Variable Step-Size APA with Decorrelation of AR Input Process

JaeWook Shin, Ju-man Song, Hyun-Taek Choi, and PooGyeon Park

***Abstract*—** This paper introduces a new variable step-size APA with decorrelation of AR input process is based on the MSD analysis. To achieve a fast convergence rate and a small steady-state estimation error, he proposed algorithm uses variable step size that is determined by minimizing the MSD. In addition, experimental results show that the proposed algorithm is achieved better performance than the other algorithms.

***Keywords*—** Adaptive filter, affine projection algorithm, variable step size.

I. Introduction

Adaptive filters have been used for solving problems in many fields in the last decades. The most popular adaptive filters that are the least mean squares (LMS) and normalized least mean square (NLMS) algorithm are employed in diverse fields such as communications, control, and acoustic processing because of their low computational complexity and ease of implementation [1]-[3]. However, when the input data is highly correlated, the convergence rate for these algorithms is highly deteriorated. To solve this drawback, the affine projection algorithm (APA) has been proposed [4]. Since [4], various APA have been devised from different perspectives such as the decorrelating algorithm [5], the regularized APA [6], the partial rank algorithm [7], and NLMS with orthogonal correction factors [8]. Because the APA updates the filter coefficients on the basis of multiple input vectors rather than a single input vector, it significantly improves the convergence rate of the NLMS algorithm for correlated input data. However, because the algorithm has larger steady-state estimation errors than NLMS, various algorithms have been proposed to overcome this drawback by controlling a step size [9].

This paper proposes a new variable step-size APA with decorrelation of AR input process. This algorithm computes the mean square deviation (MSD) of APA to obtain optimal step size.

This paper is organized as follows. Section 2 briefly reviews the APA with decorrelation of AR input process. Section 3 contains the MSD of APA with decorrelation of AR input process. The proposed algorithm derived in Section 4. Section 5 contains the experimental results and Section 6 the concluding remarks.

J. Shin and J. Song are with the Department of Electrical Engineering, Pohang University of Science and Technology, San 31, Hyojadong, Namgu, Pohang, Kyungbuk, 790-784, Korea (e-mail: shinshingo,sjm1924@postech.ac.kr)

H.T. Choi is with Korea Ocean Research and Development Institute, Korea (e-mail : htchoiphd@kordi.re.kr)

PooGyeon Park is with the Division of IT Convergence Engineering and the Department of Electrical Engineering, Pohang University of Science and Technology, San 31, Hyojadong, Namgu, Pohang, Kyungbuk, 790-784, Korea (e-mail: ppg@postech.ac.kr).

II. Affine Projection Algorithm with Decorrelation of AR Input Process

Consider data $d(n)$ originated from an unknown system :

$$d(n) = \mathbf{u}(n)^T \mathbf{w}_o + v(n), \tag{1}$$

where $\mathbf{w}_o$ is an m-dimensional column vector that is to be estimated, $v(n)$ accounts for a measurement noise that is white Gaussian noise with variance σ_v^2, and $\mathbf{u}(n)$ denotes an m-dimensional input vector, $\mathbf{u}(n) = [u(n), \cdots, u(n-m+1)]^T$. The affine projection algorithm (APA) update equation with decorrelation AR input process is given in Table I and its diagram is in Fig. 1.

TABLE I
Update equation of the APA with decorrelation of AR input process

$$\mathbf{U}(n) = [\mathbf{u}(n-1), \ldots, \mathbf{u}(n-P)]$$
$$\hat{a}(n) = (\mathbf{U}^T(n)\mathbf{U}(n))^{-1}\mathbf{U}^T(n)\mathbf{u}(n)$$
$$\phi(n) = \mathbf{u}(n) - \mathbf{U}(n)\hat{a}(n)$$
$$e(n) = d(n) - \mathbf{u}^T(n)\widehat{\mathbf{w}}(n)$$
$$\widehat{\mathbf{w}}(n+1) = \widehat{\mathbf{w}}(n) + \mu \frac{\phi(n)}{\phi^T(n)\phi(n)} e(n)$$

μ : step size

K : the number of input vector

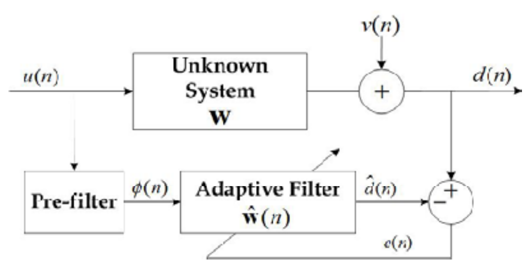

Fig. 1 Diagram of APA with decorrelation of AR input process

III. Mean Square Deviation Analysis for Decorrelated Input Signal

The APA update equation that includes decorrelation algorithm for AR environment is similar to NLMS algorithm with input signal $\phi(n)$. Therefore, we consider mean square deviation (MSD) analysis of NLMS instead of APA. We define the weight-error vector as $\widetilde{\mathbf{w}}(n) \triangleq \mathbf{w}_o - \widehat{\mathbf{w}}(n)$. We can rewrite the update equation in terms of $\widetilde{\mathbf{w}}(n)$ as

$$\begin{aligned}\widetilde{\mathbf{w}}(n+1) &= \widetilde{\mathbf{w}}(n) - \mu \frac{\phi(n)}{\phi^T(n)\phi(n)} e(n) \\ &= \widetilde{\mathbf{w}}(n) - \mu \frac{\phi(n)}{\phi^T(n)\phi(n)} (\mathbf{u}^T(n)\widetilde{\mathbf{w}}(n) + v(n)) \qquad (2) \\ &= \widetilde{\mathbf{w}}(n) - \mu \frac{\phi(n)}{\phi^T(n)\phi(n)} (\phi^T(n)\widetilde{\mathbf{w}}(n) + v_f(n))\end{aligned}$$

where $v_f(n)$ is modified noise by a linear filter. We assume that variance of $v_f(n)$ is less than variance of $v(n)$. Therefore, above equation is rewritten by

$$\widetilde{\mathbf{w}}(n+1) = \mathbf{F}(n)\widetilde{\mathbf{w}}(n) - \mu \frac{\phi(n)}{\phi^T(n)\phi(n)} v_f(n) \qquad (3)$$

where

$$\mathbf{F}(n) = \mathbf{I} - \mu \frac{\phi(n)\phi^T(n)}{\phi^T(n)\phi(n)},$$

and $\mathbf{I}$ is the identity matrix. To obtain the MSD analysis of the NLMS, we define the MSD as follows

$$\mathrm{MSD}(n) \triangleq E\left(\widetilde{\mathbf{w}}^T(n)\widetilde{\mathbf{w}}(n)\right) = \mathrm{Tr}\{\mathbf{P}(n)\} \qquad (4)$$

where $\mathrm{Tr}\{\bullet\}$ is the trace of the matrix, and $\mathbf{P}(n) \triangleq E\left\{\widetilde{\mathbf{w}}(n)\widetilde{\mathbf{w}}^T(n)\right\}$. In this paper, we consider that $\phi(n)$ is a deterministic quantity when regarding about $\mathbf{P}(n)$, and assume that the dependency of $\widetilde{\mathbf{w}}(n)$ on the noise $v_f(n)$ is negligible. Therefore, we can obtain $\mathbf{P}(n+1)$ from below a recursive equation.

$$\mathbf{P}(n+1) = \mathbf{F}(n)\mathbf{P}(n)\mathbf{F}^T(n) + \mu^2\sigma_{v_f}^2 \frac{\phi(n)\phi^T(n)}{\left(\phi^T(n)\phi(n)\right)^2} \qquad (5)$$

where $\sigma_{v_f}^2$ is variance of noise $v_f(n)$. By taking the trace of both sides of the above equation, we can achieve the MSD as follows:

$$\mathrm{Tr}\{\mathbf{P}(n+1)\} = \mathrm{Tr}\left\{\mathbf{F}^T(n)\mathbf{F}(n)\mathbf{P}(n)\right\} + \frac{\mu^2\sigma_{v_f}^2}{\phi^T(n)\phi(n)} \qquad (6)$$

If the input data is white, then $\mathbf{P}(n)$ is approximated as $\mathbf{P}(n) \simeq p(n)\mathbf{I}$. Therefore, because the decorrelated input signal $\phi(n)$ is close to white, equation (6) can be rewritten as

$$p(n+1) = \left(1 - \frac{2\mu - \mu^2}{m}\right)p(n) + \frac{\mu^2\sigma_{v_f}^2}{m\phi^T(n)\phi(n)} \qquad (7)$$

IV. Proposed Variable Step-Size APA

To obtain the variable step size that leads to a fast convergence and a low steady-state error, we minimize $p(n+1)$ with respect to $\mu(n)$ as follows:

$$\frac{\partial p(n+1)}{\partial \mu(n)} = \left(-\frac{2-2\mu(n)}{m}\right)p(n) + \frac{2\mu(n)\sigma_{v_f}^2}{m\phi^T(n)\phi(n)} \qquad (8)$$

Because we cannot directly calculate $\sigma_{v_f}^2$, it replaces $\gamma\sigma_v^2$, where $\gamma \geq 1$. Therefore, the optimum step size is derived as follows:

$$\mu^*(n) = \frac{p(n)}{p(n) + \gamma\sigma_v^2\left(\phi^T(n)\phi(n)\right)^{-1}} \qquad (9)$$

Finally, the proposed variable step-size APA is given in T able II.

TABLE II
UPDATE EQUATION OF THE PROPOSED APA WITH DECORRELATION OF AR INPUT PROCESS

$$\begin{gathered}\mathbf{U}(n) = [\mathbf{u}(n-1), \ldots, \mathbf{u}(n-P)] \\ \hat{a}(n) = (\mathbf{U}^T(n)\mathbf{U}(n))^{-1}\mathbf{U}^T(n)\mathbf{u}(n) \\ \phi(n) = \mathbf{u}(n) - \mathbf{U}(n)\hat{a}(n) \\ e(n) = d(n) - \mathbf{u}^T(n)\widehat{\mathbf{w}}(n) \\ p(n) = \left(1 - \frac{2\mu(n-1) - \mu(n-1)^2}{m}\right)p(n-1) + \frac{\mu(n-1)^2\gamma\sigma_v^2}{m\phi^T(n)\phi(n)} \\ \mu^*(n) = \frac{p(n)}{p(n) + \gamma\sigma_v^2\left(\phi^T(n)\phi(n)\right)^{-1}} \\ \widehat{\mathbf{w}}(n+1) = \widehat{\mathbf{w}}(n) + \mu^*(n)\frac{\phi(n)}{\phi^T(n)\phi(n)}e(n)\end{gathered}$$

V. Experimental Result

We illustrate the performance of the proposed algorithm by performing simulations in the channel estimation. The channel of the unknown system is randomly generated by moving average models with 16 and 128 taps. We assume that the adaptive filter and the unknown channel have the same number of taps. The input signal $u(n)$ is generated by filtering a white zero-mean Gaussian random sequence through the following system:

$$G_1(z) = \frac{1}{1-0.9z^{-1}},$$

$$G_2(z) = \frac{1}{1-0.1z^{-1}-0.8z^{-1}}.$$

The signal-to-noise ratio (SNR) used in the simulations is defined by

$$\mathrm{SNR} = 10\log_{10}\frac{E\left\{y(n)^2\right\}}{E\left\{v(n)^2\right\}},$$

where $y(n) = \mathbf{u}^T(n)\mathbf{w}_o$. The measurement noise $v(n)$ is added to $y(n)$ with SNR=10 or 30 dB. The noise variance σ_v^2 is known a priori, since it is simple to estimate during silences and online in many practical applications. The normalized mean squared deviation (NMSD) is defined as follows:

$$\mathrm{NMSD}(n) = 10\log_{10}\left(E\left\{\widetilde{\mathbf{w}}^T(n)\widetilde{\mathbf{w}}(n)\right\} / \left(\mathbf{w}_o^T\mathbf{w}_o\right)\right),$$

The NMSD is calculated to indicate the performance of the proposed algorithm, which is obtained by ensemble averaging over 30 independent trials. Variable step-size NLMS and APA (VSS-NLMS, VSS-APA) use same parameter in paper [9] and the proposed algorithm uses $\gamma = 1$. The proposed algorithm sets initial value of as 10. Fig. 2 and 3 show the NMSD learning curves for NLMS, VSS-NLMS, VSS-APA, and the proposed algorithm with colored input signal for $P = 4$ and $\mathrm{SNR} = 30$ dB. As can be seen, the proposed algorithm achieves a fast convergence rate and a small steady-state estimation error comparable to the other algorithms. Furthermore, Fig. 4 and 5 show the NMSD learning curves for NLMS, VSS-NLMS, VSS-APA, and the proposed algorithm with colored input signal for $P = 8$ and $\mathrm{SNR} = 30$ dB. As can be seen, the proposed algorithm obtains a fast convergence rate and a small steady-state estimation error.

VI. Conclusion

The proposed variable step-size APA with decorrelation of AR input process is based on the MSD analysis. To achieve a fast convergence rate and a small steady-state estimation error, he proposed algorithm uses variable step size that is determined by minimizing the MSD. In addition, experimental results show that the proposed algorithm is achieved better performance than the other algorithms.

Acknowledgment

This research was supported by the MKE(The Ministry of Knowledge Economy), Korea, under the ITRC(Information Technology Research Center) support program supervised by the NIPA(National IT Industry Promotion Agency) (NIPA-2012-(H0301-12-1003)) and NIPA-2012-(H0301-12-2002)). This research was supported by World Class University program funded by the Ministry of Education, Science and Technology through the National Research Foundation of Korea (R31-10100). This research was supported by ''Development of technologies for an underwater robot based on artificial intelligence for highly sophisticated missions", which is supported by KORDI.

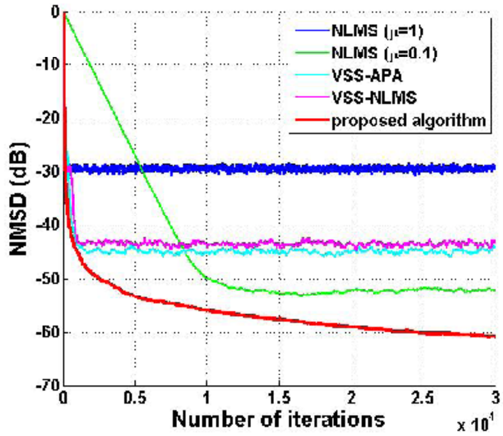

Fig. 2 NMSD learning curves for colored input that is generated $G_1(z)$ for P=4 and SNR=30dB

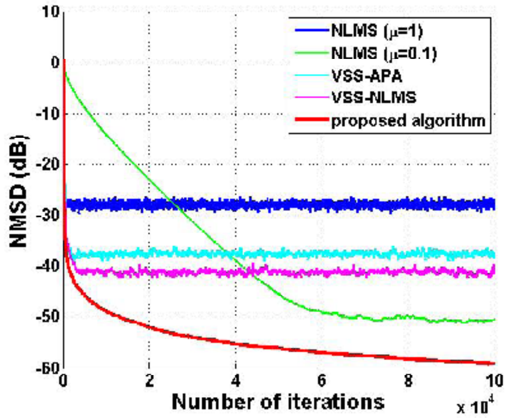

Fig. 3 NMSD learning curves for colored input that is generated $G_2(z)$ for P=4 and SNR=30dB

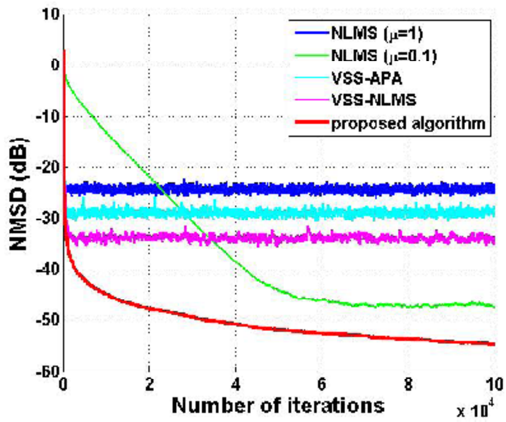

Fig. 4 NMSD learning curves for colored input that is generated $G_1(z)$ for P=8 and SNR=30dB

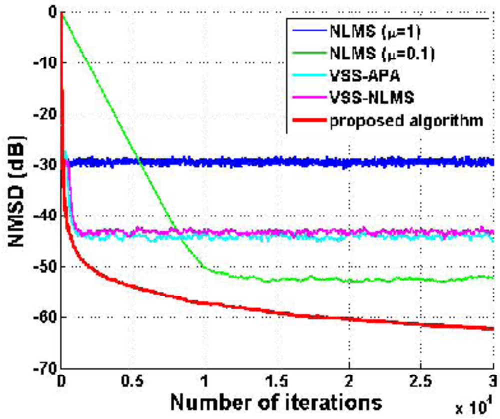

Fig. 5 NMSD learning curves for colored input that is generated $G_2(z)$ for P=8 and SNR=30dB

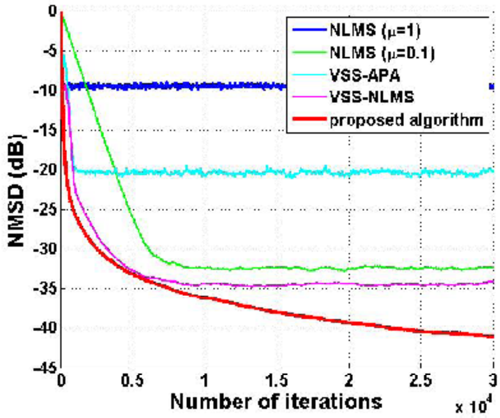

Fig. 6 NMSD learning curves for colored input that is generated $G_1(z)$ for P=8 and SNR=10dB

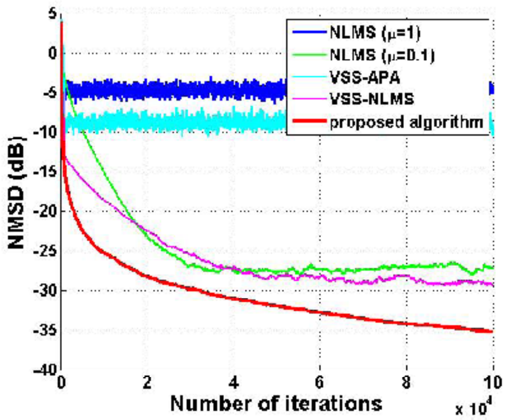

Fig. 7 NMSD learning curves for colored input that is generated $G_2(z)$ for P=8 and SNR=10dB

REFERENCES

[1] S. Haykin, ‘‘Adaptive filter theory", 4th ed. Upper Saddle River, NJ:Prentice-Hall, 2002.

[2] B. Widrow and S. D. Steams, ‘‘Adaptive Signal Processing", Englewood Cliffs, NJ:Prentice-Hall, 1985.

[3] J. Benesty and Y. Huang, Eds., ‘‘Adaptive Signal Processing-Applications to Real-World Problems", Berlin, Germany:Springer-Verlag, 2002.

[4] K. Ozeki and T. Umeda, ‘‘An adaptive filtering algorithm using an orthogonal projection to an affine subspace and its properties", Electronics and Communications in Japan, vol. 67-A, no. 5, pp. 19-27, 1984.

[5] M. Rupp, ‘‘A family of adaptive filter algorithms with decorrelating properties", IEEE Trans. Signal Processing, vol. 46, pp. 771-775, 1998.

[6] S. L. Gay and J. Benesty, ‘‘Acoustic Signal Processing for Telecommunication". Boston, MA: Kluwer, 2000.

[7] S. G. Kratzer and D. R. Morgan, ‘‘The partial-rank algorithm for adaptive beamforming", Proc. SPIE Int. Soc. Opt. Eng., vol. 564, pp. 914, 1985.

[8] S. G. Sankaran and A. A. (Louis) Beex, ‘‘Normalized LMS algorithm with orthogonal correction factors", Proc. 31st Annu. Asilomar Conf Signals, Syst., Comput., Pacific Grove, CA, Nov. pp. 1670-1673, 1997.

[9] H.-C. Shin, A.H.Sayed, and W.-J.Song, ‘‘Variable step-size NLMS and affine projection algorithms", IEEE Trans. Signal Process. Lett., vol. 11, no.2, pp. 132-135, 2004.