

# Design of angular estimator of inertial sensor using the least square method

Ji Hoon Kim, Hyung Gi Min, Jae Dong Cho, Jae Hoon Jang, Sung-Ha Kwon, Eun Tae Jeung

**Abstract**— Since MEMS gyro sensors measure not angle of rotation but angular rate, an estimator is designed to estimate the angles in many applications. Gyro and accelerometer are used to improve estimating accuracy of the angle. This paper presents a method of finding filter coefficients of the well-known estimator which is to get rotation angles from gyro and accelerometer data. In order to verify the performance of our method, the estimated angle is compared with the encoder output in a rotary pendulum system.

**Keywords**— gyro, accelerometer, estimator, least square.

## I. INTRODUCTION

RECENTLY, research related to small unmanned vehicles is attracting tremendous interest in military and aquanautics field that work in the relatively limited working space or exploration of dangerous area [1], [2]. The research for the attitude estimation system using the low priced small MEMS inertial sensor to be mounted to the small vehicles is being proceeded actively [3]. The role of gyro sensor is very important in the attitude estimation system.

However, since the gyro has drift errors caused by unstable characteristics of bias or scale factor [4], it is not easy to get the satisfactory results. Accordingly, the research to estimate the attitude by using the other sensor as compensating the gyro sensor has been proceeded. One of them is the blending the accelerometer with the gyro [5]. This conventional estimator is using the cut-off frequency method, but it is not expected satisfactory performance in all range of operating conditions.

This paper presents a method using the linear least square to get estimator coefficients. And we verify the performance of the estimator presented in this paper by applying it to the well-known pendulum system. This paper is organized as follows: Chapter 2 reviews the method to design the attitude estimation filter using the gyro and the accelerometer sensors

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and presents method to design the estimator coefficients using the least square method. We present the experiment process to get estimator coefficients using the rotary pendulum system in chapter 3 and verify the result and conclude in Chapter 5.

## II. THE ATTITUDE ESTIMATION COMPENSATOR USING THE GYRO AND THE ACCELEROMETER SENSORS

This chapter reviews the conventional attitude estimator [5] blending the accelerometer with the gyro sensor in Fig. 1. And we present the method to find compensator coefficients  $K_p$  and  $K_i$  using the least square method.

### A. Conventional attitude estimator using blending gyro and accelerometer

One of estimators to compensate gyro is the accelerometer aided mixing algorithm in Fig. 1. The role of the estimator is to compare attitude angles from the integration of the gyro with the attitude angle products from the accelerometers.

The result angle  $\theta_f$  of estimator is expressed in Laplace form as follows:

$$\theta_f = \frac{1}{s} \left[ \dot{\theta}_g - \left( K_p + \frac{1}{s} K_i \right) (\theta_f - \theta_c) \right] \quad (1)$$

where  $\dot{\theta}_g$  is the angular rate from the gyro, and  $\theta_c$  is the angle from the accelerometer,  $K_p$  and  $K_i$  are the proportional gain and integral gain, respectively. The estimator (1) yields

$$\theta_f = \frac{s^2}{s^2 + K_p s + K_i} \left( \frac{1}{s} \dot{\theta}_g \right) + \frac{K_p s + K_i}{s^2 + K_p s + K_i} \theta_c \quad (2)$$

According to (2), the output of conventional attitude estimator in Fig. 1 is sum of low pass filter output of accelerometer and high pass filter output of gyro. The estimator coefficients  $K_p$  and  $K_i$  are chosen by

$$K_i = \omega^2, \quad K_p = 2\zeta\omega \quad (3)$$

where  $\omega$  is the cut-off frequency, and  $\zeta$  is the damping ratio of estimator which the damping ratio  $\zeta$  is fixed to a suitable value of 0.707.

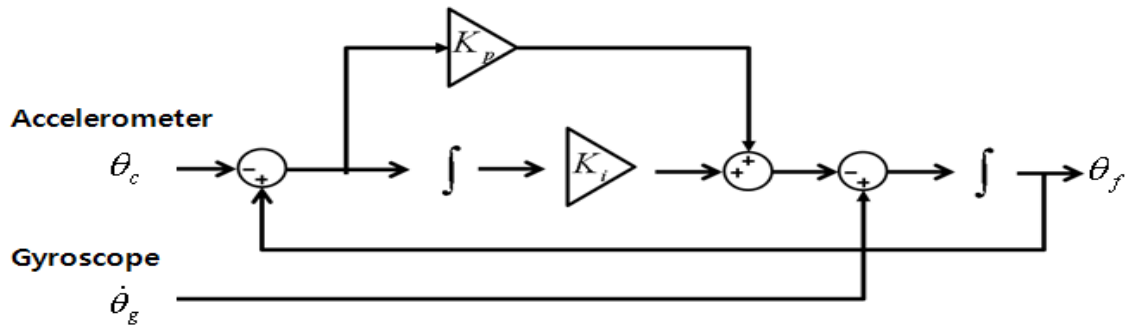


Fig. 1 Conventional attitude estimator

B. Establishment of compensator coefficients using the least square method.

In the cut-off frequency method reviewed in previous section, it is difficult to achieve satisfactory performance in all range of operating conditions, especially in case of low-cost sensors. In order to apply the linear least square method to this problem, it is necessary to integrate the output of gyro. However integrating errors are not negligible because of the presence of bias errors which cause drift. Since it had better apply the output of gyro to linear least square method, time derivative of (1) is

$$\dot{\theta}_e = \dot{\theta}_g - \left( K_p + \frac{1}{s} K_i \right) (\theta_e - \theta_c) \quad (4)$$

where,  $\theta_e$  is the angle of the encoder instead of  $\theta_f$  in (1). And (4) is represented as

$$\begin{pmatrix} \theta - \theta_c & \frac{1}{s} (\theta - \theta_c) \end{pmatrix} \begin{pmatrix} K_p \\ K_i \end{pmatrix} = \dot{\theta}_g - \dot{\theta} \quad (5)$$

The least square solution of (5) is

$$\begin{pmatrix} K_p \\ K_i \end{pmatrix} = (A^T A)^{-1} A^T B \quad (6)$$

where  $A = \begin{pmatrix} \theta_e - \theta_c & \frac{1}{s} (\theta_e - \theta_c) \end{pmatrix}$ ,  $B = \dot{\theta}_g - \dot{\theta}_e$ . And the performance index  $J$  is set the root-mean-square (RMS) of errors between  $\theta_f$  and  $\theta_e$  as follows:

$$J = \frac{1}{N} \sum_{k=1}^N (\theta_f(k) - \theta_e(k))^2 \quad (7)$$

where  $N$  is the number of sampling data.

### III. PERFORMANCE VERIFICATION IN THE ROTARY PENDULUM SYSTEM

A. The experimental setup using the rotary pendulum system.

In this section, we decide coefficients  $K_p$  and  $K_i$  of estimator using the least square method in chapter 2 through pendulum movement and verify the performance of the attitude estimator applied the coefficients derived from the experiment results in chapter 3 using the least square method.

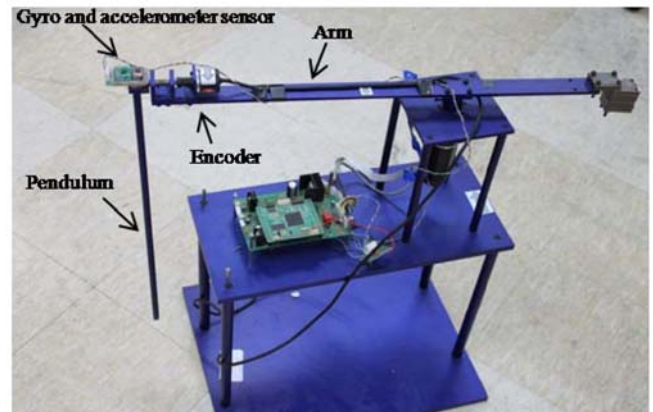


Fig. 2 The rotary pendulum system

We composed the pendulum system like Fig. 2. The encoder sensor measuring the angle of the pendulum is E30S4-500-3-2 of Autonics company. The main processor for transmission each output of sensors is DSPTMS320F2812 of TI company. And the gyro sensor for angular rate is IDG-300 of InvenSense as a low priced sensor using the MEMS technology and the accelerometer sensor is MMA7260Q of Freescale as compensating sensor.

When the pendulum and arm are rolled as shown in Fig. 3, we measure angular rate of the gyro, angle of the encoder and the accelerometer installed in between pendulum and arm in Fig. 4. Then we get the estimator coefficients  $K_p$ ,  $K_i$  with the least square method using the (6) as follows:

$$\begin{pmatrix} K_p \\ K_i \end{pmatrix} = \begin{pmatrix} 0.1973 \\ 0.00546 \end{pmatrix} \quad (8)$$

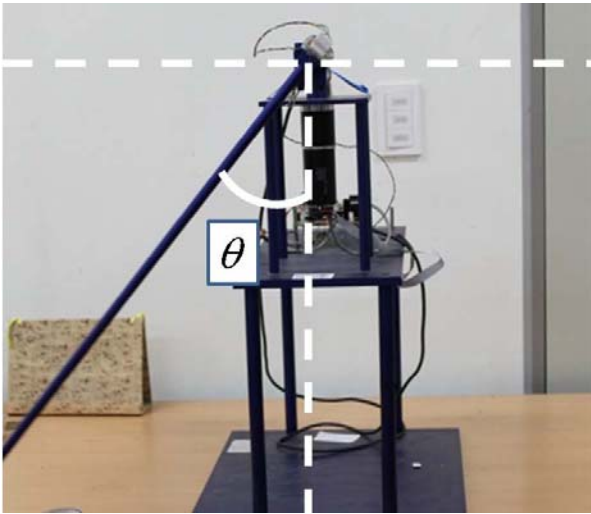


Fig. 3 Experiment using pendulum system

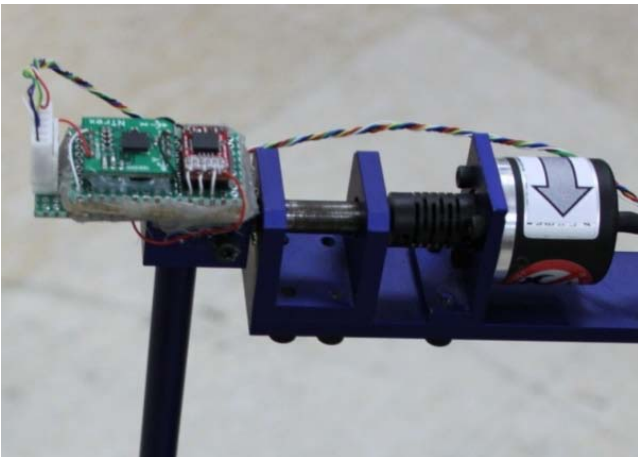


Fig. 4 Sensors (Gyro sensor, Accelerometer sensor, Encoder)

#### B. Performance verification.

There are the drift errors of gyro sensor in Fig. 5 caused by bias error. As a result of the presented estimator, it is found that there is no drift error in Fig. 6 and the response time of attitude estimator is 15ms comparing with the encoder.

The performance of compensator for each method is shown in Fig. 7 and from the top, it presented filter,  $\omega=1$ ,  $\omega=0.1$ ,  $\omega=0.01$  in order. The performance index  $J$  in (7) which is presented in this paper using the least square method is lower than the conventional method using cut-off frequency as shown in Table I.

#### IV. CONCLUSION

In order to determine the coefficients  $K_p$  and  $K_i$  of the conventional estimation compensator by blending the gyro and the accelerometer, we have used the least square method and verified the performance by the rotary pendulum system. The estimator proposed in this study has shown a good characteristic for the drift errors from the gyro sensor and verified less response time than others.

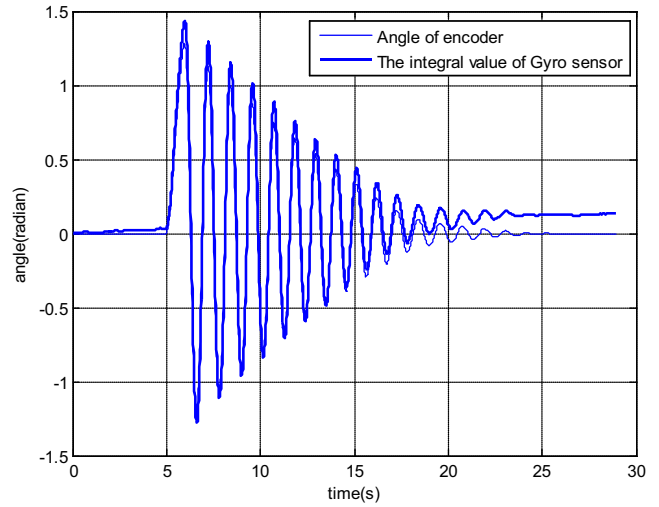


Fig. 5 Drift error of the gyro sensor

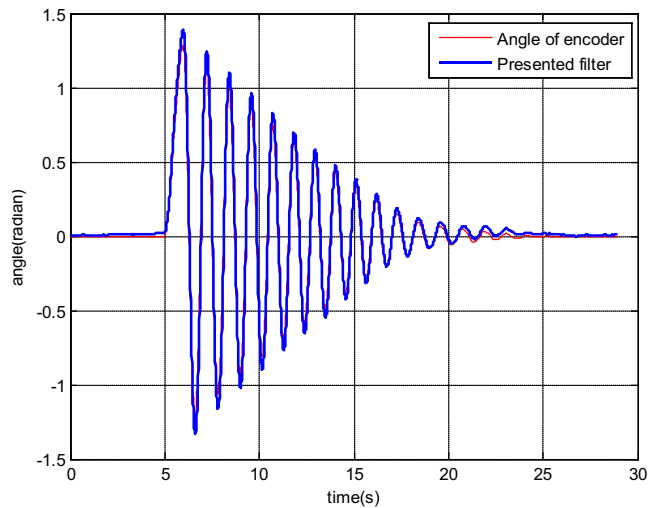


Fig. 6 Presented estimator and encoder comparison graph

TABLE I  
 THE PERFORMANCE INDEX OF ERROR

	Gyro Compensator Filter	$\omega = 1$	$\omega = 0.1$	$\omega = 0.01$
$K_i$	<b>0.1973</b>	1.414	0.1414	0.01414
$K_p$	<b>0.00546</b>	1	0.01	0.0001
Response Time(ms)	<b>15</b>	16		18
$J$	<b>0.00430</b>	0.00502	0.835	0.01

Unit = Radian

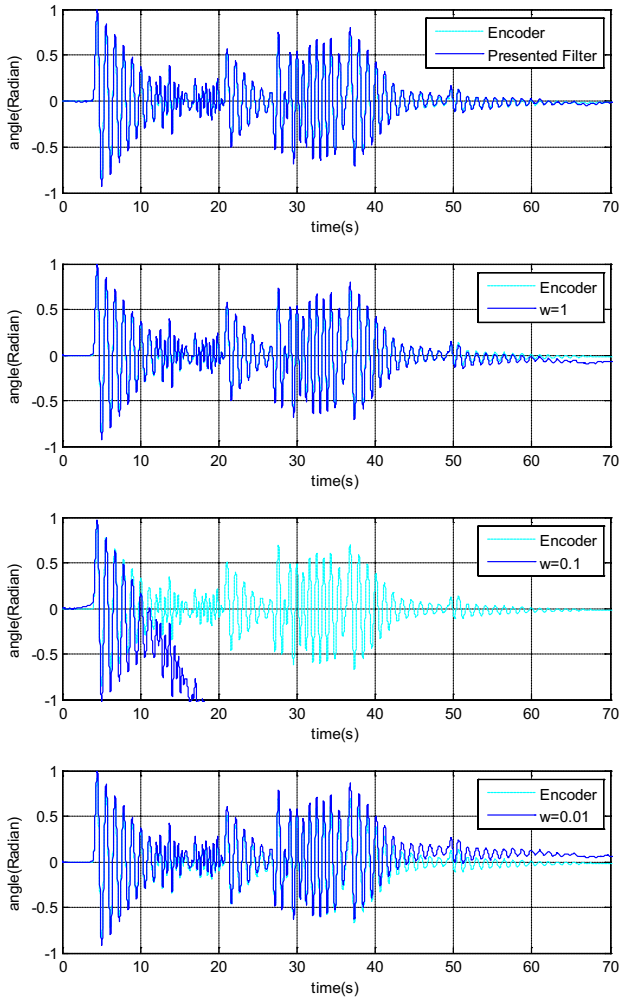


Fig. 7 Graph comparison of cut-off frequency method and the method using the Least square method

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