The Investigation of Crack's Parameters on the V-Notch using Photoelasticity Method

M. Saravani., M. Azizi

Abstract—The V-notches are most possible case for initiation of cracks in parts. The specifications of cracks on the tip of the notch will be influenced via opening angle, tip radius and depth of V-notch. In this study, the effects of V-notch's opening angle on stress intensity factor and T-stress of crack on the notch has been investigated. The experiment has been done in different opening angles and various crack length in mode (I) loading using Photoelasticity method. The results illustrate that while angle increases in constant crack's length, SIF and T-stress will decrease. Beside, the effect of V-notch angle in short crack is more than long crack. These V-notch affects are negligible by increasing the length of crack, and the crack's behavior can be considered as a single-edge crack specimen.

Finally, the results have been evaluated with numerical finite element analysis and good agreement was obvious.

Keywords—photoelasticity, Stress intensity factor, T-stress, V-notch

I. INTRODUCTION

IN In order to decrease distortions in engineering structures, improving our knowledge about crack's formation at V-notch tip seems unavoidable. V-notches usually act as a singular stress concentrator, which has a direct relation with notch geometrical configuration like notch angle, the radius of notch tip. In reality, this kind of stress singularity easily initiates fracture and crack propagation. Therefore, it is important to study the stress singularity and fracture character at the V-notches tip.

In most recent research, investigation of stress intensity coefficient (K_I, K_{II}) has been done as well as determination of constant stress parameter (T). There are many researches that tried to presents theoretical solution for determination of K_I, K_{II} and T in body under various boundary conditions, but often these bodies have simple geometry [1], [2]. If the body isn't simple structure, the experimental methods (i.e. strain gage, photoelasitcty, Moire and Caustic) can be used to determination of stress intensity factors (K_I, K_{II}) and constant stress (T).

The first study of V-notch problem for linear elastic bodies was done by Williams [3]. He described the singular stress field using eigenfunction approach. T-stress solution for two and three dimensional cracked geometries was analytically investigated by Sherry and France. [1]. In comparison to many researchers that studied on stress intensity coefficient and second parameter in simple geometries, there are a few works which have tried to investigate stress singularity at V-notch. Mahinfallah and Zackery (1995) experimentally developed an algorithm for determining Mode I and II stress intensity factors of reentrant corners using Williams stress's equations in their investigation [4]. Kondo, Kobayashi and Sekine (2001) determined the stress intensities of sharp-notched strips using strain gage [5]. But some inherent shortcomings in practical application were observed, such as the limitations of birefraction material for photoelasticity. Leguillon and Yosibash (2003) investigated crack onset at a V-notch and influence of the notch tip radius in this process [6] .Yao and Yeh. (2005) studied local deformation field and fracture characterization of Mode I V-notch tip using coherent gradient sensing (CGS) [7]. All mentioned works investigated influence of V-notch geometry (i.e. notch angle, tip radius) in crack onset, so it seems necessary to research on fracture parameters at V-notch after crack creation. Wang, Lewis and Bell (2005) presented the method of estimating T-stress for both short and long cracks at the notches [8]. This method is used to predict T-stress solution for cracks emanating from a U-shaped edge notch in a finite thickness plate.

In this paper, Photoelasticity method is used to study the stress singularity and fracture behavior of cracks at V-notch tip. Experimental and numerical simulation for different notch angle and different cracks length are performed. The stress intensity factor and T-stress of Cracks at V-notch tip are obtained in Mode I using over deterministic method. The experimental and numerical results were compared and good agreement was obvious.

II. APPLICATION OF PHOTOELASTIC METHOD IN FRACTURE ANALYSIS

Williams [3] illustrated that the stresses for symmetric and antisymmetric fields (Mode I and Mode II) can be consider as an eigen series expansion. The linear elastic stresses around the tip of a crack (where $r \to 0$) the higher order terms of the series expansion can be negligible and stresses, for Mode I, can be written as the following equations:

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$$\sigma_x = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n-2}{2}} \left\{ C_{1n} \left\{ \left(2 + (-1)^n + \frac{n}{2}\right) \cos\left(\frac{n}{2}-1\right)\theta - \left(\frac{n}{2}-1\right) \cos\left(\frac{n}{2}-3\right)\theta \right\} \right\}$$

$$\sigma_y = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n-2}{2}} \left\{ C_{1n} \left\{ \left(2 - (-1)^n - \frac{n}{2}\right) \cos\left(\frac{n}{2}-1\right)\theta + \left(\frac{n}{2}-1\right) \cos\left(\frac{n}{2}-3\right)\theta \right\} \right\}$$

$$\tau_{xy} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n-2}{2}} \left\{ C_{1n} \left\{ -\left((-1)^n + \frac{n}{2}\right) \sin\left(\frac{n}{2}-1\right)\theta + \left(\frac{n}{2}-1\right) \sin\left(\frac{n}{2}-3\right)\theta \right\} \right\} \quad (1)$$

And $C_{11} \equiv \dfrac{K_I}{\sqrt{2\pi}}, -4C_{12} \equiv T$

Where K_I is the stress intensity factor of Mode I and r, θ, x, y are co-ordinates in conventional polar and Cartesian systems. T is a constant stress parallel to the crack due to a symmetric component of loading.

One of the most popular methods in stress-strain experiments is Photoelasticity method. In this method, thick plate of polymer photoelastic material was loaded uniformly intension, and isochromatic fringe pattern were stored using image analysis system. The government for two dimensional models is:

$$\tau_m = \frac{Nf_\sigma}{2h} = \frac{\sigma_1 - \sigma_2}{2} \quad (2)$$

Where, τ_m, f_σ, N and h are maximum shear stress, material stress fringe constant, fringe order and photoelastic model thickness respectively. After substitution of N from experiment and using f_σ, the shear stress can be determined for each point and stress intensity coefficients can be extracted using 1 and solving system of equations.

There are many efforts to increase results accuracy including K_I and T. All of them use the information of one, two or three points (fringe order N and position r, θ), therefore, particular determination of stress intensity coefficients face to many difficulties [9], [10], [11]. Sanford and Dally presented a general method to determine K_I and T based on over deterministic analyze using information of many points near to crack tip [12]. According to the over deterministic method, 2 can be rewritten to the following form:

$$g_k(C_{1n}) = \left(\frac{\sigma_y - \sigma_x}{2}\right)_k^2 + (\tau_{xy})_k^2 - \left(\frac{N_k f_\sigma}{2h}\right)^2 = 0 \quad (3)$$

Where the subscript k indicates the value of g evaluated at the point (r_k, θ_k) with a fringe order N_k. By substituting 1 in to 3 to calculate the coefficients, one initially makes and estimates of the coefficients and computes g_k only to find $g_k \neq 0$. The initial estimates of C_{1n} are in error and need to be corrected. The correction process involves an iterative equation. Based on a Taylor series expansion of g_k. In order to achieve to minimum of the errors of K_I and T parameter the first six terms of 1 were considered [13], [14]. In matrix notation 3 becomes:

$$[g] = [C] \cdot [\Delta C] \quad (4)$$
Where,

$$[g] = \begin{bmatrix} -g_1 \\ \vdots \\ -g_k \end{bmatrix}, [C] = \begin{bmatrix} \frac{\partial g_1}{\partial c_{11}} & \cdots & \frac{\partial g_1}{\partial c_{16}} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_k}{\partial c_{11}} & \cdots & \frac{\partial g_k}{\partial c_{16}} \end{bmatrix}; [\Delta C] = \begin{bmatrix} \Delta C_{11} \\ \vdots \\ \Delta C_{16} \end{bmatrix} \quad (5)$$

The over determined system, given in 4, is solved in least-squares minimization process. Thus

$$[\Delta C] = \left([C]^T [C]\right)^{-1} [C]^T [g] \quad (6)$$

The solution of 6 yields correction to be applied to the previous estimates of C_{1n}.

All of the experimental methods to determine stress intensity factors employ data taken from the near field region in Fig.1. Data taken from the very near field do not yield accurate results, because of the stress field is three-dimensional (neither plane stress nor plane strain) in this region [15]. Also, if the data points are located very close to the crack tip errors in measuring r, θ are often excessive. Since high number of terms was considered to represent the stress field, data from the far field region can be useful too.
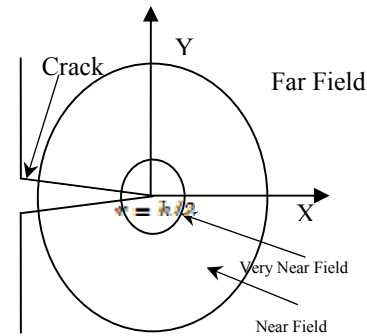

Fig. 1 Concept of three regions near a crack tip

III. EXPERIMENTAL PROCEDURE

Four specimen of Polycarbonate plate (Lexan) with 3mm in thickness and notch angle (α) 30°, 60°, 90° and 120° were used for experiment (Fig.2). The mechanical properties of Polycarbonate are given in Table I [16].
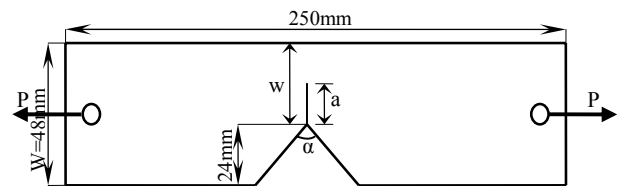

Fig. 2 Geometry of V-notch photoelastic model

According to 2, f_σ is the correlation coefficient between optical properties of material and created stress in model. Since photoelastic material properties are changed during the time and these properties can be affected by construction conditions directly, the material stress fringe value should be determined for each experimental material. There are some usual methods to measure the material stress fringe value [17]. In our study, the novel method is presented to estimate this

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property. An arbitrary geometry was loaded in polariscope to take the photograph of the fringe patterns. The principal stresses of model (σ_1, σ_2) can be calculated for each point using finite element method. By setting the principal stresses and an initial value of f_σ in 2, the fringe order N_k can be evaluated. By correlating N and the fringe order of photoelastic image, f_σ can be modified.

TABLE I
MATERIAL PROPERTIES OF POLYCARBONATE

Stress Fringe Value (kN/m)	Poisson's Ratio	Module of Elasticity (MPa)
7	0.38	2480

Therefore, single edge crack (SEC) specimen (Fig.3) was investigated and the principal stresses for each point near tip of crack was extracted using finite element model. Principal stresses and every node coordinate were entered to the computer program to calculate and compare the fringe order of every node with experimental data. The regeneration fringes for 6.5, 6.8 and 7 kN/m Stress fringe values were compared with the experimental fringe pattern.
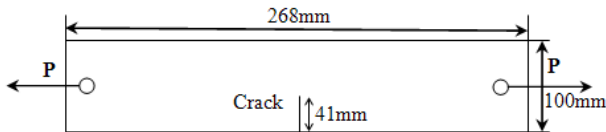

Fig. 3 Single edge crack specimen

One of these comparisons was presented in Fig.4. As it can be seen the best correlation of the stress fringe value was obtained at $f_\sigma = 7kN/m$ which is recommended in [16] and [18]. On the contrary of the usual methods there isn't any geometrical limitation in our procedure and using many data points in our evaluation decreased errors.

To simulation the crack, it was produced using special saw with 0.3mm in thickness and the crack tip was sharply introduced by a fine blade. The plates were loaded in tension in screw-type universal testing machine, and the fringed were displayed in a circular polariscope. The fringe patterns formed about the crack tips were recorded through a monochromatic filter using color digital camera. The fringe patterns were recorded with the both polariscope set of dark and light fields. The isochromatic fringe patterns for different notch angle have been depicted in Fig.5.
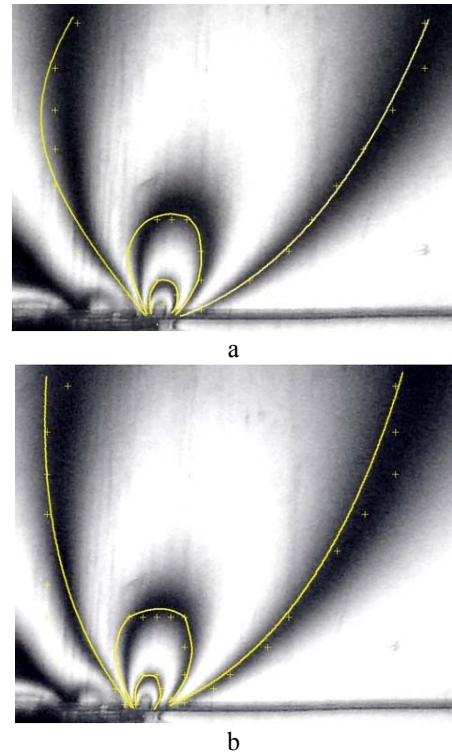

a

b

Fig. 4 The comparison of the original isochromatic fringe pattern with a computer regenerated fringe for a) $f_\sigma = 7kN/m$ and

b) $f_\sigma = 6.8kN/m$

The experiment was repeated for 0.5, 1.0, 1.5, 2.0, 3.0 and 5.0mm of length of cracks.

The symmetry of fringe pattern about crack's axis show the pure Mode I loading. The isochromatic fringe patterns were post-processed with computer code has been written in MATLAB software. This code determined the location and the number of the fringe order. The coordinates and the fringe order (r, θ and N) were established for about 80 data points for each photoelastic model. Data was taken from the near field and between the near and far field region. The full set of data points (r, θ and N) was used as input to MATLAB code has been written based on over deterministic method (6). Although the use of photoelastic data leads to a nonlinear relation among the coefficient of the series expansion for the stresses and the fringe order, the over deterministic solution converged rapidly. The convergence rate is strongly dependent to the number of data points and the accuracy of selecting them. Convergence was excellent achieving differences between iteration of the order of 10^{-4} in about twenty iterations and less

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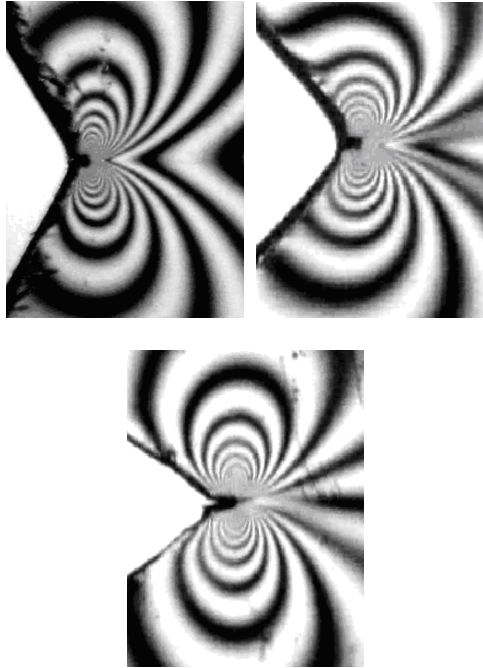Vol:5, No:1, 2011

Fig. 2 Isochromatic fringe patterns in different notch angles

To evaluate numerical computer code, a single edge crack photoelastic model (Fig.3) was considered and the results of computer code were compared to analytic solution for this geometry and a good agreement was observed. This comparison has been shown in Table II.

TABLE II
THE THEORETICAL AND EXPERIMENTAL RESULTS OF SEC

	Theory	Experimental	Error %
$K_I(MPa.m^{1/2})$	0.7348	0.6827	7%
$T(MPa)$	-0.5835	-0.5565	4.6%

IV. RESULTS AND DISCUSSION

The results obtained from four different photoelastic models with various lengths of cracks are presented in Fig.6 and7. To have a better investigation, these diagrams are normalized.
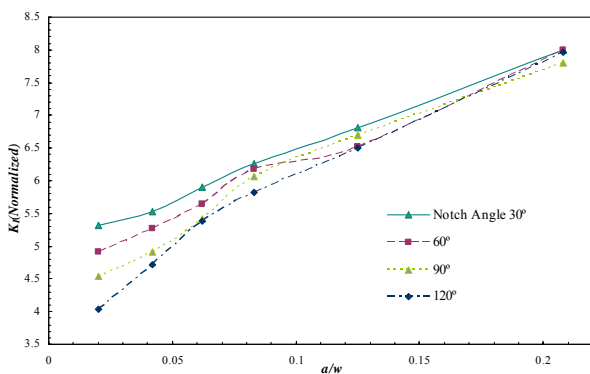

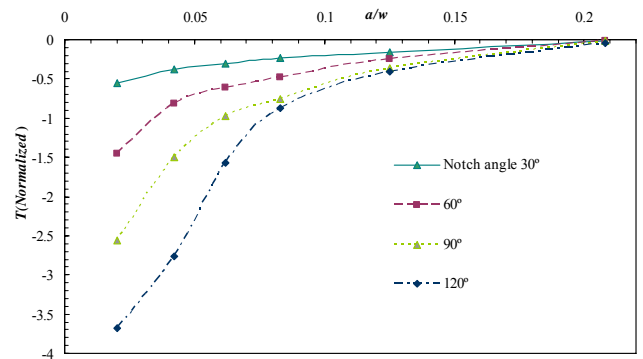Fig. 3 The variation of normalized K_I versus a/w in different Notch angles from Photoelastic

Fig. 4 The variation of normalized T versus a/w in different Notch angles from photoelastic

Where

$$K_I(Normalized) = \frac{K_I}{\sigma_\infty \sqrt{w}}, \qquad (7)$$

$$T(Normalized) = \frac{T}{\sigma_\infty}, \qquad (8)$$

a is the crack length, w is the specimen width(Fig.2) and σ_∞ is the constant stress far from the plate.

According to Fig.6 increasing crack's length in a specimen has been caused increasing of stress intensity factor, and increasing of notch-angle in constant crack's length decrease stress intensity factor. This result can be justified by decreasing the singularity effect in the root of notch. Increasing crack's length in a specimen cause to increasing T-stress, also increasing notch-angle, in a constant crack's length, T-stress reduces (Fig.7). As it can be seen, increasing crack's lengths leads to the decline the notch angle affects on the crack parameters.

Finite element analysis was performed to compare the experimental results. Since the photoelastic model was symmetrically loaded Mode I, the finite element model was considered semi symmetric. Finite element model of plate specimen was illustrated in Fig.8. Because of there isn't any geometry limitation to produce, crack in FEM the shorter crack (0.1 and 0.3mm in length) was additionally investigated in the simulation. The model includes six node triangular plane stress elements. According to the Ayatollahi, Pavier and Smith work, SIF and T-stress was calculated from FE results for the same experimental geometry and the same tension load. [18]
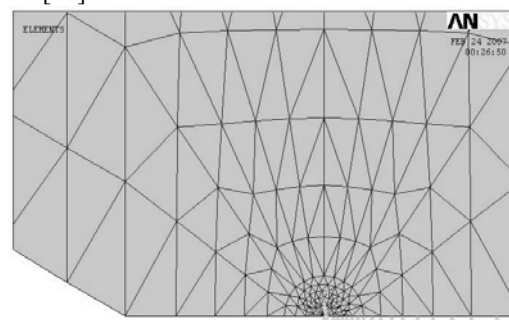

Fig. 5 FEM Mesh

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The normalized stress intensity factor and T-stress in different crack's length for different opening notch angle have been shown in Fig.9 and 10 that results are in agreement with experimental results.
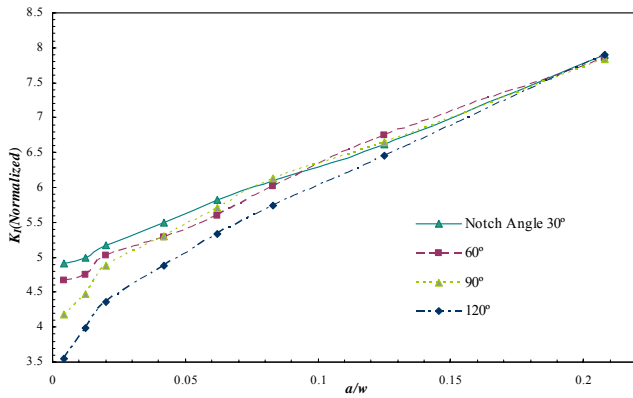

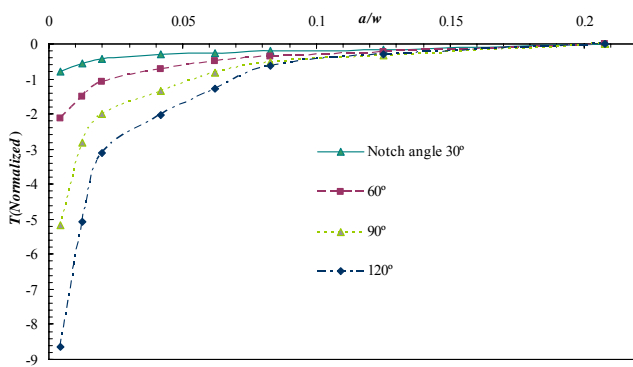Fig. 6 The variation of normalized K_I versus a/w in different Notch angles from FEM

Fig. 7 The variation of normalized T versus a/w in different Notch angles from FEM

The observed difference between experimental and FEM results is arisen of the finite element ideal modeling. While in the experimental specimen existence of V-notch tip radius is unavoidable.

Both experimental and FEM results illustrate that the increasing of the cracks length causes to reduction of the notch angle affects, insofar as, for the crack's length more than 0.125 of specimen width it will be negligible.

To have a better recognition the experimental results were compared to the results of single edge crack (notch angle $\alpha = 0$) as the length of SEC is equal to the summation of depth of notch and the experimental crack length.
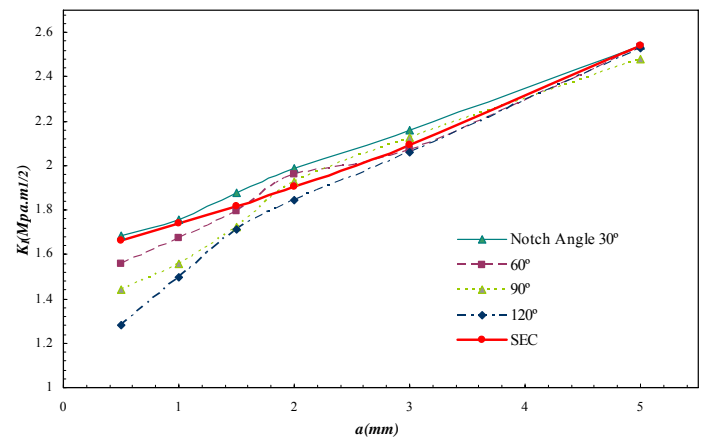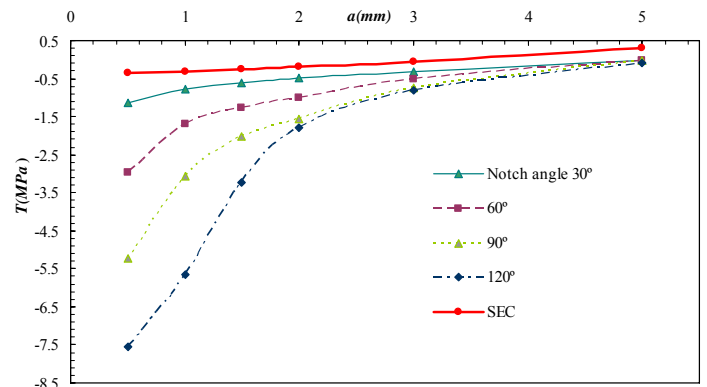

Fig. 8 The comparison of V-notch's experimental results specimen and SEC

To calculation of the results of SEC, theoretical method has been used [1], [19].

In V-notch specimen including crack, when the crack's length more than 0.125w , the behavior of crack's growth is similar to single edge crack specimen.

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