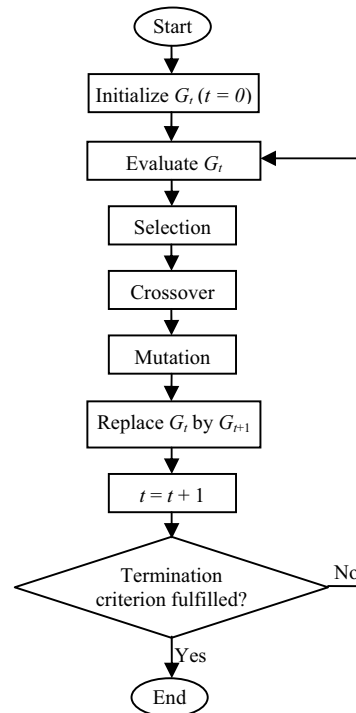

Figure 3 Flow chart of operation of GA.

As discussed before, there are some important parameters for GA. In this work, these parameters are determined through a series of experiments and algorithm performance comparison, and are listed in Table 3. From this table, the total number of potential solutions that are evaluated for a specific case is 30×30 points from the whole solution space which contains 64×128 points; that is, the total number of function evaluations is about 10% of that of a direct search approach. This means that the proposed approach is computationally efficient.

Table 1
Material properties and dimensions of the beam

Parameter	Value
Modulus of elasticity (E)	$207 \times 10^9 \ Pa$
Density	$7870 \ kg/m^3$
Beam length	$0.64 \ m$
Beam width	$0.04 \ m$
Beam height	$0.005 \ m$

Table 2
Step size and range of variations for decision variables

Variable	Lower bound	Upper bound	Step size
Location: j	1	64	1
Magnitude: P (N)	0	127	1

The first 6 modes are used here to calculate the theoretical acceleration response, while the first 20 modes are adopted to generate the experimentally measured acceleration response. The sampling frequency is 512Hz, the sensor location is

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$i = 20$, and it is assumed (in this stage) that there is no noise in the measured data.

Table 3
Parameters of GA

Parameter	Value
Population size	30
Number of generations	30
Crossover rate: p_c	0.8
Mutation rate: p_m	0.09

As mentioned before, the initial population is randomly selected from the whole solution space. In order to investigate the effect of the initial population, a particular case of impact force is considered in which $j = 25$ and $P = 15\,N$, and GA is run five times with five different random initial population. The fitness curves, which are the changes of the best fitness values during the optimization, for the five runs are depicted in Figure 4. Furthermore, the changes in the value of decision variables (location and magnitude of the force) during the optimization process are shown in Figures 5 and 6. It is clear from these figures that the effect of the initial population is trivial and there is only one incorrect identification, in which the location is correctly identified and the incorrect magnitude is very close to the actual value.

Here, we consider five different cases of impact force whose locations and magnitudes are randomly selected from the whole solution space, and apply the proposed method to determine the location and magnitude of the force. The detailed information of these five cases is summarized in Table 4. Figure 7 shows the fitness curves for these five cases. Furthermore, the changes in the value of decision variables during the optimization process are depicted in Figures 8 and 9.
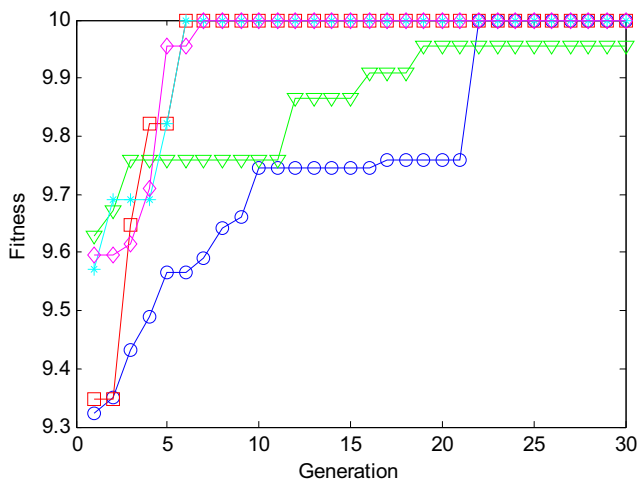

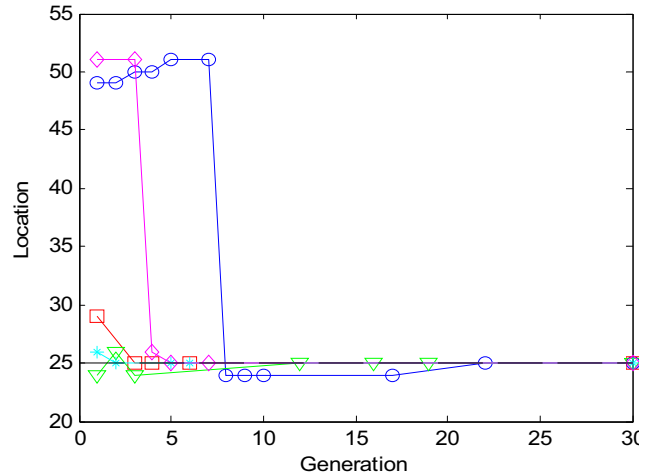Figure 4 Trace of fitness value for five runs with different initialization

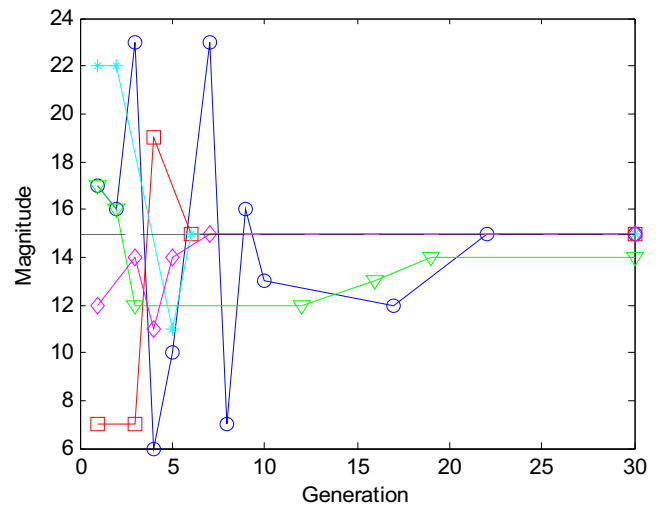Figure 5 Trace of first decision variable for five runs

Figure 6 Trace of second decision variable for five runs

Table 4
Three different cases of damage

	Location: j	Magnitude: P (N)
Case 1	9	117
Case 2	20	47
Case 3	32	39
Case 4	46	97
Case 5	58	18

A. The effect of measurement noise:

In the force identification problems, the identifiablity of the force highly depends on the accuracy of measured data. The force can be identified by the dynamic response of the structure. However, the response is sensitive to the measurement of the noise, and when measured data are corrupted with random noise, the force is not accurately identifiable by measured data.

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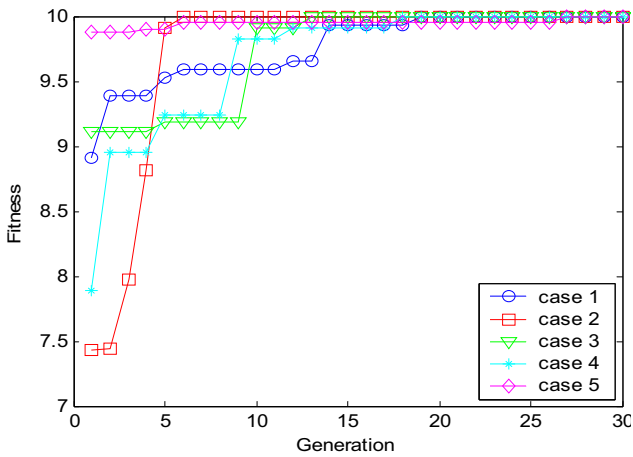Vol:1, No:12, 2007

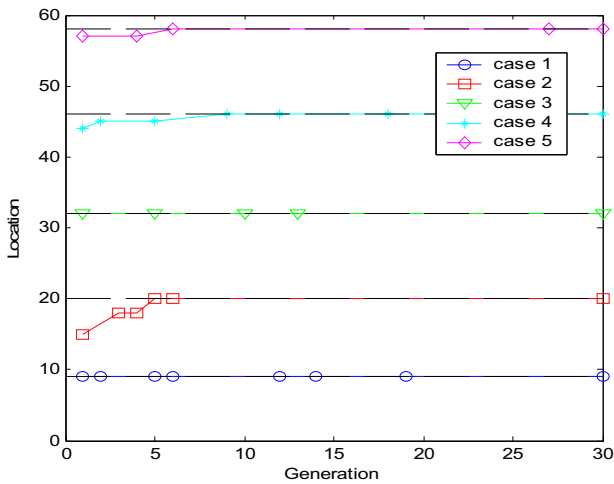Figure 7 Trace of fitness for five study cases

Figure 8 Trace of first decision variable for five study cases
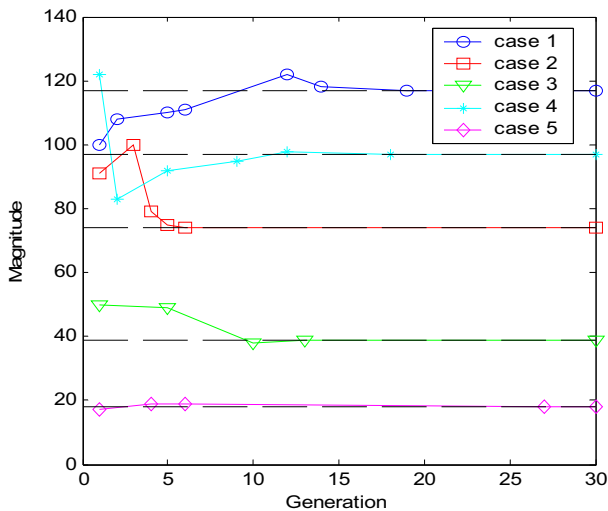

Figure 9 Trace of second decision variable for five study cases

In this work, in order to investigate the effect of measured noise, zero mean white noise is added to the measured acceleration response, as follows:

$$a_i^{noisy} = a_i\,(1 + \alpha s) \qquad (14)$$

where s is a random number between -1 and 1 from a normal distribution and α is the noise level in the data. The results for two different levels of noise are summarized in Table 5. As it is seen from this table, the robustness of the approach to measured noise is good.

Table 5
Three different cases of damage

Sample	$\alpha = 5\%$	$\alpha = 10\%$
$(j, P)_{actual}$	$(j, P)_{prediction}$	$(j, P)_{prediction}$
(58,18)	(58,18)	(58,19)
(46,97)	(46,97)	(46,96)
(32,39)	(33,38)	(32,38)
(20,47)	(20,47)	(20,48)
(9,117)	(8,105)	(9,113)

B. The effect of sensor location:

Another factor that affects the performance of the force identification algorithms is the location of sensors for measurement. The measured data may not be adequate for the force identification because certain measured modes may be less sensitive to the force. The results for different sensor locations are depicted in Figures 10-12. Close examination of these figures reveals that the proposed method is not sensitive to the sensor location.
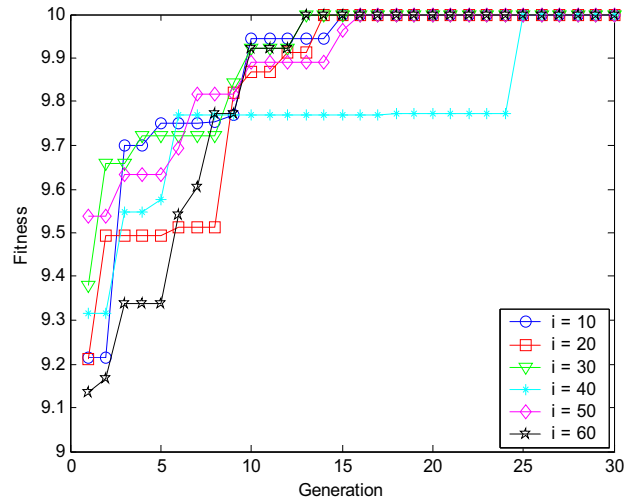

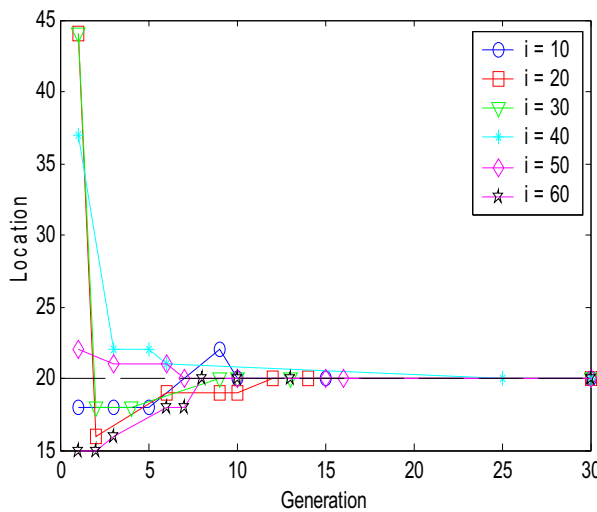Figure 10 Trace of fitness for different sensor locations

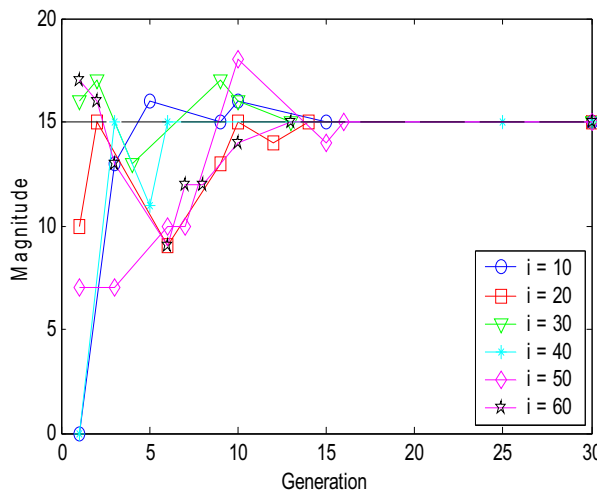Figure 11 Trace of first decision variable for different sensor locations

Figure 12 Trace of second decision variable for different sensor locations

V. CONCLUSION

The identification of impact force acting on a simply supported beam is addressed. The problem is formulated as an optimization problem and GA method is used to solve it. The proposed approach is applicable both in time and frequency domains. The simulation results show that the approach is effective and the sensitivity of the approach to the randomly selected initial population is low. The effect of measurement noise and sensor location is also investigated and the results reveal that the proposed method is adequately robust to the measured noise and is not sensitive to the location of the sensor.

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