The Intuitionistic Fuzzy Ordered Weighted Averaging-Weighted Average Operator and its Application in Financial Decision Making

Shouzhen Zeng

Abstract—We present a new intuitionistic fuzzy aggregation operator called the intuitionistic fuzzy ordered weighted averaging-weighted average (IFOWAWA) operator. The main advantage of the IFOWAWA operator is that it unifies the OWA operator with the WA in the same formulation considering the degree of importance that each concept has in the aggregation. Moreover, it is able to deal with an uncertain environment that can be assessed with intuitionistic fuzzy numbers. We study some of its main properties and we see that it has a lot of particular cases such as the intuitionistic fuzzy weighted average (IFWA) and the intuitionistic fuzzy OWA (IFOWA) operator. Finally, we study the applicability of the new approach on a financial decision making problem concerning the selection of financial strategies.

Keywords—Intuitionistic fuzzy numbers, Weighted average, OWA operator, Financial decision making

I. INTRODUCTION

DIFFERENT types of aggregation operators are found in the literature for aggregating numerical data information [1-3]. The weighted average (WA) is one of the most common aggregation operators found in the literature [1]. It can be used in a wide range of different problems including statistics, economics and engineering. Another interesting aggregation operator is the ordered weighted averaging (OWA) operator introduced by Yager [4], whose prominent characteristic is the reordering step. The OWA operator provides a parameterized family of aggregation operators that includes as special cases the maximum, the minimum and the average criteria. Since its appearance, the OWA operator has been receiving much attention from both researchers and practitioners over the last decades [5-23]. Recently, some authors have tried to unify both concepts in the same formulation such as the work developed by Torra [23] with the introduction of the weighted OWA (WOWA) operator and the work of Xu and Da [3] about the hybrid averaging (HA) operator. Both models unified the OWA and the WA because both concepts were included in the formulation as particular cases. However, these models seem to be a partial unification but not a real one because they can unify them but they cannot consider how relevant these concepts are in the specific problem considered. For example, in some problems we may prefer to give more importance to the OWA operator because we believe that it is more relevant and vice versa. To overcome this issue, Merigó [13-14] has developed the ordered weighted averaging weighted averaging (OWAWA) operator that unifies the OWA and the WA in the same formulation.

The main advantage of this approach is that it unifies the OWA and the WA taking into account the degree of importance of each case in the formulation. Thus, we are able to consider situations where we give more or less importance to the OWA and the WA depending on our interests and the problem analyzed. The OWAWA operator is applicable in a wide range of situations where it is possible to use the WA and the OWA operator. Therefore, we see that the applicability is incredibly broad because all the previous models, theories, etc., that uses the WA can be extended by using the OWAWA operator.

Usually, when using the OWAWA operator, it is assumed that the available information is clearly known and can be assessed with exact numbers. However, in the real-life world, due to the increasing complexity of the socio-economic environment and the lack of knowledge or data about the problem domain, exact numerical is sometimes unavailable. Thus, the input arguments may be vague or fuzzy in nature. Atanassov [24] defined the notion of an intuitionistic fuzzy set (IFS), whose basic elements are intuitionistic fuzzy numbers (IFNs) [25-30], each of which is composed of a membership degree and a non-membership degree. In many practical situations, particularly in the process of group decision making under uncertainty, the experts may come from different research areas and thus have different backgrounds and levels of knowledge, skills, experience, and personality. The experts may not have enough expertise or possess a sufficient level of knowledge to precisely express their preferences over the objects, and then, they usually have some uncertainty in providing their preferences, which makes the results of cognitive performance exhibit the characteristics of affirmation, negation, and hesitation. In such cases, the data or preferences given by the experts may be appropriately expressed in IFNs.

For example, in multi-criteria decision-making problems, such as personnel evaluations, medical diagnosis, project investment analysis, etc., each IFN provided by the expert can be used to express both the degree that an alternative should satisfy a criterion and the degree that the alternative should not satisfy the criterion. The IFN is highly useful in depicting uncertainty and vagueness of an object, and thus can be used as a powerful tool to express data information under various different fuzzy environments which has attracted great attentions [25-32]. In order to get a decision result, an important step is the aggregation of IFNs. Recently, some operators have been given to aggregate IFNs. Xu [27] developed some aggregation operators, such as the intuitionistic fuzzy weighted averaging (IFWA) operator, intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, intuitionistic fuzzy hybrid aggregation (IFHA) operator, and established various properties of these operators. Then Xu and Yager [29] proposed some geometric aggregation operators for IFNs, such as the intuitionistic fuzzy weighted geometric operator, the

S. Z. Zeng is with the [1]College of Computer and Information, Zhejiang Wanli University, Ningbo 315100, China (phone: 86 574 88222132; e-mail: zszzxl@63.com).

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intuitionistic fuzzy weighted geometric (IFWG) operator, intuitionistic fuzzy ordered weighted geometric (IFOGA) operator, and intuitionistic fuzzy hybrid geometric (IFHG) operator. Zeng and Su [31] developed an intuitionistic fuzzy ordered weighted distance (IFOWD) operator and applied it to group decision making about selection of strategies

The aim of this paper is to extend the OWAWA operator to accommodate the intuitionistic fuzzy situations. For doing so, we present a new intuitionistic fuzzy aggregation operator called the intuitionistic fuzzy ordered weighted averaging weighted average (IFOWAWA) operator, which unifies the OWA operator with the WA when the available information is uncertain and can be assessed with IFNs. The main advantage of this approach is that it unifies the OWA and the WA taking into account the degree of importance of each case in the formulation and considering that the information is given with IFN. Thus, we are able to consider situations where we give more or less importance to the IFOWA and the IFWA depending on our interests and the problem analyzed. We also study different properties of the IFOWAWA operator and different particular cases.

We study the applicability of the IFOWAWA and we see that it is extremely broad because all the studies that use the WA or the OWA can be revised and extended with this new approach. For example, we could use it in statistics, in economics, in engineering and in decision theory. In this paper, we focus on a financial decision making problem regarding the selection of financial strategies. We analyze a company that wants to invest some money in a country and it is looking for the optimal investment. The main advantage of the IFOWAWA in these problems is that it is possible to consider the subjective probability (or the degree of importance) and the attitudinal character of the decision maker at the same time.

II. PRELIMINERIES

In this Section, we briefly review some basic concepts about the intuitionistic fuzzy sets, the OWA and the OWAWA operator.

A. Intuitionistic Fuzzy Sets

The intuitionistic fuzzy set (IFS) introduced by Atanassov is an extension of the classical fuzzy set, which is a suitable way to deal with vagueness. It can be defined as follows.

Definition 1. Let a set $X = \{x_1, x_2, ..., x_n\}$ be fixed, an IFS A in X is given as following:

$$A = \left\{ \langle x, \mu_A(x), v_A(x) \rangle \big| x \in X \right\} \qquad (1)$$

The numbers $\mu_A(x)$ and $v_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A, $0 \leq \mu_A(x) + v_A(x) \leq 1$, for all $x \in X$. The pair $(\mu_A(x), v_A(x))$ is called an intuitionistic fuzzy number (IFN) [25-30] and each IFN can be simply

denoted as $\alpha = (\mu_\alpha, v_\alpha)$, where $\mu_\alpha \in [0,1]$, $v_\alpha \in [0,1]$, $\mu_\alpha + v_\alpha \leq 1$.

Additionally $S(\alpha) = \mu_\alpha - v_\alpha$ and $H(\alpha) = \mu_\alpha + v_\alpha$ are called the score and accuracy degree of α respectively. For any three IFNs $\alpha = (\mu_\alpha, v_\alpha), \alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$, the following operational laws are valid.

(1) $\alpha_1 + \alpha_2 = \left(\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1} \cdot \mu_{\alpha_2}, v_{\alpha_1} \cdot v_{\alpha_2} \right)$

(2) $\lambda \alpha = \left(1 - (1 - \mu_\alpha)^\lambda, v_\alpha^{\ \lambda} \right)$

To compare two IFNs α_1 and α_2, Xu and Yager [29] introduced a simple method as below:

(1) If $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;

(2) If $S(\alpha_1) = S(\alpha_2)$, then

    (a) If $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 < \alpha_2$;

    (b) If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$

B. The Weighted Average Operator

The weighted average (WA) is one of the most common aggregation operators found in the literature. It has been used in a wide range of applications. It can be defined as follows.

Definition 2. A WA operator of dimension n is a mapping WA: $R^n \to R$ that has an associated weighting W with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_j = 1$, such that:

$$WA(a_1, ..., a_n) = \sum_{i=1}^{n} w_i a_i \qquad (2)$$

where a_i represents the argument variable. The WA operator accomplishes the usual properties of the aggregation operators. For further reading on different extensions and generalizations of the WA, see for example [1-2].

C. The OWA Operator

The OWA operator [4] provides a parameterized family of aggregation operators that include the maximum, the minimum and the average criteria as special cases.. It can be defined as follows:

Definition 3. An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA(a_1, ..., a_n) = \sum_{j=1}^{n} w_j b_j \qquad (3)$$

where b_j is the j th largest of the a_i

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The OWA operator aggregates the information according to the attitudinal character (or degree of orness) of the decision-maker [4]. The attitudinal character is represented according to the following formula:

$$\alpha(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} \right) \qquad (4)$$

Note that $\alpha(W) \in [0,1]$. The more weight W is located close to the top, the closer α is to 1. In decision-making problems, the degree of orness is useful for representing the attitudinal character of the decision-maker by using it as the degree of optimism or pessimism.

D. The Intuitionistic Fuzzy OWA operator

The intuitionistic fuzzy OWA (IFOWA) operator was introduced by Xu [27]. It is an extension of the OWA operator for uncertain situations where the available information can be assessed with IFNs. Let Ω be the set of all IFNs, it can be defined as follows:

Definition 4. Let $\alpha_i = (\mu_i, v_i)(i = 1, 2, ..., n)$ be a collection of IFNs, an IFOWA operator of dimension n is a mapping *IFOWA*: $\Omega^n \to \Omega$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$IFOWA(\alpha_1, \alpha_2, ..., \alpha_n) = \sum_{j=1}^{n} w_j \beta_j$$
$$= \left(1 - \prod_{j=1}^{n} (1 - \mu_j)^{w_j}, \prod_{j=1}^{n} v_j^{w_j} \right) \qquad (5)$$

where $\beta_j = (\mu_j, v_j)$ is the jth largest of the α_i and α_i is the argument variable represented in the form of IFN.

E. The OWAWA Operator

The ordered weighted averaging – weighted average (OWAWA) operator is a new model that unifies the OWA operator and the weighted average in the same formulation. Its main advantage is that it can unify both concepts considering the degree of importance that each one has in the aggregation process. It can be defined as follows.

Definition 5. An OWAWA operator of dimension n is a mapping *OWAWA*: $R^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$OWAWA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} \hat{v}_j b_j \qquad (6)$$

where b_j is the jth largest of the a_i, each argument a_i has an associated weight (WA) v_j with $\sum_{j=1}^{n} v_j = 1$ and $v_j \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$ and v_j is the weight (WA) v_i ordered according to b_j, that is, according to the jth largest of the a_i.

As we can see, if $\beta = 1$, we get the OWA operator and if $\beta = 0$, the WA. The OWAWA operator accomplishes similar properties than the usual aggregation operators. Note that we can distinguish between descending and ascending orders, extend it by using mixture operators, and so on.

III. THE INTUITIONISTIC FUZZY OWAWA OPERATOR

The intuitionistic fuzzy ordered weighted averaging-weighted average (IFOWAWA) operator is an extension of the OWAWA operator that uses uncertain information in the aggregation represented in the form of IFNs. Note that the IFOWAWA can also be seen as an aggregation operator that uses the OWA operator, the WA and IFNs in the same formulation. The reason for using this operator is that sometimes, the uncertain factors that affect our decisions are not clearly known and in order to assess the problem we need to use IFNs. This operator can be defined as follows.

Definition 6. An IFOWAWA operator of dimension n is a mapping *I-IFOWAWA*: $\Omega^n \to \Omega$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$IFOWAWA(\alpha_1, \alpha_2, ..., \alpha_n) = \sum_{j=1}^{n} \hat{v}_j \beta_j \qquad (7)$$

where β_j is the jth largest of the α_i, each argument α_i has an associated weight (WA) v_j with $\sum_{j=1}^{n} v_j = 1$ and $v_j \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$ and v_j is the weight (WA) v_i ordered according to β_j, that is, according to the jth largest of the α_i.

Note that it is also possible to formulate the IFOWAWA operator separating the part that strictly affects the OWA operator and the WA.

Definition 8. An IFOWAWA operator of dimension n is a mapping *IFOWAWA*: $\Omega^n \to \Omega$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, and a

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weighting vector V that affects the WA, with $\sum_{i=1}^{n} \upsilon_i = 1$ and $\upsilon_i \in [0,1]$, such that:

$$IFOWAWA(\alpha_1, \alpha_2, ..., \alpha_n) = \beta \sum_{j=1}^{n} w_j \beta_j + (1-\beta) \sum_{i=1}^{n} \upsilon_i \alpha_i \qquad (8)$$

where β_j is the j th largest of the arguments α_i and $\beta \in [0,1]$. Note that if $\beta = 1$, we get the IFOWA operator and if $\beta = 0$, the IFWA operator.

In the following, we are going to give a simple example on how to aggregate with the IFOWAWA operator. We consider the aggregation with both definitions.

Example 1. Assume the following arguments in an aggregation process $((0.5,0.3),(0.4,0.5),(0.8,0.1),(0.6,0.3))$. Assume the following weighting vector $W = (0.2, 0.2, 0.3, 0.3)$ and the following probabilistic weighting vector $V = (0.3, 0.2, 0.4, 0.1)$. Note that the WA has a degree of importance of 70% while the weighting vector W of the OWA a degree of 30%. If we want to aggregate this information by using the IFOWAWA operator, we will get the following result. The aggregation can be solved either with Eq. (7) or Eq. (8).
With Eq. (6) we calculate the new weighting vector as:

$$\hat{\upsilon}_1 = 0.3 \times 0.2 + 0.7 \times 0.4 = 0.34$$
$$\hat{\upsilon}_2 = 0.3 \times 0.2 + 0.7 \times 0.1 = 0.13$$
$$\hat{\upsilon}_3 = 0.3 \times 0.3 + 0.7 \times 0.3 = 0.3$$
$$\hat{\upsilon}_4 = 0.3 \times 0.3 + 0.7 \times 0.2 = 0.23$$

And then, we calculate the aggregation process as follows:

$$IFOWAWA = 0.34 \times (0.8,0.1) + 0.13 \times (0.6,0.3) + $$
$$+0.3 \times (0.5,0.3) + 0.23 \times (0.4,0.5) = (0.63,0.23)$$

With Eq. (8), we aggregate as follows:

$$IFOWAWA = 0.3 \times (0.2 \times (0.8,0.1) + 0.2 \times (0.6,0.3) + $$
$$0.3 \times (0.5,0.3) + 0.3 \times (0.4,0.5)) + 0.7 \times (0.3 \times (0.5,0.3) + $$
$$0.2 \times (0.4,0.5) + 0.4 \times (0.8,0.1) + 0.1 \times (0.6,0.3)) = $$
$$(0.63,0.23)$$

Obviously, we get the same results with both methods.
From a generalized perspective of the reordering step, we can distinguish between the descending IFOWAWA (D-IFOWAWA) operator and the ascending IFOWAWA (A-IFOWAWA) operator by using $w_j = w_{n-j+1}^*$, where w_j is the j th weight of the D-IFOWAWA and $w_{n-j+1\ 1}^*$ the j th weight of the A-IFOWAWA operator.

Note that if the weighting vector is not normalized, i.e., $\hat{V}$ $= \sum_{j=1}^{n} \hat{\upsilon}_j \neq 1$, then, the IFOWAWA operator can be expressed as:

$$IFOWAWA(\alpha_1, \alpha_2, ..., \alpha_n) = \frac{1}{\hat{V}} \sum_{j=1}^{n} \hat{\upsilon}_j \beta_j \qquad (9)$$

The IFOWAWA operator is monotonic, bounded and idempotent. These properties can be proved with the following theorems. It is monotonic because $\alpha_i \geq \alpha_i'$ for all i, then

$$IFOWAWA(\alpha_1, \alpha_2, ..., \alpha_n) \geq IFOWAWA(\alpha_1', \alpha_2', ..., \alpha_n')$$

. It is commutative because any permutation of the arguments has the same evaluation. That is,

$$IFOWAWA(\alpha_1, \alpha_2, ..., \alpha_n) \geq IFOWAWA(\alpha_1', \alpha_2', ..., \alpha_n')$$

, where $(\alpha_1', \alpha_2', ..., \alpha_n')$ is any permutation of the arguments $(\alpha_1, \alpha_2, ..., \alpha_n)$. It is bounded because the IFOWAWA aggregation is delimitated by the minimum and the maximum. That is, $Min(\alpha_i) \leq IFOWAWA(\alpha_1, ..., \alpha_n) \leq Max(\alpha_i)$. It is idempotent because if $\alpha_1 = \alpha$, for all i $IFOWAWA(\alpha_1, \alpha_2, ..., \alpha_n) = \alpha$.

IV. FAMILIES OF IFOWAWA OPERATORS

In this Section we analyze different families of IFOWAWA operators. The main advantage is that we can consider a wide range of particular cases that can be used in the IFOWAWA operator leading to different results. Thus, we are able to provide a more complete representation of the aggregation process.

Remark 1. First of all we are going to consider the two main cases of the IFOWAWA operator that are found by analyzing the coefficient β. Basically:

- If $\beta = 0$, we get the IFWA
- If $\beta = 1$, we get the IFOWA.
- Note that when β increases, we are giving more importance to the IFOWA operator and when β decreases, we give more to the IFWA.

Remark 2. Another group of interesting families are the maximum-IFWA, the minimum-IFWA, the step-IFOWAWA operator and the usual average.

- The maximum-IFWA is found when $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$.

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- The minimum-IFWA is found when $w_n = 1$ and $w_j = 0$, for all $j \neq 1$.

- More generally, the step- I-IFOWAWA is formed when $w_k = 1$ and $w_j = 0$, for all $j \neq k$.

Remark 3. For the median- IFOWAWA, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_j = 0$ for all others. If n is even, then we assign $w_{n/2} = w_{(n/2)+1} = 0.5$.

Remark 4. Other families of IFOWAWA operators can be constructed by choosing a different weighting vector. For example, when $w_j = 1/m$ for $k \leq j \leq k+m-1$ and $w_j = 0$ for $j > k+m$ and $j < k$, we obtain the window-IFOWAWA operator. Note that k and m must be positive integers such that $k+m-1 \leq n$.

Remark 5. Another particular case is the Olympic-IFOWAWA. This operator is found when $w_1 = w_n = 0$ and for all others $w_j = 1/(n-2)$. Note that if $n = 3$ or $n = 4$, the Olympic- IFOWAWA is transformed in the median-IFOWAWA and if $m = n-2$ and $k = 2$, the window-IFOWAWA is transformed in the Olympic- IFOWAWA.

Remark 6. Note that other families of IFOWAWA operators could be used following the recent literature about different methods for obtaining the OWAWA weights such as [1, 11-17].

V. APPLICATION IN BUSINESS DECISION-MAKING

In the following, we are going to develop an illustrative example of the new approach in a decision making problem. We will study an investment selection problem where an investor is looking for an optimal investment. Note that other decision making applications could be developed such as the selection of human resources [18], etc.

Assume an investor wants to invest some money in an enterprise in order to get high profits. Initially, he considers five possible alternatives.

- A_1 is a computer company.
- A_2 is a chemical company.
- A_3 is a food company.
- A_4 is a car company.
- A_5 is a TV company.

In order to evaluate these investments, the investor uses a group of experts. This group of experts considers that the key factor is the economic environment of the economy. After careful analysis, they consider five possible situations:

- S_1 = Negative-growth rate.
- S_2 = Growth rate near 0.
- S_3 = Low-growth rate.
- S_4 = Medium-growth rate.
- S_5 = High-growth rate.

The expected results depending on the situation S_i and the alternative A_k are shown in Tables I. Note that the results are IFNs.

In this problem, the decision maker assumes the following degrees of importance (IFWA) of the characteristics: $V = (0.1, 0.2, 0.2, 0.2, 0.3)$. He assumes that the IFOWA weight is: $W = (0.1, 0.1, 0.2, 0.2, 0.4)$. Note that IFWA has an importance of 70% and the IFOWA an importance of 30% ($\beta = 0.3$) because he believes that the IFWA is more relevant in the problem.

TABLE I
AVAILABLE STRATEGIES

	S_1	S_2	S_3	S_4	S_5
A_1	(0.5,0.4)	(0.6,0.2)	(0.4,0.4)	(0.7,0.1)	(0.5,0.5)
A_2	(0.6,0.2)	(0.9,0.1)	[(0.7,0.2)	(0.3,0.5)	(0.4,0.5)
A_3	(0.4,0.4)	(0.6,0.3)	(0.8,0.1)	(0.5,0.4)	(0.6,0.3)
A_4	(0.7,0.2)	(0.4,0.6)	(0.5,0.3)	(0.3,0.6)	(0.7,0.1)
A_5	(0.8,0.2)	(0.5,0.3)	(0.6,0.3)	(0.6,0.4)	(0.4,0.5)

With this information, we can make an aggregation to make a decision. In this example, we will consider the Max-IFWA, the Min- IFWA, the IFWA, the IFOWA and the IFOWAWA operators. The results are shown in Table II.

TABLE II
AGGREGATION RESULTS

	IFWA	IFOWA	IFOWAWA
A_1	(0.55,0.28)	(0.50,0.34)	(0.59,0.30)
A_2	(0.64,0.28)	(0.54,0.32)	(0.64,0.29)
A_3	(0.62,0.26)	(0.54,0.32)	(0.66,0.28)
A_4	(0.55,0.27)	(0.46,0.39)	(0.54,0.30)
A_5	(0.56,0.36)	(0.54,0.37)	(0.60,0.36)

As we can see, depending on the aggregation operator used, the ordering of the strategies may be different. Therefore, the decision about which strategy select may be also different.

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If we establish an ordering of the investments, a typical situation if we want to consider more than one alternative, we will get the following orders shown in Table III. Note that the first alternative in each ordering is the optimal choice.

TABLE III
ORDERING OF THE INVESTMENTS

	Ordering
IFWA	$A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$
IFOWA	$A_2 = A_3 \succ A_5 \succ A_1 \succ A_4$
IFOWAWA	$A_3 \succ A_2 \succ A_1 \succ A_5 \succ A_4$

As we can see, depending on the aggregator operator used, the ordering of the investments may be different. Then, it is clear that each particular case of the IFOWAWA may lead to different results and decisions. Obviously, the decision maker will select the particular case that it is in accordance with its interests.

VI. CONCLUSIONS

We have introduced the IFOWAWA operator. It is an aggregation operator that uses the OWA operator, the WA and uncertain information represented in the form of IFNs. It is very useful for uncertain situations where the decision maker can not assess the information with exact numbers or singletons but it is possible to assess it with IFNs. By using the IFOWAWA, we are able to e get a generalization that includes a wide range of intuitionistic fuzzy operators such as the IFWA and the IFOWA operators. We have also developed an application of the new approach in a financial decision making problem. We have studied an investment selection problem where a company is looking for its optimal investment. We have also seen that depending on the particular case of the IFOWAWA operator used the results may lead to different decisions. In future research we expect to develop further extensions by adding new characteristics in the problem such as the use of probabilistic aggregations. We will also consider other decision making applications such as human resource management, investment selection and product management.

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Zeng Shouzhen (M'07) was born inJiangxi (China) in 1981.He graduated from the Tianjin University and obtained the Master degree in applied mathematics in 2007. At present, he is studying his in-service doctor of applied statistics in Zhejiang Gongshang University. He has published more than 30 papers in journals, books and conference proceedings including journals such as *Statistics Research, Economic Research, Knowledge-based Systems and Group decision and Negotiation*. His main research fields are Aggregation Operators, Decision Making, Comprehensive evaluation and Uncertainty. Now he is a full-time Lecture in Zhejiang Wanli University.