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Robust Nonlinear Control of Two Links Robot manipulator and Computing Maximum Load

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***Abstract*—**A new robust nonlinear control scheme of a manipulator is proposed in this paper which is robust against modeling errors and unknown disturbances. It is based on the principle of variable structure control, with sliding mode control (SMC) method. The variable structure control method is a robust method that appears to be well suited for robotic manipulators because it requers only bounds on the robotic arm parameters. But there is no single systematic procedure that is guaranteed to produce a suitable control law. Also, to reduce chattring of the control signal, we replaced the *sgn* function in the control law by a continuous approximation such as tangant function. We can compute the maximum load with regard to applied torque into joints. The effectivness of the proposed approach has been evaluated analitically demonstrated through computer simulations for the cases of variable load and robot arm parameters.

***Keywords*—**Variable structure control, robust control, switching surface, robot manipulator.

I. INTRODUCTION

THE robotic control and its application are very popular research topics in control field as well as in industry automation. A robot manipulator is a highly nonlinear and dynamically coupled system, which is subject to disturbances and model uncertainties. The general control methods, such as computed torque method, PD control method, etc., will not render the expected performance with the presence of disturbances and model uncertainties. Also, a number of approaches have been proposed to develop controllers that are more robust so that their performance is not sensitive to modeling errors.

The sliding mode control (SMC) theory has been applied to robot manipulators for the last decade [1-5]. SMC is commonly favored as a powerful robust control method for its independence from parametric uncertainties and external disturbances under matching conditions. In general, SMC comprises a discontinuous control input that drives the control system toward a specified sliding surface. Usually, a large control gain formula is applied to handle the unknown parametric variations and external disturbances [6].

Here, we develop a class of sliding mode controllers to the case of two link elbow robot manipulator with variable structure control method [7, 8].

This paper organized as follows: the basic concept of sliding mode is presented in section III the manipulator dynamics is introduced and a new control structure is proposed developed. Simulation results are presented in section IV. Section V gives the conclusion.

II. SLIDING MODE CONCEPT

The basic idea behind adaptive control is that the controller gains gradually changes as parameters of the system being controlled evolve.[9] It is also possible to change the control signal abruptly on the basis of the state of system being controlled. Control systems of this type are referred to as variable structure systems (VSS)[10-12]. A block diagram of a variable structure controller for a robotic arm is shown in Fig. 1.

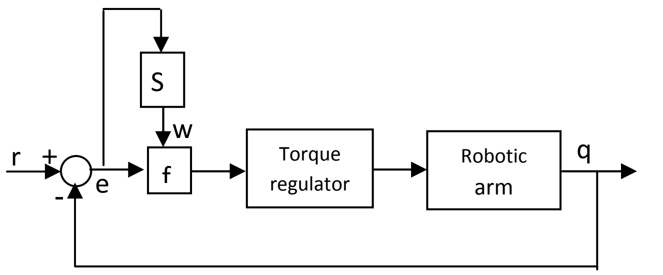


Fig. 1 Variable structure control of a robotic arm

To apply variable structure control, we do not have to know the exact robotic arm parameters, instead only bounds on these parameters. Variable structure controllers are robust in the sense that they are insensitive to errors in the estimates of the parameters as long as reliable bounds on the parameters are known. To formulate a variable structure control law, it is helpful to first recast the state equations in terms of the tracking error and its derivative. Suppose the reference input $r(t)$ is sufficiently smooth that is has at least one derivative. Define the state vector as $x \triangleq \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ where:

$$\begin{cases} x_1 = e_1 = r_1 - q_1 \quad , \quad x_2 = \dot{e_1} = \dot{r_1} - \dot{q_1} \\ x_3 = e_2 = r_2 - q_2 \quad , \quad x_4 = \dot{e_2} = \dot{r_2} - \dot{q_2} \end{cases} \tag{1}$$

Given a reference trajectory $r(t)$, the objective is to find a variable structure control law $\tau = f(x)$ such that the solution of the closed loop system satisfies $\lim_{t\to\infty} x(t) = 0$. Consider the following linear constraint on the state variables:

$$\sigma(x) \triangleq F\boldsymbol{e} + \dot{\boldsymbol{e}} = 0 \quad (2)$$

Here F can be any positive definite matrix. For example, F might be a diagonal matrix with positive diagonal elements:

$$F = diag\{f_1, f_2, \dots, f_n\} \qquad f_i > 0 \quad (3)$$

The set of all x such that $\sigma(x) = 0$ is a (*2n-1*)_dimensional subspace or "hyperplane" in $\mathbb{R}^{2n}$ which we refer to as the *switching surface*. The switching surfaces divide the state space into two regions. If $\sigma(x) > 0$, then we are on one side of the switching surface and the control law will have one form; if $\sigma(x) < 0$, then we are on the another side of the switching surface and the control law will have a different form. Thus the controller changes the structure when the state of the system crosses the switching surface. In this paper, the switching surface is a plane for *n=2*.

Our objective is to devise a control law $\tau = f(x)$ which will drive the system to the switching surface in a finite time and then constrain the system to stay on the switching surface. When the system is operating on the switching surface, we say that it is in the sliding mode. The dynamics of the system simplify substantially in the sliding mode. If $\sigma = 0$, then from Eq.2 the state equation for $\dot{e}$ reduced to:

$$\dot{e} + Fe = 0 \quad (4)$$

Thus when the system is in the sliding mode the tracking error is independent of the robotic arm parameters. The solution depends only on the matrix *F*, which is a design parameter called the sliding mode gain matrix. Recall from Eq.3 that *F* is a diagonal matrix with positive diagonal elements. Consequently, the sliding mode equation is not only linear but also uncoupled and the solution is:

$$e_k(t) = \exp(-f_k t) \,.\, e_k(0) \quad (5)$$

Clearly $\begin{cases} e(t) \to 0 \\ t \to 0 \end{cases}$. not only does the error go to zero by *sliding* down the switching surface, but the rate at which the error decreases can be controlled through the specification of the gain *F*, which controls the slope of the surface. Since Eq.(5) is totally independent of the robotic arm parameters, the variable structure control system is robust when it is operating in the sliding mode. There remains the problem of developing a control law $\tau = f(x)$ which will ensure that the system operates in the sliding mode. To develop a suitable control law, we make use of liapunov techniques. Consider, in particular, the following function:

$$V_L(x) = \frac{\sigma^T(x).\sigma(x)}{2} \quad (6)$$

This is a liapunov – type function in the sense that $V_L(x)$ is continuously differentiable and $\begin{cases} V_L(x) \geq 0 \\ V_L(x) = 0 \Leftrightarrow \sigma(x) = 0 \end{cases}$

To show that the solution of the closed loop system approaches the switching surface, it is sufficient to show that, along solution of the dynamics robot:

$$\dot{V}_L(x(t)) \leq 0$$
$$\dot{V}_L(x(t)) \equiv 0 \Rightarrow \sigma(x(t)) \equiv 0 \quad (7)$$

Where $\dot{V}_L(x(t)) = \sigma^T(x).\dot{\sigma}_L(x)$.
However, this would only guarantee that the solution approaches the switching surface in the limit as $t \to \infty$.

III. System Dynamics and the Control Structure

We must find the control law $\tau = f(x)$ such that $\sigma^T(x).\sigma(x) \leq -\gamma\|\sigma(x)\|$ for some $\gamma > 0$. For an *n-axis* robot, this is challenging task. To illustrate the derivation of a control law, we examine a special case, the two-axis robot in Fig. 2.

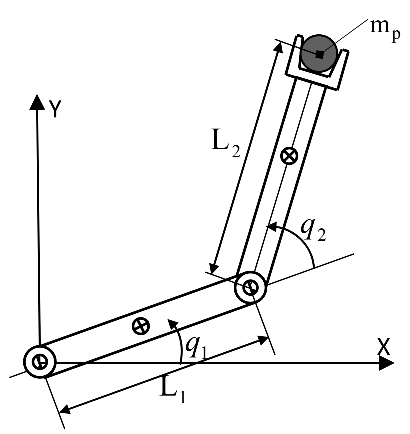


Fig. 2 Schematic of two axis robot

The dynamic equation of a rigid robot manipulator is shown as follows:

$$D\ddot{q} + H + G = \tau$$

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}, \ddot{q} = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}, H = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}, \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (8)$$

Where $q(t) \in R^2$ and suppose the links mass m_1 and m_2 , load mass m_p, links length L_1 and L_2, center of links length L_{c_1} and L_{c_2} and moment inertia I_1 and I_2:

$$d_{11} = a_1 + a_2 + 2a_3 \cos q_2 \;, d_{12} = d_{21} = a_2 + a_3 \cos q_2$$
$$d_{22} = a_2$$

$$h_1 = -a_3 \sin q_2 (2\dot{q}_1\dot{q}_2 + \dot{q}_2^2) \;, \; h_2 = a_3 \sin q_2 (\dot{q}_1^2)$$

$$g_1 = g(a_4 \cos q_1 + a_5 \cos(q_1 + q_2))$$

$$g_2 = g a_5 \cos(q_1 + q_2)$$

$$a_1 = I_1 + m_1 L_{c_1}^2 + (m_2 + m_p) L_1^2$$

$$a_2 = I_2 + m_2 L_{c_2}^2 + m_p L_2^2$$

$a_3 = L_1(m_2 L_{c_2} + m_p L_2)$, $a_4 = m_1 L_{c_1} + (m_2 + m_p) L_1$,
$a_5 = m_2 L_{c_2} + m_p L_1$ (9)

Using the Chang of variables:

$$\begin{cases} x_1 = e_1 = r_1 - q_1 \quad , \quad x_2 = \dot{e}_1 = \dot{r}_1 - \dot{q}_1 \\ x_3 = e_2 = r_2 - q_2 \quad , \quad x_4 = \dot{e}_2 = \dot{r}_2 - \dot{q}_2 \end{cases} \quad (10)$$

the state space is as follows:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{r}_1 - \frac{d_{22}\tau_1 + d_{22}h_1 - d_{22}g_1 - d_{12}\tau_2 + d_{12}h_2 + d_{12}g_2}{\Delta}$$

$$\dot{x}_3 = x_4 \quad (11)$$

$$\dot{x}_4 = \ddot{r}_2 - \frac{-d_{12}\tau_1 - d_{12}h_1 + d_{12}g_1 + d_{11}\tau_2 - d_{11}h_2 - d_{11}g_2}{\Delta}$$

Where:

$$h_1 = (m_2 L_1 L_{c_2} + m_p L_1 L_2).\sin(r_2 - x_3).[2(\dot{r}_1 - x_2).(\dot{r}_2 - x_4) + (\dot{r}_2 - x_4)^2]$$

$$h_2 = (m_2 L_1 L_{c_2} + m_p L_1 L_2)\sin(r_2 - x_3)(\dot{r}_1 - x_2)^2$$

$$g_1 = g_0\left((m_1 L_{c_1} + m_2 L_1 + m_p L_1)\right)\cos(r_1 - x_1) + (m_2 L_{c_2} + m_p L_2)\cos(r_1 - x_1 + r_2 - x_3)$$

$$g_2 = g_0(m_2 L_{c_2} + m_p L_2) + \cos(r_1 - x_1 + r_2 - x_3)$$

$$\Delta = d_{11} d_{22} - d_{12}^2 \quad (12)$$

To develop a control law at first, bounds on the values of the robot parameters must be established, including the rate of change of the reference trajectory. Suppose:

$$|\dot{r}_1| < R_{11} \;, \quad |\dot{r}_2| < R_{21} \;, \quad |\ddot{r}_1| < R_{12} \;, |\ddot{r}_2| < R_{22}$$

$$|d_{11}| < d_{11}^* \;, |d_{12}| < d_{12}^* \;, |d_{22}| < d_{22}^* \quad (13)$$

Now the follow inequality can be written:

$$|h_1| < h_{10}(2R_{11}R_{21} + 2R_{11}|x_4| + 2R_{21}|x_2| + 2|x_2||x_4| + R_{21}^2 + 2R_{21}|x_4| + |x_4|^2)$$

$$h_{10} = max\{m_2 L_1 L_{c_2} + m_p L_1 L_2\}$$

$$|h_2| < h_{20}(R_{11}^2 + 2R_{11}|x_2| + |x_2|^2)$$

$$h_{20} = h_{10}$$

$$|g_1| < g_{10} = max\{g_0(m_1 L_{c_1} + m_2 L_1 + m_p L_1 + m_2 L_{c_2} + m_p L_2)\}$$

$$|g_2| < g_{20} = max\{g_0(m_2 L_{c_2} + m_p L_2)\}$$

$$\Delta_0 < |\Delta| < \Delta_1 \quad (14)$$

Using $V(x) = \frac{1}{2}\sigma_1^2(x) + \frac{1}{2}\sigma_2^2(x)$ as liapunov function and considerig the sliding surface as follow:

$$\sigma(x) = \begin{bmatrix} \sigma_1(x) \\ \sigma_2(x) \end{bmatrix} = Fe + \dot{e} = Fx + \dot{x} = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} f_1 e_1 + \dot{e}_1 \\ f_2 e_2 + \dot{e}_2 \end{bmatrix} = \begin{bmatrix} f_1 x_1 + x_2 \\ f_2 x_3 + x_4 \end{bmatrix} \quad (15)$$

now $\dot{V}(x) = \sigma_1(x)\dot{\sigma}_1(x) + \sigma_2(x)\dot{\sigma}_2(x)$ can evaluated along solution of (11) and it must be negative. In the other hand, the following inequality must be shown:

$$\dot{V}(x) \le -\gamma(x)$$

Thus:

$$\sigma_1(x)\dot{\sigma}_1(x) = \sigma_1(x)[f_1\dot{x}_1 + \dot{x}_2] = \sigma_1(x)\left[f_1 x_2 + \ddot{r}_1 - \frac{d_{22}\tau_1 + d_{22}h_1 - d_{22}g_1 - d_{12}\tau_2 + d_{12}h_2 + d_{12}g_2}{\Delta}\right] = -\frac{\sigma_1(x)}{\Delta}d_{22}\tau_1 + \frac{\sigma_1(x)}{\Delta}d_{12}\tau_2 + \sigma_1(x)\left[f_1 x_2 + \ddot{r}_1 - \frac{d_{22}h_1 - d_{22}g_1 + d_{12}h_2 + d_{12}g_2}{\Delta}\right] \quad (16)$$

With considering $-\frac{d_{22}}{\Delta}\tau_1 + \frac{d_{12}}{\Delta}\tau_2 = \tau^*$ and using triangular inequality ($|x \mp y| < |x| + |y|$), and Eq. (13),(14):

$$\sigma_1(x)\dot{\sigma}_1(x) < \tau^*\sigma_1(x) + |\sigma_1(x)|\{p_1|x_2| + p_2|x_4| + p_3|x_2|^2 + p_4|x_4|^2 + p_5|x_2||x_4| + p_6\} \quad (17)$$

Where:

$$p_1 = f_1 + \frac{d_{22}^*}{\Delta_0}h_{10}(2R_{21}) + \frac{d_{12}^*}{\Delta_0}h_{20}(2R_{11}),$$

$$p_2 = \frac{d_{22}^*}{\Delta_0}h_{10}(2R_{11} + 2R_{21}) \;, p_3 = \frac{d_{12}^*}{\Delta_0}h_{20} \;, p_4 = \frac{d_{22}^*}{\Delta_0}h_{10} \;, p_5 = 2\frac{d_{22}^*}{\Delta_0}h_{10} \;,$$

$$p_6 = R_{12} + \frac{d_{22}^*}{\Delta_0}h_{10}(2R_{11}R_{21} + R_{21}^2) + \frac{d_{22}^*}{\Delta_0}g_{10} + \frac{d_{12}^*}{\Delta_0}h_{20}(R_{11}^2) + \frac{d_{12}^*}{\Delta_0}g_{20} \quad (18)$$

And with considering $\frac{d_{12}}{\Delta}\tau_1 - \frac{d_{11}}{\Delta}\tau_2 = \tau^{**}$ and using triangular inequality ($|x \mp y| < |x| + |y|$), and Eq. (13),(14):

$$\sigma_2(x)\dot{\sigma}_2(x) < \tau^{**}\sigma_2(x) + |\sigma_2(x)|\{p_7|x_2| + p_8|x_4| + p_9|x_2|^2 + p_{10}|x_4|^2 + p_{11}|x_2||x_4| + p_{12}\} \quad (19)$$

Where:

$$p_7 = \frac{d_{12}^*}{\Delta_0}h_{10}(2R_{21}) + \frac{d_{11}^*}{\Delta_0}h_{20}(2R_{11}) \;,$$

$$p_8 = f_2 + \frac{d_{12}^*}{\Delta_0}h_{10}(2R_{11} + 2R_{21}) \;,$$

$$p_9 = \frac{d_{11}^*}{\Delta_0}h_{20} \;, \quad p_{10} = \frac{d_{12}^*}{\Delta_0}h_{10} \;, p_{11} = 2\frac{d_{12}^*}{\Delta_0}h_{10}$$

$$p_{12} = R_{22} + \frac{d_{12}^*}{\Delta_0}h_{10}(2R_{11}R_{21} + R_{21}^2) + \frac{d_{12}^*}{\Delta_0}g_{10} + \frac{d_{11}^*}{\Delta_0}h_{20}(R_{11}^2) + \frac{d_{11}^*}{\Delta_0}g_{20} \quad (20)$$

Thus in general:

$\dot{V}(x) = \sigma_1(x)\dot{\sigma_1}(x) + \sigma_2(x)\dot{\sigma_2}(x) < \tau^*\sigma_1(x) + \tau^{**}\sigma_2(x) + |\sigma_1(x)|\{p_1|x_2| + p_2|x_4| + p_3|x_2|^2 + p_4|x_4|^2 + p_5|x_2||x_4| + p_6\} + |\sigma_2(x)|\{p_7|x_2| + p_8|x_4| + p_9|x_2|^2 + p_{10}|x_4|^2 + p_{11}|x_2||x_4| + p_{12}\}$ (21)

Now the virtual control law $\tau^*(x)$ and $\tau^{**}(x)$ must be selected in such a way that it dominates the other positive terms. As a candidate, consider the following virtual variable structure control law:

$\tau^*(x) = -\frac{d_{22}}{\Delta}\tau_1 + \frac{d_{12}}{\Delta}\tau_2 = -k_1\sigma_1(x).sgn\{\sigma_1(x)\} - k_2\sigma_1(x).sgn\{\sigma_1(x).x_2\}.x_2 - k_3\sigma_1(x).sgn\{\sigma_1(x).x_4\}.x_4 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_2^2 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_4^2 - k_4\sigma_1(x).sgn\{\sigma_1(x).x_2.x_4\}.x_2.x_4$

$\tau^{**}(x) = \frac{d_{12}}{\Delta}\tau_1 - \frac{d_{11}}{\Delta}\tau_2 = -k_5\sigma_2(x).sgn\{\sigma_2(x)\} - k_6\sigma_2(x).sgn\{\sigma_2(x).x_2\}.x_2 - k_7\sigma_2(x).sgn\{\sigma_2(x).x_4\}.x_4 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_2^2 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_4^2 - k_8\sigma_2(x).sgn\{\sigma_2(x).x_2.x_4\}.x_2.x_4$ (22)

Here $k_i > 0$ are controller gains that remain to be determined.

By substiuting Eq. (22) in (21) and using identity $sgn(z).z = |z|$ yeilds:

$\dot{V}(x) < -k_1\sigma_1(x).sgn\{\sigma_1(x)\} - k_2\sigma_1(x).sgn\{\sigma_1(x).x_2\}.x_2 - k_3\sigma_1(x).sgn\{\sigma_1(x).x_4\}.x_4 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_2^2 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_4^2 - k_4\sigma_1(x).sgn\{\sigma_1(x).x_2.x_4\}.x_2.x_4 - k_5\sigma_2(x).sgn\{\sigma_2(x)\} - k_6\sigma_2(x).sgn\{\sigma_2(x).x_2\}.x_2 - k_7\sigma_2(x).sgn\{\sigma_2(x).x_4\}.x_4 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_2^2 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_4^2 - k_8\sigma_2(x).sgn\{\sigma_2(x).x_2.x_4\}.x_2.x_4 + |\sigma_1(x)|\{p_1|x_2| + p_2|x_4| + p_3|x_2|^2 + p_4|x_4|^2 + p_5|x_2||x_4| + p_6\} + |\sigma_2(x)|\{p_7|x_2| + p_8|x_4| + p_9|x_2|^2 + p_{10}|x_4|^2 + p_{11}|x_2||x_4| + p_{12}\} = |\sigma_1(x)|\{-k_1 - k_1x_2^2 - k_1x_4^2 + p_3|x_2|^2 + p_4|x_4|^2 + p_6\} + |\sigma_1(x).x_2|\{-k_2 + p_1\} + |\sigma_1(x).x_4|\{-k_3 + p_2\} + |\sigma_1(x).x_2.x_4|\{-k_4 + p_5\} + |\sigma_2(x)|\{-k_5 - k_5x_2^2 - k_5x_4^2 + p_9|x_2|^2 + p_{10}|x_4|^2 + p_{12}\} + |\sigma_2(x).x_2|\{-k_6 + p_7\} + |\sigma_2(x).x_4|\{-k_7 + p_8\} + |\sigma_1(x).x_2.x_4|\{-k_8 + p_{11}\}$ (23)

It is clear from expression (23) that $\dot{V}(x)$ can be made negative if the controller gains $k_i > 0$ are sufficiently large, that is, if :

$\{k_1 > p_3\} \cap \{k_1 > p_4\} \cap \{k_1 > p_6\}$, $k_2 > p_1$, $k_3 > p_2$, $k_4 > p_5$

$\{k_5 > p_9\} \cap \{k_5 > p_{10}\} \cap \{k_5 > p_{12}\}$, $k_6 > p_7$, $k_7 > p_8$, $k_8 > p_{11}$ (24)

From Eq. (22), the variable structure control law, is as follow:

$\tau_1 = -d_{11}[-k_1\sigma_1(x).sgn\{\sigma_1(x)\} - k_2\sigma_1(x).sgn\{\sigma_1(x).x_2\}.x_2 - k_3\sigma_1(x).sgn\{\sigma_1(x).x_4\}.x_4 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_2^2 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_4^2 - k_4\sigma_1(x).sgn\{\sigma_1(x).x_2.x_4\}.x_2.x_4] - d_{12}[-k_5\sigma_2(x).sgn\{\sigma_2(x)\} - k_6\sigma_2(x).sgn\{\sigma_2(x).x_2\}.x_2 - k_7\sigma_2(x).sgn\{\sigma_2(x).x_4\}.x_4 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_2^2 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_4^2 - k_8\sigma_2(x).sgn\{\sigma_2(x).x_2.x_4\}.x_2.x_4]$

$\tau_2 = -d_{12}[-k_1\sigma_1(x).sgn\{\sigma_1(x)\} - k_2\sigma_1(x).sgn\{\sigma_1(x).x_2\}.x_2 - k_3\sigma_1(x).sgn\{\sigma_1(x).x_4\}.x_4 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_2^2 - k_1\sigma_1(x).sgn\{\sigma_1(x)\}.x_4^2 - k_4\sigma_1(x).sgn\{\sigma_1(x).x_2.x_4\}.x_2.x_4] - d_{22}[-k_5\sigma_2(x).sgn\{\sigma_2(x)\} - k_6\sigma_2(x).sgn\{\sigma_2(x).x_2\}.x_2 - k_7\sigma_2(x).sgn\{\sigma_2(x).x_4\}.x_4 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_2^2 - k_5\sigma_2(x).sgn\{\sigma_2(x)\}.x_4^2 - k_8\sigma_2(x).sgn\{\sigma_2(x).x_2.x_4\}.x_2.x_4]$ (25)

Thus if the controller gains satisfy inequalities (24), then the variable structure control law in Eq. (25) will drive the two axis robot to the switching surface and then keep it there. Consequently, x_1 will track $r_1(t)$ and x_2 will track $r_2(t)$ with a performance that is independent of the robotic arm parameters.

IV. Simulation and Result

Consider the bouds of robotic parameters as follows:

$1.8 \le m_1 \le 2.2$, $1.8 \le m_2 \le 2.2$, $0.4 \le L_1 \le 0.6$, $0.4 \le L_2 \le 0.6$

$0.2 \le L_{c_1} \le 0.3$, $0.2 \le L_{c_2} \le 0.3$, $0.024 \le I_1 \le 0.066$, $0.024 \le I_2 \le 0.066$

and suppose the reference trajectory as follows:

$r_1(t) = \pi\left[1 - exp\left(-\frac{t}{2}\right)\right] \Rightarrow \dot{r}_1(t) = \frac{\pi}{2}\left(exp\left(-\frac{t}{2}\right)\right) \Rightarrow \ddot{r}_1(t) = -\frac{\pi}{4}\left(exp\left(-\frac{t}{2}\right)\right)$

$r_2(t) = \pi\left[1 - exp\left(-\frac{t}{2}\right)\right] \Rightarrow \dot{r}_2(t) = \frac{\pi}{2}\left(exp\left(-\frac{t}{2}\right)\right) \Rightarrow \ddot{r}_2(t) = -\frac{\pi}{4}\left(exp\left(-\frac{t}{2}\right)\right)$

Then the following bounds can calculated:

$|\dot{r}_1| < R_{11} = 1.6$, $|\dot{r}_2| < R_{21} = 1.6$

$|\ddot{r}_1| < R_{12} = 0.8$, $|\ddot{r}_2| < R_{22} = 0.8$

$|d_{11}| < d_{11}^* = 11$, $|d_{12}| < d_{12}^* = 5$, $|d_{22}| < d_{22}^* = 3$

$h_{10} = max\{m_2L_1L_{c_2} + m_pL_1L_2\} = 2.56$

$h_{20} = h_{10}$

$|g_1| < g_{10} = max\{g_0(m_1L_{c_1} + m_2L_1 + m_pL_1 + m_2L_{c_2} + m_pL_2)\} = 97$

$|g_2| < g_{20} = max\{g_0(m_2 L_{c_2} + m_p L_2)\} = 42$

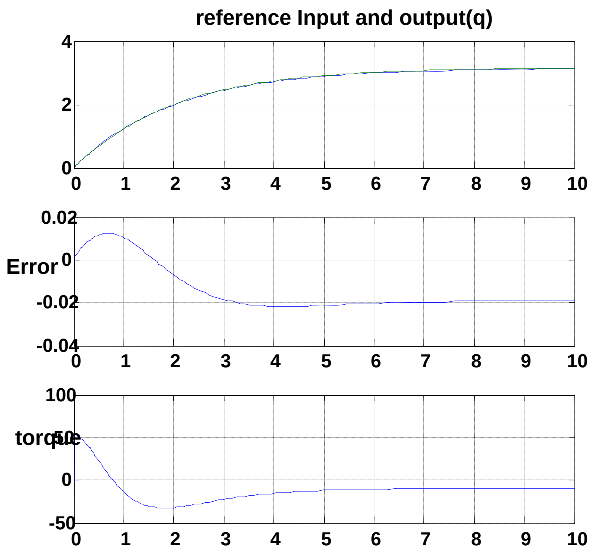
$\Delta_0 = 1.2$

To determine the constraints on the controller gains, first the slope of the switching surface have to be choosen. Suppose the sliding mode gains are $f_1 = 1$ and $f_2 = 1$. Then the constraints on the variable structure controller gains are:

$k_1 = 425$, $k_2 = 55$, $k_3 = 35$, $k_4 = 10$, $k_5 = 875$, $k_6 = 105$, $k_7 = 70$, $k_8 = 25$

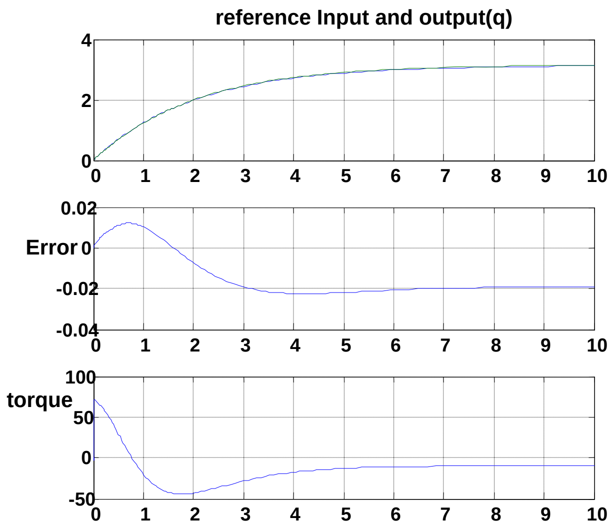
By applying the control law, the tracking error, applied torque and output are simulated at the Fig. 3 for load mass $m_p = 4, 6, 8\ Kg$:

$m_p = 4\ Kg$



(a)

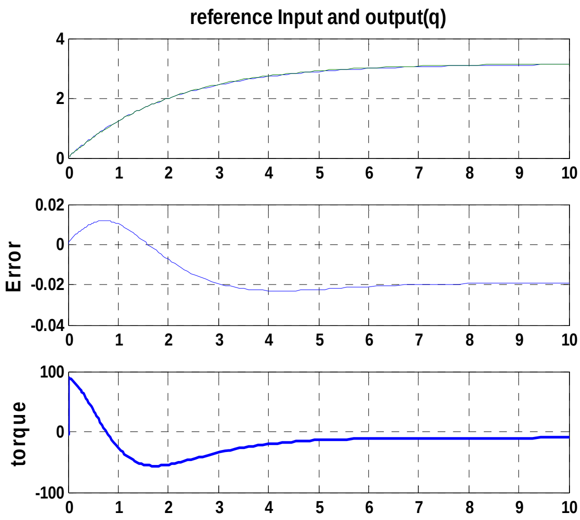
$m_p = 6\ Kg$



(b)

Fig. 3 The tracking error, applied torque and output

$m_p = 8\ Kg$



(c)

Now the values of robotic arm parameters as follows are changed:

$m_p = 6$, $m_1 = 2.2$, $m_2 = 2.2$, $L_1 = 0.6$, $L_2 = 0.6$, $L_{c_1} = 0.3$, $L_{c_2} = 0.3$, $I_1 = 0.066$, $I_2 = 0.066$

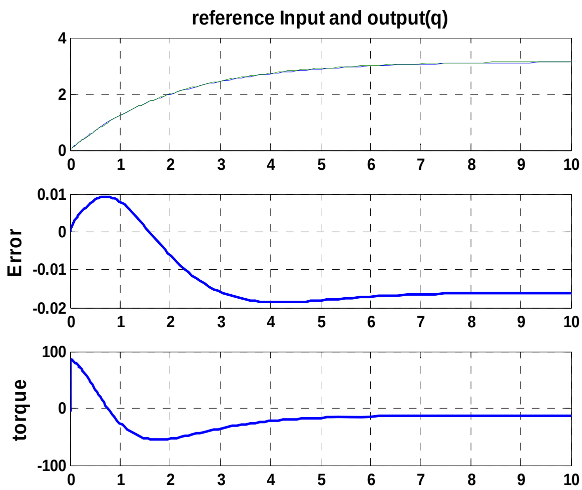


Fig. 4 Performance with variation of parameters

As shown in Fig. 4, for acceptable values of robotic arm parameters, the performance is unchanged. In the other hand, performance is robust against uncertainties and modelling error.

V. Conclusion

The variable structure control method is a robust method that appears to be well suited for robotic manipulators because it requers only bounds on the robotic arm parameters. However, the variable structure control method does have its drawback. One is that there is no single systematic procedure that is guaranteed to produce a suitable control law. Also, to reduce chattring of the control signal, we replaced the *sgn* function in the control law by a continuous approximation such as tangant function.

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