

# Optimal Power Allocation to Diversity Branches of Cooperative MISO Sensor Networks

Rooholah Hasanizadeh, and Saadan Zokaei

**Abstract**—In the context of sensor networks, where every few dB saving counts, the novel node cooperation schemes are reviewed where MIMO techniques play a leading role. These methods could be treated as joint approach for designing physical layer of their communication scenarios. Then we analyzed the BER performance of transmission diversity schemes under a general fading channel model and proposed a power allocation strategy to the transmitting sensor nodes. This approach is then compared to an equal-power assignment method and its performance enhancement is verified by the simulation. Another key point of the contribution lies in the combination of optimal power allocation and sensor nodes' cooperation in a transmission diversity regime (MISO). Numerical results are given through figures to demonstrate the optimality and efficiency of proposed combined approach.

**Keywords**—Optimal power allocation, cooperative MISO scheme, sensor networks, diversity branch.

## I. INTRODUCTION

NETWORKED microsensors technology is a key technology for the future [1]. In September 1999, Business Week signed it as one of the 21 most important technologies for the 21<sup>st</sup> century. A sensor network is composed of a large number of tiny devices, called sensor nodes. Since these nodes have limited size capabilities to maintain high capacity power supplies and mainly operate on batteries, power conservation would be an important benchmark in designing different parts of a sensor networks, including physical layer entities by modulation, coding and other related techniques. Amongst these techniques is the Multi-Input Multi-Output (MIMO) methods which dramatically increase channel capacity in fading environments; thus subsiding the required power for transmission.

It has been shown [2] that multiple antenna systems can support higher data rates under the same transmit power budget and bit-error-rate performance requirements as a Single-Input Single-Output (SISO) system. An alternative view is that for the same throughput requirement, MIMO systems require less transmission energy than SISO systems. So applying such techniques in the case of a sensor network may seem to be an obvious improvement to their highly restricted resources of power; however, direct application of

multi-antenna techniques to sensor networks is impractical due to the limited physical size of a sensor node which typically can only support a single antenna.

Another inherent benefit of MIMO systems is their spatial diversity establishment throughout the structure of communication scenario. If the antennas are placed sufficiently far apart, the channel gains between different antenna pairs fade more independently, and independent signal paths are created [3]; therefore we achieve sufficient diversity orders to maintain the signal quality. Each of these independent paths is called a diversity branch. In this paper we used a new technique to assign different amounts of power to diversity branches for the sake of minimizing total probability of error, so that the sensor nodes would dissipate less energy to operate longer. We also just discussed MISO case to employ transmission diversity.

The reminder of this paper is organized as follows. In Section II we will introduce the previous work on cooperating schemes of sensor nodes using MIMO techniques. Then our proposal for un-equal power allocation to different diversity branches is discussed in Section III. Section IV clearly combines the methods mentioned in previous sections and Section V summarizes our conclusions.

## II. COOPERATIVE MISO TECHNIQUE

### A. The Origins

The broadcast nature of wireless communications suggests that a source signal transmitted towards the destination can be "overheard" at neighboring nodes. Cooperative communication [4] refers to processing of this overheard information at the surrounding nodes and retransmission towards the destination to create spatial diversity, thereby to obtain higher throughput and reliability. In a cooperative communication system two or more active users in a network share their information and jointly transmit their messages, either at the different times or simultaneously, to obtain greater reliability and efficiency than they could obtain individually. User-cooperation [5]-[6] provides spatial diversity by enabling two or more active sensor nodes to jointly transmit their messages towards their destinations. Each wireless transmission can be overheard by neighboring nodes (which we call "partners" here) which then process this signal and retransmit to provide additional reliability. Hence the partnering sensor nodes can be thought of as an antenna array. However, inter-user channels are noisy and the

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cooperation schemes need to take into account that the information received by the partner may not be accurate. It is found that cooperative space-time coding is beneficial for both partners when they have similar, as well as different channel qualities to the destination. Also it is demonstrated that the cooperative space-time coding approach is robust and still provides gains with respect to no cooperation, even when there is a severe degradation in the inter-user channel quality [7].

### B. MISO Sensor Networks

As we highlighted in Section I, direct application of multi-antenna techniques to sensor networks is impractical due to the limited physical size of a sensor node which typically can only support a single antenna. Fortunately, if we allow individual single-antenna nodes to cooperate on information transmission and/or reception, a cooperative MIMO system can be constructed such that energy-efficient MIMO schemes can be deployed [8]. This problem (in short-range applications such as sensor networks where the circuit energy consumption is comparable to or even dominates the transmission energy) becomes more significant in MIMO systems since the circuit complexity of MIMO structures is much higher than that of SISO structures and it is not clear whether MIMO systems are more energy efficient than SISO systems due to the high circuit complexity associated with the MIMO structure.

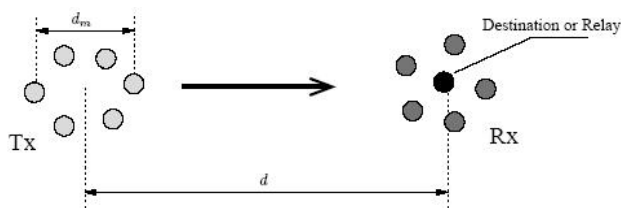


Fig. 1 Information flow in a sensor network [9]

Thus, if we allow cooperative transmission among multiple nodes, we can treat them as multiple antennas to the destination node such that an equivalent MISO system can be constructed. By using this equivalent MISO system, the requirement on transmission energy for the long-haul transmission can be reduced. However, in order to make the cooperative transmission possible, local data exchange is necessary before the long-haul transmission. The local information flow costs energy, which should be less than the energy saved by using the MISO structure. Another tradeoff is the transmission delay since the MISO approach has different delay characteristics than non-cooperative approaches. As shown in Fig. 1 [9], the  $M_t$  nodes on the transmitting side will cooperate. Each node first broadcasts its information to all the other local nodes using different time slots. After each node receives all the information bits from other nodes, they encode the transmission sequence according to the Alamouti diversity codes [10]. Since each node has a pre-assigned index  $i$ , they will transmit the sequence which the  $i^{\text{th}}$  antenna should transmit in an Alamouti MIMO system. On the receiving side,

there are  $M_r$  nodes (including one destination node and  $M_r-1$  assisting nodes) joining the cooperative reception. The  $M_r-1$  assisting nodes first quantize each symbol they receive into  $n_r$  bits, and then transmit all the bits using uncoded MQAM to the destination node to do the joint detection. A comparison of total consumed energy has been shown in the Fig. 2. Since a constant power which is needed for transmitting the desired number of bits, even in a noise and fading free environment, is common for the two non- and MISO-cooperative schemes, we omitted that. Because it forms the major and a high ratio (namely 90%) of total consumed power and also is empty of comparison information for its commonality in two cases.

### C. Simulation Results

We will consider a 2x1 MISO case where two nodes cooperate to send a number of information symbols to a sink. Using BPSK modulation and a constant SNR, we assume a Rayleigh flat fading channel for long-haul transmissions and an error-free channel for local exchange of raw data. Space-time coding will be Alamouti and the nodes try to send a number bits to the sink; hence the retransmission would be the main, and in this work, the only source of energy waste. Afterwards, a simple 2x2 cooperative MIMO scheme using Alamouti space-time codes is also considered. The plots are versus  $d$ , which is the long-haul transmission distance and is defined in Fig. 1.

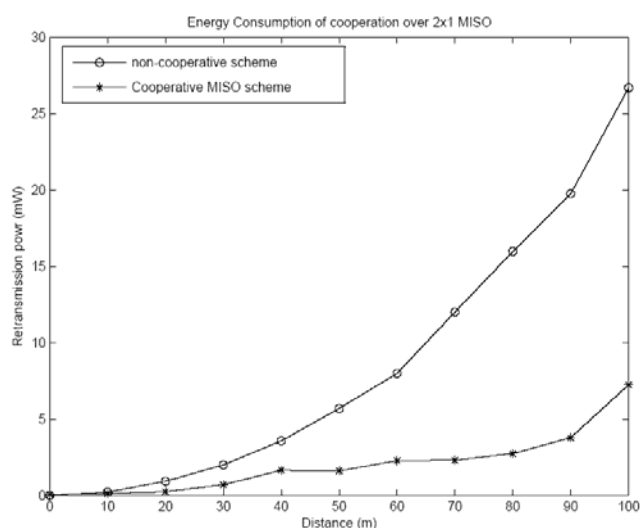


Fig. 2 Power consumption comparison of non-cooperative and MISO-cooperative schemes

## III. OPTIMAL POWER ALLOCATION TO DIVERSITY BRANCHES

### A. System Model

We consider a MISO system where  $L$  transmitting sensor nodes cooperate to send some information bits to another node. Of course the geographical separation of these nodes makes  $L$  independent diversity paths to mitigate fading.

But how to define the fading environment? Rayleigh is a widely used and well-known statistical channel fading model

which most of performance studies have taken place on. However, in many practical scenarios, especially for the case of sensor networks, this model is far too rare than to be realistic and would make inaccurate results in comparison with practical efforts. This model is based on the assumption that there are a large number of statistically independent reflected and scattered paths with random amplitudes and uniform incoming angles [3]. In contrary, the distribution of sensor nodes on different surfaces would not let the incoming signals to enter the receiver with uniform phases and from all angles equally. So we tried to use a more general model to accommodate all fading scenarios by just choosing a number, say, fading parameter. The Nakagami- $m$  fading model, then, is chosen for its routine mathematical manipulations and generality. By choosing, for example,  $m=1$  the Rayleigh scenario is present. It can also approximate Rician fading model with parameter  $K$ , when we notice its relation to  $m$ . For every  $K$ , we can easily choose  $m$  as  $m = (K + 1)^2 / (2K + 1)$  [11].

Finally by using maximal ratio combining rule at the receiver, we will get the transmitted signal.

#### B. Error Probability of $L$ Branches Diversity System

Defining the system model, we are ready to derive a comparison metric which will be used to evaluate the performance improvement of our proposed power allocation strategy on the next part. Clearly we use error probability as the desired metric.

We assume that the transmitted signal on the  $l^{\text{th}}$  antenna undergoes Nakagami- $m$  fading with fading parameter  $m_l$  and average fading power  $\Omega_l = E[\alpha_l^2]$ ,  $l = 1, 2, \dots, L$ . As such the instantaneous signal-to-noise ratio (SNR) per symbol of the  $l^{\text{th}}$  diversity channel is given by

$$\gamma_l = (\alpha_l^2 T_s P_l) / N_0, \quad (1)$$

where  $N_0$  is the AWGN power spectral density,  $\alpha_l$  is the fading amplitude of the  $l^{\text{th}}$  diversity path,  $P_l$  is the power allocated to the  $l^{\text{th}}$  node, and  $T_s$  is the symbol duration. Denoting  $G_l = \Omega_l T_s / N_0$ ,  $\bar{\gamma}_l$  the average SNR of the  $l^{\text{th}}$  path, can be written as  $\bar{\gamma}_l = G_l P_l$ .

We assume that the signal is transmitted over the  $L$  sensor nodes representing antennas using a modulation scheme such that its conditional bit error rate (BER) (conditioned on the SNR) is well approximated by,  $P_b(E/\gamma) = ae^{-b\gamma}$  where  $a$  and  $b$  are constants. For example,  $a$  and  $b$  are 0.00852 and 0.4030, respectively, for the 16-QAM case [11].

For computing average BER in the receiver, we first calculate the error probability of each branch. This would be obtained by integrating (averaging) the conditional BER on the fading distribution. In a Nakagami- $m$  fading environment, the power density function is well-known as [11]

$$p_{\gamma_l}(\gamma_l) = \left(\frac{m_l}{\bar{\gamma}_l}\right)^{m_l} \frac{\gamma_l^{m_l-1}}{\Gamma(m_l)} e^{-\frac{m_l \gamma_l}{\bar{\gamma}_l}}. \quad (2)$$

Now we step forward for calculating average BER of every path as

$$\begin{aligned} P_{bl}(E) &= \int_0^\infty P_b(E/\gamma_l) p_{\gamma_l}(\gamma_l) d\gamma_l \\ &= \int_0^\infty ae^{-b\gamma_l} \times \left(\frac{m_l}{\bar{\gamma}_l}\right)^{m_l} \frac{\gamma_l^{m_l-1}}{\Gamma(m_l)} e^{-\frac{m_l \gamma_l}{\bar{\gamma}_l}} d\gamma_l \\ &= \left(\frac{m_l}{\bar{\gamma}_l}\right)^{m_l} \frac{a}{\Gamma(m_l)} \int_0^\infty \gamma_l^{m_l-1} e^{-\gamma_l \left(b + \frac{m_l}{\bar{\gamma}_l}\right)} d\gamma_l. \end{aligned}$$

By changing the variable to  $x$  as

$$x = \gamma_l \left(b + \frac{m_l}{\bar{\gamma}_l}\right) \Rightarrow \gamma_l = x \left(\frac{\bar{\gamma}_l}{b\bar{\gamma}_l + m_l}\right), d\gamma_l = \left(\frac{\bar{\gamma}_l}{b\bar{\gamma}_l + m_l}\right) dx$$

we will have

$$\begin{aligned} P_{bl}(E) &= \left(\frac{m_l}{\bar{\gamma}_l}\right)^{m_l} \frac{a}{\Gamma(m_l)} \int_0^\infty \left(\frac{\bar{\gamma}_l}{b\bar{\gamma}_l + m_l}\right)^{m_l} x^{m_l-1} e^{-x} dx \\ &= \left(\frac{m_l}{b\bar{\gamma}_l + m_l}\right)^{m_l} \frac{a}{\Gamma(m_l)} \int_0^\infty x^{m_l-1} e^{-x} dx \\ &= \left(\frac{m_l}{b\bar{\gamma}_l + m_l}\right)^{m_l} \frac{a}{\Gamma(m_l)} \Gamma(m_l) \\ &= a \left(\frac{m_l + b\bar{\gamma}_l}{m_l}\right)^{-m_l} \\ &= a \left(1 + \frac{b\bar{\gamma}_l}{m_l}\right)^{-m_l}. \quad (3) \end{aligned}$$

Now when the fading is assumed on  $L$  channels to be mutually statistically independent, the average BER of  $L$  combined paths would be the product of BERs for each path where we have from (3)

$$P_b(E) = a \prod_{l=1}^L \left(1 + \frac{b\bar{\gamma}_l}{m_l}\right)^{-m_l}. \quad (4)$$

#### C. Power Allocation Strategy

We have a constant amount of power to be allocated to the transmitted signals of different paths. Minimizing the average BER would be favorable and we use it to calculate the optimal allocation of power to the branches. The whole power is

$$P_t = \sum_{l=1}^L P_l. \quad (5)$$

For minimizing  $P_b(E)$ , we use the Lagrange multiplier method. But since the intended function in (4) is too complex, we minimize  $\log(P_b(E))$  which of course maintains its curviness to find the minimum point correctly. Now by the equation (5) we will form the following optimization problem using Lagrange multiplier  $\lambda$ :

$$\mathcal{L}(\lambda, P_1, P_2, \dots, P_L) = \log(P_b(E)) + \lambda \sum_{l=1}^L P_l$$

$$= \log(a) - \sum_{l=1}^L m_l \log\left(1 + \frac{bG_l P_l}{m_l}\right) + \lambda \sum_{l=1}^L P_l. \quad (6)$$

The last equation is derived by noting that  $\bar{\gamma}_l = G_l P_l$ . Then by differentiating (6) with respect to  $P_l$  and equating  $\mathcal{S}'(\lambda, P_1, \dots, P_L) = 0$ , we will have the statement to be

$$\begin{aligned} \mathcal{S}'(\lambda, P_1, \dots, P_L) &= \frac{\partial}{\partial P_l} \mathcal{S}(\lambda, P_1, \dots, P_L) \\ &= \lambda - \frac{bG_l}{1 + \frac{bG_l P_l}{m_l}} = 0 \\ \Rightarrow P_l^* &= m_l \left( \frac{1}{\lambda} - \frac{1}{bG_l} \right)^+ \end{aligned} \quad (7)$$

where  $x^+ = \max(0, x)$  forces  $P_l$  to be a positive value. Then from (5)

$$\begin{aligned} \sum_{k=1}^L P_k &= P_t \Rightarrow \sum_{k=1}^L m_k \left( \frac{1}{\lambda} - \frac{1}{bG_k} \right) = P_t \\ \Rightarrow \frac{1}{\lambda} &= \frac{P_t}{\sum_{k=1}^L m_k} + \frac{\sum_{k=1}^L \frac{m_k}{G_k}}{b \sum_{k=1}^L m_k}. \end{aligned} \quad (8)$$

Substituting (8) into (7) results in

$$P_l^* = m_l \left( \frac{P_t}{\sum_{k=1}^L m_k} + \frac{\sum_{k=1}^L \frac{m_k}{G_k}}{b \sum_{k=1}^L m_k} - \frac{1}{bG_l} \right)^+ \quad (9)$$

#### D. Simulating the Proposed Scheme

In this part we see the BER performance improvement that optimal allocation of power to different transmitting sensor nodes can produce. Firstly consider the previously stated assumptions about a Nakagami fading environment with parameter  $m$ , and  $L$  cooperating sensor nodes in transmission. Then we use 16-QAM modulation scheme which its conditional BER performance could be best mentioned by  $P_b(E/\gamma) = ae^{-b\gamma}$ , where  $a = 0.00852$  and  $b = 0.4030$  [11]. So we can use equation (4) as the whole system performance.

For our simulations, we considered two power allocation strategies: the first one sends out data from sensor nodes with equal power and in the second one power is assigned according to (9) for each node as a diversity branch. The fading environment is assumed to be identical for all branches, where we chose all  $m$  parameters to be equal, but we make a reasonable difference in channel conditions for different paths. To be specific, in Fig. 3 we showed the comparison for  $L = 3$  cooperating nodes where for example we set  $\Omega_1 = 2\Omega_2 = 10\Omega_3$ . Fig. 4 also shows the improvement for  $L = 4$  diversity branches.

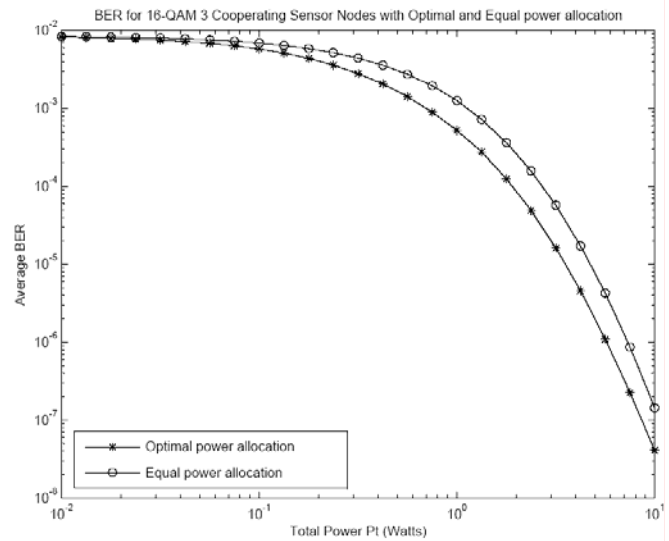


Fig. 3 BER performance comparison for different power allocation strategies with 3 cooperating nodes

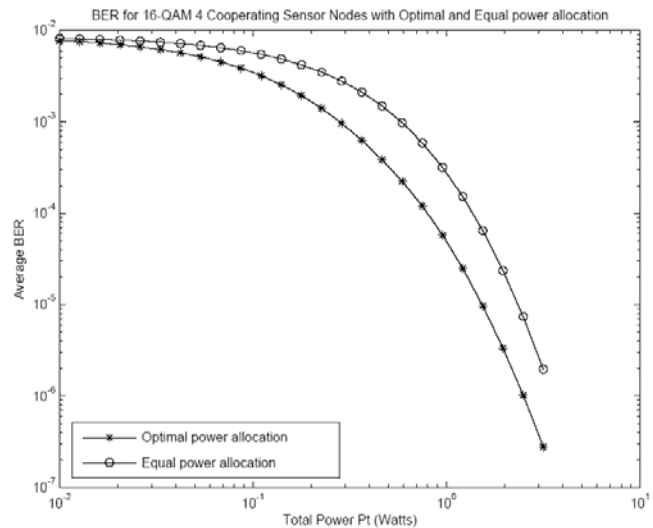


Fig. 4 BER performance comparison for different power allocation strategies with 4 cooperating nodes

Simulation outputs show an improvement in BER when using optimal power allocation to different diversity branches, for example when  $L = 3$  sensor nodes cooperate (Fig. 3), at  $BER=10^{-4}$  optimal allocation needs 42% less power. It also can be seen that when the total power is low, for example  $P_t = 0.5W$ , the optimal allocation tends to assign whole power to the best branch, leaving the other branches with zero power (i.e.  $P_1=P_t$  and  $P_2=P_3=0$ ). As  $P_t$  increases, a portion of which would be allocated to next rich branch, again leaving the poor channel branch with no power (e.g. when  $P_t = 1.5W$  we have  $P_1 = 1.25W$ ,  $P_2 = 0.25W$ , and  $P_3 = 0$  where at this point the BER for optimal allocation strategy is 174% lower).

#### IV. PERFORMANCE ENHANCEMENT BY COMBINING MERITS

We now get to the point which we can declare to gain more efficiency by combining the above mentioned methods,

applying our optimal power assignment to cooperative scheme in [9]. The simulation is done in a 100 meter data transmission with 2x1 MISO channel where two sensor nodes cooperate. Channel gains are of course non-symmetric, say,  $\Omega_1 = 10\Omega_2$ . The results show, as expected, a drastic improvement in the BER performance of data transmission. Fig. 5 compares different possible schemes including cooperation-only, optimal allocation-only, the combined proposal, and the traditional approach.

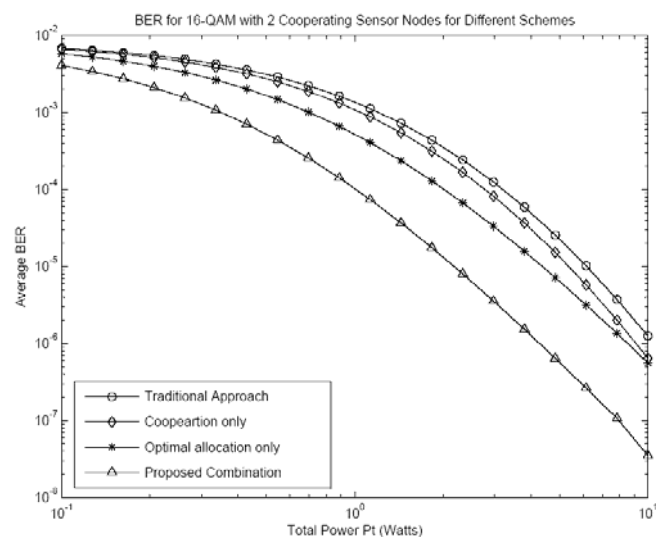


Fig. 5 Comparing performance of different possible schemes

## V. CONCLUSION

We showed that by a power allocation strategy which tends to minimize bit-error rate of system, we will achieve better performance in the fading environment. This leads to less power consumption for transmitting a number of data bits, so that a highly limited resource sensor network could remain active a longer time. Previously cooperative approach to sensor networks is also reviewed and finally combined with the proposed optimal power assignment. The results show a tremendous saving in energy compared to traditional schemes.

A more important contribution of this work is that the aforesaid obtained performance does not sacrifice any other important metrics to achieve this success. All the overhead produced in these schemes is with the primary software concerns of sensor nodes and the energy exchanged in the initial stages of transmission scenario. Our proposed scheme even doesn't make an additional overhead, but just few mathematical software calculations at the startup.

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