Bounds on Reliability of Parallel Computer Interconnection Systems

Ranjan Kumar Dash and Chita Ranjan Tripathy

Abstract—The evaluation of residual reliability of large sized parallel computer interconnection systems is not practicable with the existing methods. Under such conditions, one must go for approximation techniques which provide the upper bound and lower bound on this reliability. In this context, a new approximation method for providing bounds on residual reliability is proposed here. The proposed method is well supported by two algorithms for simulation purpose. The bounds on residual reliability of three different categories of interconnection topologies are efficiently found by using the proposed method

Keywords—Parallel computer network, reliability, probabilistic graph, interconnection networks.

I. INTRODUCTION

ADVANCES in computer technology and the need to have the computers communicating with each other have led to an increased demand for reliable parallel computing systems. Parallel processing systems typically consists of an ensemble of processors. They have found wide applications real time environments and are becoming increasingly popular for large commercial applications as well. Such systems use various interconnection techniques to support communication between its resources. As the size of the interconnection structure increases, the presence of greater number of components makes the failure of some components increasingly likely. Consequently, the need for prediction of the system reliability becomes essential. An important performance metric in the design of highly reliable parallel computing system is provided by its reliability. The most conventional reliability modeling approaches use either graph theoretic technique [1], [3], [4], [5], [6], [7] and [18] or state space method [2]. However, the exact reliability computation of fault-tolerant parallel computing networks of large size becomes intractable with existing tools. For example, in using sum of disjoint techniques for a fault tolerant parallel computer interconnection networks of size higher than (32x32) the path enumeration and disjointing process takes enormous amount of computer time and memory. Similarly, in using Markov approaches the exponential growth of state space for network of larger size makes the construction and solution of a Markov chain computationally prohibitive [9]. Even if network decomposition techniques as proposed in [10] for finding reliability of very large size networks

become ineffective. In such a case, the only option left is to go for some simple and approximation method for computing the reliability of parallel computing systems. However, it is possible to obtain bounds on reliability much more efficiently for parallel computer interconnection networks. Finding the upper bound on reliability of parallel computer interconnection networks presents an optimistic view of the real world. The lower bound on the other hand, provides the probabilities that the parallel computer interconnection networks will still be operational at some specified time and it can be expected that the system is at least that much reliable. If the upper bound provides sufficient assurance that the network will be operational over the time interval of interest then, no further effort at obtaining a better approximation on finding the exact reliability expression is necessary. In the literature, some works have been reported on the upper and lower bounds of computer networks. There are quite a number of papers dealing with approximation algorithms for estimating reliability under the edge fault model and under the node fault model [11],[12], [13] and [14]. Colbourn [12] proposed a polynomial algorithm for certain restricted classes of graphs, including trees, series-parallel graphs, and permutation graphs. Ramanathan et al. [11] and Chen et al. [14] developed algorithms for arbitrary graphs. Chen et al. [14] estimated bounds on reliability of networks. But their method is unsuitable for parallel computer interconnection systems. However, the same with appropriate modifications can be used for parallel systems. The paper is organized as follow: Section II presents a background for the rest portion of this paper. In section III, a method has been proposed to compute the bounds on residual reliability of multicomputers. Three multicomputers namely Hypercube, Star Graph, Alternating Group graphs have been considered for the purpose of estimation of their bounds on residual reliability. Results and discussion are presented in Section IV. Section V concludes the paper with its future scope.

II. BACKGROUND

A. Network Details

Here, we present the details of two multicomputer networks: Alternating group graph (AGG) [16] and the Star graph [17].

1) Alternating Group Graph (AGG): The n-dimensional Alternating Group Graph, AGG_n (refer Fig.1) is a regular graph with $n!/2$ nodes, $n!(n-2)/2$ edges, node degree 2(n-2), and diameter $\lfloor 3n/2 \rfloor - 3$.

2) Star Interconnection Topology (S): The n-dimensional star graph also referred to as S_n or n-star (refer Fig.5.2) is

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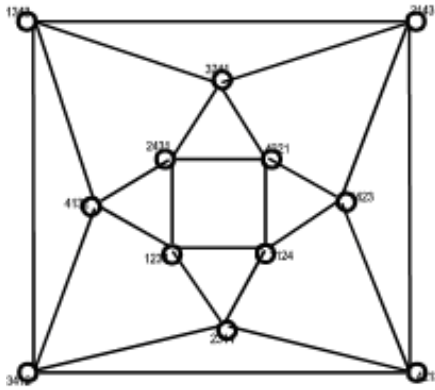
Fig. 1.   Alternating Group Graph (n=3)

an edge and node symmetric graph consisting of $n!$ vertices labeled with $n!$ permutations on n symbols. The network contains $\frac{(n-1)n!}{2}$ edges or links. Two nodes are joined with a link labeled i if and only if the label of one can be obtained from the label of the other by swapping the first digit and i^{th} digit, where $1 \le i \le n$. The n star can be partitioned into n copies of (n-1) stars.

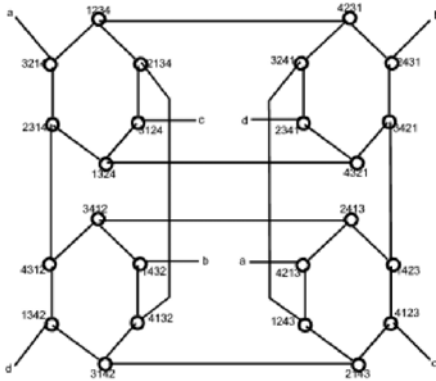
Fig. 2.   Star Interconnection Topology (n=3)

III. PROPOSED BOUNDS ON RESIDUAL RELIABILITY OF MULTICOMPUTER NETWORKS

In this section, the details of the proposed method for evaluation of bounds on reliability have been presented. It has been supported with two theorems along with proofs. Two algorithms, one for computation of upper bound measures and the other for computation of lower bounds measures have been proposed.

The following notation and assumptions have been made throughout this section.

A. Notations

    G *Probabilistic graph corresponding to the parallel computer system*

V *vertex set of graph G*
E *edge set of graph G*
$R_r(G)$ *Residual reliability*
N *number of nodes in the graph*
n *dimension of the graph*
p_p *node success probability*
p_l *edge success probability*
$N_G(v)$ *neighbor node set of node v in graph G.*
δR *difference between LB and UB i.e. $\delta R = UB - LB$*
t *time in hours*
A,B *event*
l *distance between nodes u, v*

B. Assumptions

1. Both the nodes (processors) and links are imperfect.
2. All failures are statistically independent.
3. Initially all the components are in good state and repair facility is not available.
4. The components may fail at random with constant link failure rate and processor failure rate.

C. Proposed Bounds on Multicomputers

Definition 1 The residual reliability is the probability of existence of a minimal set of good links such that all the nodes of the network remain connected. The Residual Reliability of a graph G is defined as Rr (G) = Pr the sub graphs induced by the surviving edges is connected

Definition 2 A k-distance set in multicomputer denoted by S, is such a subset of nodes that each pair of nodes has a distance of at least k in graph G, where Definition 3 A maximal k-distance set, denoted by Sk, is a k-independent set such that any extra node adding to set Sk will result in a network, where it will no longer be k-distance set. The following theorem proposes an upper bound on residual reliability of multicomputer networks: .

Theorem 1: If S_3 be a maximal 3-distance set and $r_3 = |S_3|$. Then the upper bound on residual reliability of multicomputer is given by

$$R_r(G) \le \prod_{i=1}^{r_3}((1-p_p)(1-(p_pp_l)^{f_i})) \qquad (1)$$

where $f_i = |N_G(u_i)|$, $N_G(u_i)$ being the neighbouring set of u_i in G
Proof:
Let A_i be the event that v_i is isolated in the induced graph, i=1,2,.n. Then the occurrences of any $A_1, A_2...A_n$ imply that does not occur. Consequently, we have
$\bigcup_{i=1}^{n} A_i \subseteq \overline{A}$
Let B_i be the event that v_i is isolated in the remaining sub graph of$G, i = 1, 2, 3, ..., n$ Then,
$\bigcup_{i=1}^{r_3} B_i \subseteq \bigcup_{i=1}^{n} A_i \subseteq \overline{A}$
$B_i$occurs if and only if v_i does not fail and for each neighbour

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node w of v_i , either w or edge wv_i fails i.e.

$$\begin{aligned} Pr(B_i) &= p_p(1-p_pp_l)^{|N_G(v_i)|} \\ &= p_p(1-p_pp_l)^{f_i}, i=1,2,3...,r_3 \end{aligned}$$

Since $B_1, B_2 \cdots B_{r_3}$ are independent of each other,

$$\begin{aligned} 1-R_r(G) &= Pr\{\overline{A}\} \geq Pr\{\bigcup_{i=1}^{r_3}\} \\ &= 1-Pr\{\bigcap_{i=1}^{r_3}\overline{B_i}\} \\ &= 1-\prod_{i=1}^{r_3}Pr\{\overline{B_i}\} \\ &= 1-\prod_{i=1}^{r_3}(1-p_p(1-p_pp_l))^{f_i} \end{aligned}$$

which follows,
$R-r(G) = 1-\prod_{i=1}^{r_3}(1-p_p(1-p_pp_l))^{f_i}$
The following theorem proposes lower bounds on residual reliability of multicomputer networks.

Theorem 2: The lower bounds on residual reliability of multicomputer networks is given by

$$\begin{aligned} R_r(G) \geq 1 &+ (N-)(1-p_p)^N - N(1-p_p)^{N-1} \\ &- \prod_{j\geq 1}(1-p_p^{j-1}p_i^j)^{m_j} \\ &+ (1-p_p)^{2m_3}((1-p_p)(1-p_p^2))^{m_4} \\ &\quad ((1-p_p^2)^2)^{m_5}(1-(1-p_p^2))^2 p_p)m_6 \cdots \\ &+ ((1-p_p)^{l-1})^{m_l}((1-p_p)^{l-2})^{m_{l+1}} \\ &\quad (1-p_p^2))((1-p_p)^{l-3})^{m_{l+2}}(1-p_p^2)) \\ &\quad (1-(1-(1-p_p)^2))^2(1-(1-p_p)^{l-2})^{m_l} \end{aligned}$$

Proof: The probability that all the nodes are failed is

$$P_0 = (1-p_p)^N \tag{2}$$

and the probabilities that out of N nodes one is working is

$$P_1 = Np_p(1-p_p)^{N-1} \tag{3}$$

The probability that two nodes u, v out of N nodes are working is given as bellow Let the distance between the two node u, v be l
Case-I: l=1 i.e. u, v adjacent nodes. Let there be m_j be the number of inner node disjoint paths between u, v with distance $j \geq 1$ and $m_j \geq 0$. For each path joining u and v , Case-I holds, if either at least one of the j-1 node fails or at least one of the j edge fails, so the probability is . Since there are m_j number of paths between u and v, so

$$P_{2_{l=1}} = \prod_{j\geq 2}(1-p_p^{j-1}p_l^j)^{m_j} \tag{4}$$

Case-II: l=2 Let there be m_j be the number of inner node disjoint paths between u, v with distance . For each path

joining u and v, case-II holds, if at least one inner node fails, so the probability is $(1-p_p^{j-1})^{m_j}$. Since there are mj number of paths between u and v, so

$$P_{2_{l=1}} = \prod_{j\geq 2}(1-p_p^{j-1})^{m_j} \tag{5}$$

Case-III: l=3 holds if
i. For every length-3 path, its two inner nodes have failed and the probability is $(1--p_p)^2$
ii. For every length-4 path, its two inner nodes have failed, while one of the 1st and 3rd nodes have failed, so the probability is $(1-p_p)(1-p_p)^2$
iii. For every length 5 path, either its 1st or its 3rd inner nodes has failed, while one of the 2nd and 4th inner node has failed, thus the probability will be $(1-p_p^2)^2)$.
iv. For every length-6 path, the five inner node must satisfy that the following event do not occur simultaneously: (i) the 3rd node is fault free (ii) either the 1st or 2nd node is fault free, and (iii) either the 4th or 5th node is fault free. Then the probability is $1-(1-(1-p_p)^2)^2 p_p$.
Let there be m_3, m_4, m_5 and m_6 number of paths of length 3,4,5,6 respectively. So,

$$\begin{aligned} P_3 &= (1-p_p)^{2m_3}((1-p_p)(1-p_p^2))^{m_4} \\ &\quad ((10p_p^2)^2)^{m_5}(1-(1-(1-p_p)^2)^2 p_p)^{m_4} \end{aligned}$$

Case-IV can be proved similar to Case-III.
So, from the Case-I to IV and from equations 3 and 4, it follows that

$$\begin{aligned} R_r(G) \geq 1 &+ (N-)(1-p_p)^N - N(1-p_p)^{N-1} \\ &- \prod_{j\geq 1}(1-p_p^{j-1}p_i^j)^{m_j} \\ &+ (1-p_p)^{2m_3}((1-p_p)(1-p_p^2))^{m_4} \\ &\quad ((1-p_p^2)^2)^{m_5}(1-(1-p_p^2))^2 p_p)m_6 \cdots \\ &+ ((1-p_p)^{l-1})^{m_l}((1-p_p)^{l-2})^{m_{l+1}} \\ &\quad (1-p_p^2))((1-p_p)^{l-3})^{m_{l+2}}(1-p_p^2)) \\ &\quad (1-(1-(1-p_p)^2))^2(1-(1-p_p)^{l-2})^{m_l} \end{aligned}$$

The following subsection proposes an algorithm for evaluation of upper bounds on residual reliability of Multicomputers. mds

D. Algorithm-I (Evaluation of Upper bounds on Residual Reliability of multicomputer networks)

Upper_Bound (G, pp)
S=NULL;
U=V(G);
While (U ¡¿NULL)
u=the node of V with minimum degree in G;
for every v of $N_G(u)$
$U = U (N_G(v)\bigcup v)$;
$R_r(G) = 1$;
For every node v_i of S
$R_r(G) = R_r(G)(1-p_p(1-p_pp_l)^{f_i}$

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Efficiency: To find an extreme 3-distance node set requires $O(N^2)$ operations and dominates other computational processes of the procedure. So the running time of the algorithm is $O(N^2)$.

In the following subsection, an algorithm has been proposed to compute lower bounds on residual reliability of multicomputer networks.

E. Algorithm-II (Evaluation of Lower bounds on Residual Relibility of multicomputer networks)

Lower_Bound(G, p_p, p_l)
k=3;
$R_r(G) = 1 + (N - 1) - (1 - p_p)^N - N(1 - p_p)^{N-1}$
for every pair of u,v in G
Path= shortest path between u and v
Dist=∥Path∥-1
l[Dist]=1
D[u,v]=Dist
U=V(G)
for(i=0;i¡=k;I++)
l[Dist+i]=0
while $(D[u, v] \leq Dist + k)$
U=(U Path) u,v
Path=the shortest path between u and v of U
j=∥P-1∥
l[j]=l[j]+1
D[u,v]=j
$R_r(G) = max\{0, Rr(G)\}$
$R_r(G) = R_r(G) - p(l, u, v)$

Efficiency: In the above procedure every short path needs $O(N^2)$ operations and there are at most N paths, so it requires $O(N^3)$ operations for every pair of nodes. Since there are $O(N^2)$ pairs of nodes. This procedure requires $O(N^5)$ operations. So the running time of this algorithm is $O(N^5)$.

The proposed algorithms I and II have been applied to different categories of multicomputers viz Hypercube [15], AGG and Star graph for the computation of the bounds on residual reliability. The results along with discussion are presented in the next section to follow.

IV. RESULTS AND DISCUSSIONS

The upper bound and lower bound on Residual reliability of hypercube (n=8) are plotted against the time in Fig.3. From this Figure, it is quite clear that at time 100 hrs with link failure rate λ=0.0001 and processor failure rate λ_p=0.001, the bounds on residual reliability of hypercube are having a small difference in their values. With increase in time, both the upper bound and lower bound decrease exponentially and approach to be the same at time 800 hrs.
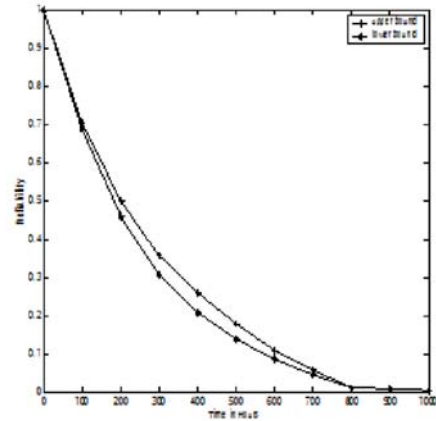

Fig. 3. Bounds on Residual Reliability of Hypercube (n=8)

Sl. No.	n	Upper bound on R_r	Lower bound on R_r	δR_r
1	3	1.0000	0.3958	0.6042
2	4	0.9243	0.4906	0.4337
3	5	0.8289	0.5579	0.2710
4	6	0.7579	0.6141	0.1450
5	7	0.7201	0.6723	0.0478
6	8	0.7038	0.6873	0.0166
7	9	0.6996	0.6987	0.0009
8	10	0.6977	0.6965	0.012
9	11	0.6965	0.6956	0.0009

At a constant time t=100 hrs, the value of upper bound of hypercube decreases exponentially with its dimension (n) and approaches to a constant value with dimension, n =9 or more (Refer Fig. 4). On the other hand, the value of the lower bound increases with dimension and attains its maximum with dimension, n=9, henceforth, it has a constant value.
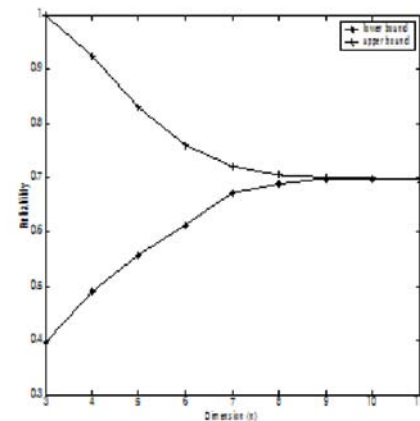

Fig. 4. Bounds on Residual Reliability of Hypercube (t=100 hrs)

The Upper bound and lower bound on Residual reliability of Alternating Group Graph (n=6) is plotted against the time under link failure rate λ=0.0001 and processor failure rate λ_p=0.001 in Fig. 5. Initially, with time t=100 hrs, the value

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of upper bound of AGG is observed to be 0.76. At the same mission time, the lower bound of AGG is found to be 0.45. But, the difference in their values gradually decreases with increase in time, and at mission time 800 hrs. both have found to be the same value.
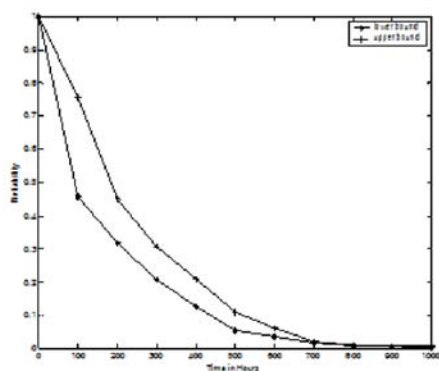

Fig. 5.   . Bounds on Residual Reliability of AGG (n=6)

Fig. 6 plots the bounds on Residual Reliability of Alternating group graph against its dimension (n) at constant time t=100 hrs. The upper bound starting with a value of 1 decreases exponentially with increase of its dimension and approaches to constant value after the size of AGG =9. But, the reverse happens with lower bound. It starts with a value of 0.09, increases and has its highest value at dimension n=9, after that, it has constant value irrespective of the size AGG.
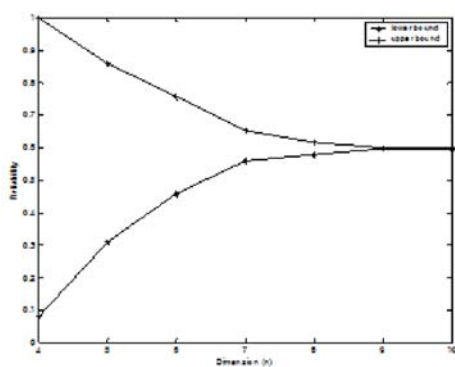

Fig. 6.   Bounds on Residual Reliability of AGG (t=100 hrs)

TABLE I DIFFERENCES (δR_r) BETWEEN BOUNDS ON RESIDUAL RELIABILITY OF HYPERCUBE

Sl. No.	n	Upper bound on R_r	Lower bound on R_r	δR_r
1	4	1.0000	0.0795	0.9204
2	5	0.8579	0.3091	0.5488
3	6	0.7561	0.4585	0.2976
4	7	0.6518	0.5578	0.0940
5	8	0.6169	0.5779	0.0390
6	9	0.5996	0.5958	0.0038
7	10	0.5995	0.5945	0.0050

The upper bound on residual reliability of Star graph (S) has a value of 0.7 at time 100 hrs under the link

failure rate λ=0.0001 and processor failure rate λ_p =0.001 (Refer Fig.7). Its value decreases exponentially with time and approaches to zero at time 700 hrs. Similarly, the value of lower bound on residual reliability of Star graph is observed to be 0.4 at time 100 hrs under the same environment. After that, it suffers a decrease in its value and has the same value as that of the upper bound at time 600 hrs.
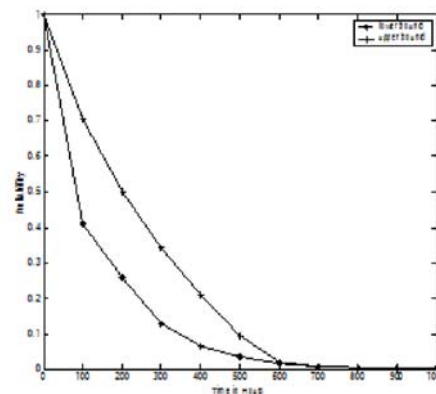

Fig. 7.   Bounds on Residual Reliability of Star Graph (n=6)

The bounds on residual reliability of star graph are plotted against its dimension (n) (Refer Fig. 8). The value of upper bound decreases while that of lower bound increases with increase in size of star graph. Both have the same value at dimension n=9.
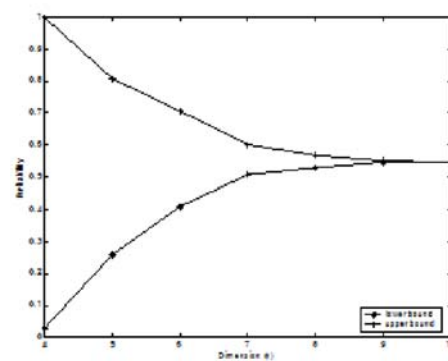

Fig. 8.   Bounds on Residual Reliability of Star graph (t=100 hrs)

TABLE II DIFFERENCES (δR_r) BETWEEN BOUNDS ON RESIDUAL RELIABILITY OF AGG

Sl. No.	n	Upper bound on R_r	Lower bound on R_r	δR_r
1	4	1.0000	0.0296	0.9705
2	5	0.8079	0.2591	0.5488
3	6	0.7061	0.4085	0.2976
4	7	0.6018	0.5078	0.0940
5	8	0.5669	0.5279	0.0390
6	9	0.5490	0.5458	0.0032
7	10	0.5447	0.5445	0.0003

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V. Conclusion

In this paper, a new method has been approached to provide the upper bound and the lower bound on residual reliability of parallel computer. Two polynomial algorithms are proposed for simulating the proposed method. The bounds on residual reliability of few important parallel computer interconnection systems have been bound by using the proposed method. From these simulated results, it can be concluded that the proposed method is a simple, general and efficient method for finding bounds on reliability of any type of interconnection networks. The work carried out in this paper can be further extended to find the bound on network reliability of parallel computer interconnection systems.

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