# Numerical Simulation of the Liquid-Vapor Interface Evolution with Material Properties

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*Abstract*—A satured liquid is warmed until boiling in a parallelepipedic boiler. The heat is supplied in a liquid through the horizontal bottom of the boiler, the other walls being adiabatic. During the process of boiling, the liquid evaporates through its free surface by deforming it. This surface which subdivides the boiler into two regions occupied on both sides by the boiled liquid (broth) and its vapor which surmounts it. The broth occupying the region and its vapor the superior region. A two- fluids model is used to describe the dynamics of the broth, its vapor and their interface. In this model, the broth is treated as a monophasic fluid (homogeneous model) and form with its vapor adiphasic pseudo fluid (two-fluid model). Furthermore, the interface is treated as a zone of mixture characterized by superficial void fraction noted $\alpha^*$. The aim of this article is to describe the dynamics of the interface between the boiled fluid and its vapor within a boiler. The resolution of the problem allowed us to show the evolution of the broth and the level of the liquid.

*Keywords*—Two-fluid models, homogeneous model, interface, averaged equations, Jumps conditions, void fraction.

## I. INTRODUCTION

SOME numerous problems linked to thermal transfers in fluids in transition's phase are still posed. For instance, the transfer of mass through a liquid-vapor interface and the effects of imbalance that arise from it are not cleared up in a satisfactory way. Also the dynamic of bubbly vapor within a liquid remains a difficult problem because of macroscopic phenomenons put in game such as thermal transfer, sticky dissipation, compressibility, and non-linear interactions between the bubble and le liquid. The problem becomes more complex when it is question of several bubbles. The complete formulation of such problems requires an important number of variables (spatio-temporal), that must check up a system of equations strongly non linear and including a lot of discontinuities. The numerical study of these problems is delicate, near the interfaces (fluid-fluid and solid-fluid) where we take part in the degradation the accuracy of these diagrams of integration (finite difference type). Despite those difficulties, some studies most of the times experimental have allowed to obtain some diagram or abacus extremely useful for the use; and also allowed to put into evidence phenomenons of instabilities linked to the coupling flow-change of phase [10].

We also find a number of articles treating in a basic way those subjects. For a dynamic of bubbly liquids, models for the obtaining of averaged equation (of continuities) are proposed by Drew and Segal and William [5 et 13] whereas Drumheller and Bedford [6] use a varitional approach. For the jumps' conditions at a fluid-fluid interface, Gatignol [7] use an asymptotic model to obtain jumps equations where [4] use the theory of non-linear distributions. We also know that for an interface without mass it is impossible to define a material velocity [11]. For interface between two interfering fluids Bedeaux [1] define the tangential component of the material velocity as being the average of tangential component of velocity in every fluid near the interface.

In this study we will use a different approach of those that have been made till now to define the material quantities of the interface.

This present article records itself in the thematic of numerical resolution of problems of thermal transfer in a two-phase flow. Therefore it will be articulated around a following point: modelling and numerical simulation of the evolution, a fluid-fluid interface with material properties.

The study will be led in respecting the following plan: In section 2 position of problem and basic hypothesis, section 3 general equation of the dynamic, section 4 equations of the dynamic, section 5 definition of material properties of the interface, section 6 additional modelling and closed equations, section 7 numerical modelling.

## II. PROBLEM POSITION AND BASIS HYPOTHESIS

The fundamental hypotheses that we will have to do all along this study are the followings:

**Interface:**
- Doted of material properties like the mass and interfacial energy
- Is a zone of mixture characterized by a sign of superficial space noted $\alpha^*$.
- Material surface that coincides every time with the geometrical surface of equation $z = h(t, x, y)$. We will note $\vec{W}$ the geometrical velocity of the interface. The spacial variations of $h$ are weak, therefore we will pose $h = \bar{h}$ where $\bar{h}$ indicates the average level of the interface.

**Vapor:**
- Check up the equation of perfect gas.

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**Broth:**

- The broth is considered as an incompressible pseudo fluid monophonic (homogeneous model) although formed of water and bubbly water.

**In the equations:**

- We will neglect the terms deriving from process of average in front of the other terms.

## III. GENERAL EQUATIONS OF THE DYNAMIC

The fluid system considered is formed of two separated phases. In most of the works marking on the general formulation on the dynamic of diphasic fluids, we begin to begin to give a local instantaneous formulation of fluid flow, then at middle quantities in order to obtain a system of average equations. The step that we adopt here is different from that classical step.

In fact we go from a global formulation of equations of the dynamic for each phase then we deduct a system of average equations. Let us go from the general equation of the following transfer

$$\frac{d}{dt}\int_V \beta_a \psi dv + \int \beta_a (\psi \vec{V} + \vec{f}).\vec{n}ds + \int_{\partial V} \beta_a \vec{f}.\vec{n}ds + \int_A \beta_a \psi (\vec{V} - \vec{V}_A).\vec{n}ds = \int_V \beta_a^A s dv \quad \text{a} = 1, \text{g} \quad (1)$$

In the equation (1), $\psi$ represents the density of a physical property of a fluid, $(\psi \vec{V} + \vec{f})$ is the flow of $\psi$, $\vec{V}$ is the local velocity of the fluid, $s$ is the external source of density, $\vec{V}_A$ is the geometrical velocity of the interface, $\vec{n}$ is a normal unit vector pointed external to $\partial V$, $\beta_a$ the function of presence defined by :

$$\beta_a(\vec{X},t) = \begin{cases} 1, \text{if } \vec{X} \text{ is internal to the phase } a \\ \quad \text{at instant } t \\ 0, \text{si } \vec{X} \text{ is external to the phase } a \\ \quad \text{at instant } t \end{cases} \quad (2)$$

The expressions of $\psi$, $\vec{f}$ and $s$ are given in the table below

TABLE I
EXPRESSION OF PHYSICAL QUANTITY IN FUNCTION OF PHYSICAL LAWS

| Quantity | $\psi$ | $\vec{f}$ | s |
|---|---|---|---|
| Mass | $\rho$ | 0 | 0 |
| Quantity of movement | $\rho\vec{V}$ | $-\Sigma$ | $\rho\vec{b}$ |
| Energy | $\rho(e + \frac{1}{2}\vec{V}^2)$ | $\vec{q} - \Sigma.\vec{V}$ | $\rho\vec{b}.\vec{V} + \rho Q$ |

## IV. AVERAGED EQUATIONS OF DYNAMIC

### A. Definition of Averaged Quantities

Let us design the average of all quantity $f(t,Y)$ by $\lfloor f(t,Y) \rfloor$. The spatio temporal average of $f$ is defined like this:

$$\lfloor f \rfloor_\tau (X,t) = \frac{1}{2\tau}\int_{t-\tau}^{t+\tau} f(\sigma, X)d\sigma \quad (3)$$

$$\lfloor g \rfloor_D (t) = \frac{1}{D}\int_D g(t, \vec{X})dX \quad (4)$$

In the above equalities $D$ represents domain occupied by the broth-vapor (internal volume of the boiler) and $D$ its volume.

Let us design $\psi^k$ a physical quantity attached to phase $k$. We define the intrinsic averages of vapor and broth respectively like this:

$$\overline{\psi}^+ (t) = \frac{1}{2\tau|D^+|}\int_{D^+}\int_{t-\tau}^{t+\tau} \psi^+ d\sigma dv \quad (5)$$

$$\overline{\psi}^- (t) = \frac{1}{2\tau|D^-|}\int_{D^-}\int_{t-\tau}^{t+\tau} \psi^- d\sigma dv \quad (6)$$

In the above relations $D^+$ and $D^-$ design respectively the domain occupied by the vapor-phase and the broth at this instant.

We also have the following relation:

$$\lfloor \psi^k \rfloor = \frac{1}{2\tau|D|}\int_D\int_{t-\tau}^{t+\tau} \alpha^k \psi d\sigma dv = \overline{\alpha}^k \overline{\psi}^k \quad (7)$$

where $\overline{\alpha}^k$ is the average void fraction of phase $k$.

### B. Homogeneous General Equations

In re-writing the equation (1) in the terms of our problem, it becomes:

$$\frac{d}{dt}\int_D \alpha^k \psi dv + \int \alpha^k (\psi\vec{V} + \vec{f}).\vec{n}ds + \int \alpha^k \vec{f}.\vec{n}ds + \int_F \alpha^k \psi (\vec{V} - \vec{V}_A).\vec{n}ds = \int_D \alpha^k s dv \quad (8)$$

When the boundaries conditions at the boiler are those of viscous fluid (adherence conditions), in applying the temporal average (3), then in dividing the result by the volume $D$ of the boiler, equation (8) becomes:

$$\frac{1}{|D|}\lfloor \frac{d}{dt}\int_D \alpha^k \psi dv \rfloor_\tau + \frac{1}{D}\lfloor \int_{\partial D} \alpha^k \vec{f}.\vec{n}dS \rfloor_\tau + \frac{1}{D}\lfloor \int_F \alpha^k \vec{f}.\vec{n}dS \rfloor_\tau + \frac{1}{D}\lfloor \int_F \alpha^k \psi (\vec{V} - \vec{W}).\vec{n}dS \rfloor_\tau = 0 \quad (9)$$

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We show that the normal unit vector $\vec{n}$ expresses itself in function of the interface displacement $h$ by the above relation:

$$\vec{n} = \frac{\vec{\nabla}h - \vec{k}}{\sqrt{1 + \left\|\vec{\nabla}h\right\|^2}} \approx \vec{\nabla}h - \vec{k} \qquad (10)$$

In substituting (10) in (9) it comes,

$$\frac{d}{dt}\lfloor \alpha^k \psi \rfloor + \frac{1}{D}\left\lfloor \int_{\partial D} \alpha^k \vec{f}.\vec{n}dS \right\rfloor_\tau + \frac{1}{D}\left\lfloor \int_\Pi \alpha^k \vec{f}.(\vec{k} + \vec{\nabla}h)dS \right\rfloor_\tau$$
$$+ \frac{1}{D}\left\lfloor \int_F \alpha^k \psi(\vec{V} - \vec{W})(\vec{k} + \vec{\nabla}h)dS \right\rfloor_\tau \approx 0 \qquad (11)$$

The spatial variations of $h$ being weak, we can neglect the terms comprising $\vec{\nabla}h$ in front of the other terms In this case equation (11) becomes

$$\frac{d}{dt}\lfloor \alpha^k \psi \rfloor + \frac{1}{D}\left\lfloor \int_{\partial D} \alpha^k \vec{f}.\vec{n}dS \right\rfloor_\tau + \frac{1}{H}\lfloor \alpha^k \vec{f}\vec{k} \rfloor_*$$
$$+ \frac{1}{H}\lfloor \alpha^k \psi(\vec{V} - \vec{W})\vec{k} \rfloor_* \approx 0 \qquad (12)$$

In the above equation $\lfloor . \rfloor_*$ designes a surfacic average defined by a quantity $g$ attached at the interface and is defined like this

$$\lfloor g \rfloor_*(t) = \frac{1}{2\tau|F|}\int_F \int_{t-\tau}^{t+\tau} g d\sigma d\eta \approx \frac{1}{2\tau|\Pi|}\int_\Pi \int_{t-\tau}^{t+\tau} g d\sigma dY \qquad (13)$$

Equation (12) will be modelled for each phase and for each law (mass, QM and energy) in order to obtain a homogeneous average equations system.

*C. Balance Equations of Emitted Vapor*

- Mass

In the case where $\psi$, $\vec{f}$ and $s$ take their corresponding valor at the mass (Box 1) equation (12) becomes

$$\frac{d}{dt}(\overline{\alpha\rho}^+) = \lfloor \alpha^+ \rho(\vec{V} - \vec{W}) \rfloor_* \qquad (14)$$

Where $\alpha^+$ is the presence function of the superior vapor near the interface

- Quantity of Movement

Now let's identify $\psi$, $\vec{f}$ and $s$ at their corresponding valor at the quantity of movement (box 1) then (12) becomes

$$\frac{d}{dt}(\overline{\alpha\rho}^+\overline{\vec{V}}^+) - \frac{1}{|D|}\sum_{k=0}^5 \int_{\Pi_k} \alpha_k^+ \Sigma.\vec{n}dY$$
$$+ \frac{1}{H}\lfloor \alpha^+\Sigma.\vec{k} \rfloor_* - \frac{1}{H}\lfloor \alpha^+\rho\vec{V}(\vec{V} - \vec{W}).\vec{k} \rfloor_* = 0 \qquad (15)$$

where $\alpha_k^+$ represents the presence function of vapor on face $k$ *of the* boiler. In taking account the adherence condition of the vapour phase on the faces, equation (15) is written:

$$\frac{d}{dt}(\overline{\alpha\rho}^+\overline{\vec{V}}^+) + \frac{1}{|D|}\sum_{k=0}^5 \int_{\Pi_k} \alpha_k^+ p\delta_{ij}n_{kj}dY$$
$$+ \frac{1}{H}\lfloor \alpha^+\Sigma.\vec{k} \rfloor_* - \frac{1}{H}\lfloor \alpha^+\rho\vec{V}(\vec{V} - \vec{W}).\vec{k} \rfloor_* = 0$$

be

$$\frac{d}{dt}(\overline{\alpha\rho}^+\overline{\vec{V}}^+) + \frac{1}{|H|}\sum_{k=0}^5 \lfloor \alpha^+ p \rfloor_{\partial\Pi} \delta_{ij}n_{kj}$$
$$+ \frac{1}{H}\lfloor \alpha^+\Sigma.\vec{k} \rfloor_* - \frac{1}{H}\lfloor \alpha^+\rho\vec{V}(\vec{V} - \vec{W}).\vec{k} \rfloor_* = 0 \qquad (16)$$

- Energy

Lastly in giving to $\psi$, $\vec{f}$ and $s$ their corresponding valor to the energy (box 1), and taking account $\vec{b} = \vec{0}$ (neglect the external body forces) then equation (12) becomes:

$$\frac{d}{dt}(\overline{\alpha\rho}^+\overline{e}^+) + \frac{1}{D}\sum_{k=0}^5 \int_{\Pi_k} \alpha^+ \vec{q}.\vec{n}dY$$
$$- \frac{1}{H}\lfloor \alpha^+(\vec{q} - \Sigma.\vec{V}).\vec{k} \rfloor_* - \frac{1}{H}\lfloor \alpha^+\rho e(\vec{V} - \vec{W}).\vec{k} \rfloor$$

now

$$\frac{1}{D}\int_{\Pi_k} \alpha^+ \vec{q}.\vec{n}_k dY = \overline{\alpha}_k^+ \gamma_k(T_{pk} - \overline{T}^+) \qquad (17)$$

therefore,

$$\frac{d}{dt}(\overline{\alpha\rho}^+\overline{e}^+) + \sum_{k=0}^5 \overline{\alpha}_k^+ \gamma_k(T_{pk} - \overline{T}^+)$$
$$- \frac{1}{H}\lfloor \alpha^+(\vec{q} - \Sigma.\vec{V}).\vec{k} \rfloor_* - \frac{1}{H}\lfloor \alpha^+\rho e(\vec{V} - \vec{W}).\vec{k} \rfloor \qquad (18)$$

- State Law

It will be a perfect gas given to the following relation:

$$\overline{p}^+ = M\overline{\rho}^+ R\overline{T}^+ \qquad (19)$$

where $\overline{p}^+$, $\overline{\rho}^+$ and $\overline{T}^+$ are respectively, the pressure, the density mass and the absolute temperature of the vapor. *R* and *M* are the constant of perfect gas and molar mass.

*D. Balance Equations of the Broth*

The balance equations and mass transfer, the quantity of movement and energy are respectively:

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$$\frac{d}{dt}[(1-\overline{\alpha})\overline{\rho}^{-}] = \left\lfloor (1-\alpha^{+})\rho(\vec{V}-\vec{W}) \right\rfloor_{*} \quad (20)$$

$$\frac{d}{dt}[(1-\overline{\alpha})\overline{\rho}^{-}\overline{\vec{V}}^{-}] + \frac{1}{|D|}\sum_{k=0}^{5}\left\lfloor \alpha_{k}^{-}p \right\rfloor_{\pi\Pi}\delta_{ij}n_{kj} \\ -\frac{1}{H}\left\lfloor \alpha^{-}\Sigma.\vec{k} \right\rfloor_{*} + \frac{1}{H}\left\lfloor \alpha^{-}\rho\vec{V}(\vec{V}-\vec{W}).\vec{k} \right\rfloor_{*} = 0 \quad (21)$$

$$H\frac{d}{dt}[(1-\overline{\alpha})\overline{\rho}^{-}\overline{e}^{-}] + H\sum_{k=0}^{5}\overline{\alpha}_{k}^{-}\gamma_{k}(T_{pk}-\overline{T}^{+}) \\ + \left\lfloor \alpha^{-}(\vec{q}-\Sigma.\vec{V}).\vec{k} \right\rfloor_{*} + \left\lfloor \alpha^{-}\rho e(\vec{V}-\vec{W}).\vec{k} \right\rfloor_{*} = 0 \quad (22)$$

However the state law of the broth is not classical but when the differences of temperature are weak, it is equal to [3]:

$$\frac{\overline{\alpha}^{-}}{x}\rho^{+} = \frac{1-\overline{\alpha}^{-}}{1-x}\rho^{-} \quad (23)$$

## V. Definition of Material Properties of the Interface

In our hypothesis the interface is a material surface formed of liquids and vapors zones (zone of mixture), characterized by a sign of superficial space noted $\overline{\alpha}^{*}$. This model allows us to define the interfacial body mass, the material velocity and the surfacic internal energy respectively as in [9] by the relations below:

$$\overline{\rho}^{*} = \overline{\alpha}^{*}\rho^{*} + (1-\overline{\alpha}^{*})\rho^{-*} \quad (24)$$

$$\overline{\rho}^{*}\vec{V}_{s} = \overline{\alpha}^{*}\overline{\rho}^{+}\vec{V}^{+} + (1-\overline{\alpha}^{*})\overline{\rho}^{-}\vec{V}^{-} \quad (25)$$

Furthermore we define the superficial enthalpy like this:

$$h_{e}^{*} = h_{e}^{-} + \overline{x}^{*}(h_{e}^{-}-h_{e}^{+}) \quad (26)$$

Where

$$\overline{x}^{*} = \frac{\overline{\rho}^{+}}{\rho^{-}}\overline{\alpha}^{*}$$

We deduct the expression of the following interfacial internal energy:

$$e_{s} = \frac{\overline{p}^{*}}{\rho^{*}} - h_{e}^{*} \quad (27)$$

where $\overline{p}^{*}$ is the average pressure of phase [9] we will define by the relation

$$\overline{p}^{*} = \overline{\alpha}^{*}\overline{p}^{+} + (1-\overline{\alpha}^{*})p^{-} \quad (28)$$

## VI. Additional Modellings and Closing Equations

### A. Additional Modellings
- Losses and flows through the interface

$$\frac{1}{H}\left\lfloor \alpha^{-}\vec{q}.n. \right\rfloor_{*} = \frac{1}{D}\int_{\Pi}\alpha^{-}\vec{q}.\vec{n}dY = \overline{\alpha}^{*}\gamma^{*}(T^{*}-\overline{T}^{-}) \quad (29)$$

- Calorific Capacity

The emitted vapor is considered as a perfect gas, its thermal capacity is given by the above relation

$$\overline{c}^{+} = \frac{7R}{2\Pi\overline{\rho}^{+}(H-h)}$$
$$\overline{c}^{+} = \frac{29,1}{\Pi\overline{\rho}^{+}(H-h)} \quad (30)$$

The thermal capacity of the water's vapor will be defined by the following expression:

$$c^{+} = 30.4 + 9.63*10^{-3}\overline{T}^{-} + 1.18*(\overline{T}^{-})^{2} \quad (31)$$

Heat transfer through the boiler's base is given the following relation of Jens and Lottes, that given [8]

$$\overline{\gamma}_{0}^{-}(T_{P_{0}}-\overline{T}^{-})^{n} = \frac{\exp^{4}(\overline{P}^{-}/62)}{25^{4}}(T_{P_{0}}-\overline{T}^{-})^{4} \quad (32)$$

We will take as expression of latent heat, the formula given by Regnault below,

$$S_{m} = 3340 - 2.93T . \quad (33)$$

We adopt for the pressure the following relation of Dupré (legitimate between 100°C and 200°C),

$$\overline{p}^{-} = \left(\frac{-273,15+\overline{T}^{-}}{100}\right)^{4} \quad 34)$$

### B. Modelling Dynamic Equations

In taking account hypothesis of the section 2 and the relations from (29) to (34) the averaged homogeneous equations of broth dynamic and its vapour, equations (14), (16), (18) and (20) to (22) becomes:
- Vapor phase equation:
*Mass:*

$$\overline{\rho}^{+}(\overline{\alpha}+\frac{1}{H})\frac{d\overline{h}}{dt} - \overline{\alpha}\frac{d\overline{\rho}^{+}}{dt} = \overline{\alpha}\rho^{+}\overline{V}_{z}^{+} \quad (36)$$

with

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$$\overline{\alpha} = 1 - \frac{\overline{h}}{H} \quad \text{and} \quad \overline{h} = \frac{1}{2\tau\Pi} \int\limits_{t-\tau}^{t+\tau}\int\limits_{\Pi} h\, dY \tag{37}$$

where $\overline{h}$ is the average level of broth. In continuing we will write simply $h$

*Quantity of movement:*

$$-\frac{\overline{\rho}^+\overline{V}_z^+}{H}(1-\overline{\alpha}^*)\frac{dh}{dt} + \overline{\alpha}\,\overline{V}_z^+\frac{d\overline{\rho}^+}{dt}$$
$$+ \overline{\alpha}\,\overline{\rho}^+\frac{d\overline{V}_z^+}{dt} = \frac{1}{H}[(1-\overline{\alpha}^*)\,\overline{p}^+ + \overline{\alpha}^*\,\overline{\rho}^+(\overline{V}_z^+)^2] \tag{38}$$

*Energy:*

$$-\frac{\overline{\rho}^+\overline{c}^+\overline{T}^+}{H}(1-\overline{\alpha}^*)\frac{dh}{dt} + \overline{\alpha}\overline{c}^+\overline{T}^+\frac{d\overline{\rho}^+}{dt} + \overline{\alpha}\overline{\rho}^+\overline{c}^+\frac{d\overline{T}^+}{dt}$$
$$= \frac{\overline{\alpha}^*}{H}\overline{\rho}^+\overline{c}^+\overline{T}^+V_z^+ + \overline{\alpha}^*\overline{p}^+V_z^+ - \overline{\alpha}^*\gamma^*(T^*-\overline{T}^+) \tag{39}$$

- Broth equation:

*Mass:*

$$\frac{\overline{\rho}^-}{H}(1-h)\frac{dh}{dt} + \frac{h}{H}\frac{d\overline{\rho}^-}{dt} = 0 \tag{40}$$

with

$$\overline{\rho}^- = \overline{\alpha}^-\rho^+ + (1-\overline{\alpha}^-)\rho^- \tag{41}$$

where $\overline{\alpha}^-$ is the average void fraction of vapor phase within the broth.

*Energy:*

$$\frac{h}{H}\overline{c}^-\overline{T}^-\frac{d\overline{\rho}^-}{dt} + \frac{h}{H}\overline{\rho}^-\overline{c}^-\frac{d\overline{T}^-}{dt} + \phi_0 = 0 \tag{42}$$

with

$$\phi_0 = \Pi\varphi_0 = \Pi\frac{\exp^4(\overline{P}^-/62)}{25^4}(T_{P_0} - \overline{T}^-)^4 \tag{43}$$

*C. Closing Equations*
- Interfacial balance equation of mass.

$$\frac{d}{dt}\overline{\rho}^* = \left\lfloor\alpha^*\rho(\vec{V}-\vec{W}).\vec{n}\right\rfloor_* + \left\lfloor(1-\alpha^*)\rho(\vec{V}-\vec{W}).\vec{n}\right\rfloor_*$$

becomes

$$\frac{d}{dt}\overline{\rho}^* = -\overline{\alpha}^*\overline{\rho}^+(\vec{V}_z^+ - \frac{dh}{dt}) - (1-\overline{\alpha}^*)\overline{\rho}^-\frac{dh}{dt} \tag{44}$$

where $\alpha^* = \alpha^+(t,x,y,h)$ is defined as in the relation (2) but on the interface. In fact $\alpha^*$ is a presence function of vapor

phase within the interfacial layer (homogenous model for the interface). Thus $\overline{\alpha}^*$ is the average void fraction of interface.

- Equation linking the evolutions of the interface and the axial velocity $\vec{V}_z^+$

We know that the quantity of evaporated water through the interface by unity of time (debit) is equal to the mass of vapor having left the interface during this time. This statement is translated by the following relation:

$$-(1-\overline{\alpha}^-)\rho^-\frac{dh}{dt} = \overline{\rho}^+\overline{V}_z^+ \tag{45}$$

- Relation between the broth void fraction and its quality
The expression of a mixture quality in function of void fraction is [2]

$$\overline{x}^- = \frac{\rho^+}{\rho^-}\overline{\alpha}^- \tag{46}$$

-Modelling $\gamma^*(T^*-T)$ term in the energy equation

We do hypothesis by that the heat received by the surmounting vapour is useful to rise temperature only. This is translated by the following relation:

$$\gamma^*(T^*-T) = \overline{\rho}^+\Pi(H-h)\overline{c}^+\frac{d\overline{T}^+}{dt} \tag{47}$$

## VII. NUMERICAL INVESTIGATION

In considering that $\overline{\alpha}^- \approx 0$, then equation (41) becomes:

$$\overline{\rho}^- \approx \rho^- \tag{48}$$

In this case, we can close the equations system. In fact, in taking account the previous relation, the equations (40), (42) and (45) become respectively

$$\frac{h}{H}\frac{d\overline{\rho}^-}{dt} \approx 0 \Rightarrow \overline{\rho}^- = C = \rho^- \tag{49}$$

$$\frac{h}{H}\rho^-c^-\frac{dT^-}{dt} + \Pi\frac{\exp^4(p^-/62)}{25^4}(T_{P_0}-T^-)^4 = 0 \tag{50}$$

$$-\rho^-\frac{dh}{dt} = \overline{\rho}^+\overline{V}_z^+ \tag{51}$$

The modelling equations ((36), (38), (39), (44), (47), (49), (50) et (51)) form a closed system of equations (of order 8) noted in a matrix way like this:

$$A(t,Y)\frac{dY}{dt} = b(t,Y) \tag{52}$$

where

$$Y = {}^t(h \quad \overline{\rho}^+ \quad \overline{V}_z^+ \quad \overline{T}^+ \quad \overline{T}^- \quad \rho^+ \quad \overline{\rho}^* \overline{\alpha}^*)$$

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In order to solve system (53) we must add the following initial conditions:

$$\overline{\rho}_0^{\,-} = 10^3 \ \text{kg/m}^3, \overline{\rho}_0^{\,+} = 0,590 \ kg/m^3,$$

$$\overline{T}_0^{\,-} = \overline{T}_0^{\,+} = 100°, \overline{p}_0^{\,+} = \overline{p}_0^{\,-} = 1 \, \text{atm}, \overline{\alpha}_0 = 1 - \frac{e}{H},$$

$$\overline{\alpha}_0^{\,-} = 0; \ \overline{\alpha}_0^{\,*} = \frac{1}{2}, \vec{W}_0 = \vec{V}_0^{\,+} = \vec{V}_0^{\,-} = \vec{V}_s = \vec{0}, h_0 = 0,1$$

Let us:

$$Y_0 = \begin{pmatrix} h_0 & \overline{\rho}_0^{\,+} & \overline{V}_{0z}^{\,+} & \overline{T}_0^{\,+} & \overline{T}_0^{\,-} & \rho_0^{\,+} & \overline{\rho}_0^{\,*} & \overline{\alpha}_0^{\,*} \end{pmatrix}$$

then problem (52) to be solved is written:

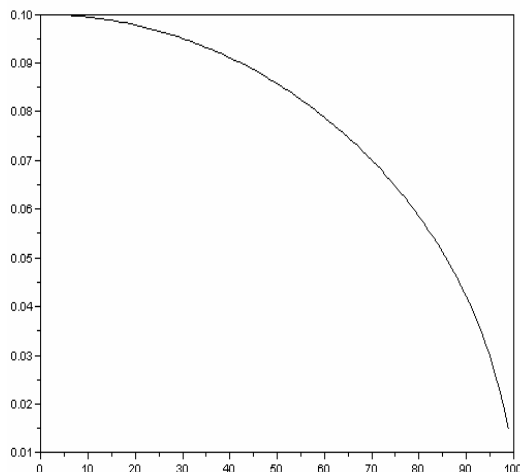$$\begin{cases} A(t,Y)\dfrac{dY}{dt} = b(t,Y) \\ Y_0 \end{cases} \tag{53}$$



Fig. 3 Graph of the evolution according to time

## VIII. ANALYSIS OF SOLUTIONS

The homogeneous problem is a simplified model of the general problem. In fact we had to solve an evolution problem (11) needing the initial conditions and boundaries.

We will note that in the present model the conditions to boundary appear directly in the equations. The equations system (53) is obtained under the implicit form, we solve it therefore by an implicit scheme with of *dassl* primitive. The resolution of the system (53) has allowed us to obtain the graph of function *h* (Fig. 1). We can see that the graph translating the interface evolution is decreasing. That expresses the mass transfer through the interface (loss of liquid).

## IX. CONCLUSION

This article presents a model to describe a dynamic of an interface between a boiled fluid (broth) and its vapor within a boiler. The system form by a broth is treated with the help of a two fluids model; the broth being considered as a pseudo-homogenous fluid (homogeneous model). Averaged quantities have been defined in each domain occupied by phases in interaction and their interface. Afterwards homogeneous equations (depending only of time t) translating phases in presence and their interface have been obtained. Additional hypothesis have been posed in order to obtain a closing of equation system obtained. The numerical resolution of equation of movement has been made, and the law of evolution of the interface and some parameters of the problem has been obtained. We have considered the interface as a material surface formed of liquid and vapor zones characterized by indication void fraction $\overline{\alpha}^{\,*}$. The definition of the interfacial void fraction has allowed us to define some material properties of the interface as the superficial density of mass, the material velocity, and surfacic internal energy.

The resolution of system of modelling equation, the problem has allowed us to obtain the evolution law of average level of the broth.

REFERENCES

[1] Bedaux D., Albano A. M., Mazur P., Boundary conditions and non equilibrium thermodynamics, Physica 82A, 1996, pp. 438-462.
[2] Bilicki Z., Michaelides E. E., Thermodynamic of bubbly none equilibrium in liquid-vapor flows, Valter Degruyer, 1997.
[3] Boure J. A., 1978, Two-phase flows and Heat transfer with application to Nuclear reactor Design problem, Hemisphere Pub Corp. Washington, 1978.
[4] Danho E., Non linear distributions theory for jump conditions in two-phase flows instabilities, In journal of Africa Mathematical Union, 1993.
[5] Drew D. A. and Segal L. A., Averaged equations for two-phase flow, Studies in Appl. Math. L, 1971, pp. 205-231.
[6] Drumheller, D. S. and Bedford, A., A theory of liquids with vapor bubbles. J. Acoust. Soc. Am. 667, 1971, pp. 186-200.
[7] Gatignol R., Dynamics of multiphase flows across Interfaces, edited by Annie Steichen, Springer-Verlag, 1996.
[8] Hewitt G. H., Two phase flows and heat transfer with application to nuclear reactor design problems, pp. 1-15.
[9] Hsieh D. Y., On dynamics of bubbly liquids, Advances in applied mechanics 26, 63-13, Academic Press, 1988.
[10] Lombard B, Piraux J., Numerical treatment of two-dimensional interfaces for acoustic and elastic waves, Journal of computational physics, 195-1, 2004, pp. 90-116.
[11] Properriti, A., Boundary conditions at liquid-vapor Interfaces, Meccanica 19, 1979 p p. 123-136.
[12] Roger P. and Thomas D. T., Computational Methods for Fluids Flows, Springer-Verlag New York, Heidelberg Berlin, 1983.
[13] William G. and Kevin O., On the general Equations for Flow in Porous media and their Reduction to Darcy's Law'' Water Ressources Research, Vol. 12. , n°2, April 1976 pp. 148-154.